CONFERENCE ON TAX RESEARCH 1975

OFFICE OF TAX ANALYSIS • DEPARTMENT OF THE TREASURY • WASHINGTON, D.C. 20226

CONFERENCE ON TAX RESEARCH 1975.

Sponsored by The Office
of Tax Analysis
U.S. Department of the Treasury.
Washington D. C.
July 17 and 18, 1975

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price $3

Stock No. 048-000-00297-8

ACKNOWLEDGMENT

The Treasury Department has an established program which sponsors economic research on the tax system for use by the Office of Tax Analysis in policy-analysis work. While the studies commissioned by the Treasury have traditionally been employed primarily for internal purposes, George Tolley, while Director of the Office of Tax Analysis, recognized the value of making this research more readily available. The conference on tax research, July 17–18, 1975, was planned with this objective in mind. Most of the studies presented at the conference were undertaken under contract with the Treasury and they constitute a sample of modern tax analysis. The publication of this volume represents a further step in disseminating this research.

I wish to thank George Tolley for his efforts on the volume, which continued beyond his term of office. Special thanks are also due to Michael J. Kaufman of the Office of Tax Analysis, who bore the brunt of the job of organizing the practical details of the conference and this publication, and to Augustine Johnson, who assisted in the editorial production of the volume.

David F. Bradford

DAVID F. BRADFORD
Deputy Assistant Secretary (Tax Analysis)

FOREWORD

The studies in this volume contribute to an improved understanding of the economic effects of some of the more important and controversial features of our tax system.

During 1975, the economists represented in this volume served as consultants to the Office of Tax Analysis. On July 17–18, 1975, a conference on tax policy was held at the Treasury. It was designed to enable presentation of important results of the Treasury's consultants to an audience which included both policymakers and members of the academic community whose evaluations could raise the value of this work for policy purposes. The authors of the papers are distinguished researchers who have brought advanced theoretical and empirical analysis to bear on a number of key issues in tax policy.

William Fellner argues that the postponement analysis developed by the Austrian capital theorists gives insight in understanding the existing tax code and particularly current proposals to change it. Fellner demonstrates the applicability of this analysis to the issue of modifying the treatment of inventory profits in the tax system.

In his paper on the effects of taxation on charitable contributions, Martin Feldstein presents quantitative estimates of the income and substitution effects of taxes on charitable giving. His results make it possible to develop measures of the efficiency of the current system of allowing itemized deductions for charitable contributions. Generally Feldstein's results indicate that giving is substantially increased by the deduction. Feldstein is also able to use his analysis to examine the effects of replacing the current system of deductibility with a 30 percent tax credit. He finds that although such a policy would increase total charitable giving, gifts to educational institutions and hospitals would be cut roughly in half.

There has been much interest in the effects of budget deficits on the level of interest rates. This became a topic of debate in connection with the President's 1975 proposed anti-recessionary tax cut. Patric Hendershott has examined this question with his financial flow of funds model. The results indicate that rates of interest in particular sectors and crowding out in those sectors can be influenced by the response of the economy to the tax cut, the method of its finance, and the current position of the economy.

The effects of tax policy on investment in the oil and gas industries were studied by James Cox and Arthur Wright. Their paper reports on

the effects of tax and other public policies on investments to increase energy supply. Examples of policies analyzed include Federal taxation, market demand prorationing, and Federal price controls. The model should be useful in examining the extent to which tax policy can be used to achieve greater domestic energy production.

A number of proposals have been made to liberalize depreciation allowances. However, relatively little quantitative information is available on economic depreciation of structures of various types or on the correspondence of these magnitudes to the depreciation deductions now available for tax purposes. The paper by Frank Wykoff and Charles Hulten presents econometric work aimed at removing this lack of information. The report contains estimates of economic depreciation for two types of structures, office and medical buildings, using two sophisticated econometric approaches. The paper also points out the advantages and drawbacks of each of the techniques as a research device for this particular problem. When completed, this study can be expected to expand knowledge of the allocative effects of taxes on capital.

Recent increases in leasing have led to concern that the tax system is unduly influencing buying and leasing decisions. The effects of taxes on the lease-buy decision have been largely ignored in most studies of the cost of capital. Such studies have generally assumed that users of durable equipment would be indifferent between buying and leasing. Charles Upton, in his work with Merton Miller, suggests that economic analysis can provide some broad generalizations about lease or buy policy. The paper shows that under present tax laws, a taxable corporation is encouraged to rent rather than buy, while the reverse conclusion will hold for a tax-exempt institution.

Integration of the corporate and personal tax systems has been urged as a tax reform that would stimulate capital formation. Warren Weber and Nicolaus Tideman have analyzed the economic distortions caused by the corporate and personal taxes and have developed a framework of analysis that will allow a quantitative study of the effect of corporate integration on saving. By lowering the tax on capital, they argue, integration would increase the rate of return on saving. By reducing the incentive to use debt finance, the riskiness of the return on saving would be reduced. However, integration would also reduce the incentives of firms to retain earnings. Tideman and Weber argue that this effect of integration will cause firms to save less, but that aggregate savings should fall for this reason only if consumers fail to see through the corporate veil. The final effect of integration would come about because the marginal personal tax rates might have to be raised to avoid losing revenue. Weber and Tideman sketch out how a life cycle model of savings might be adapted to analyze these behavioral effects of tax integration.

In recent years, economists have found that investments in human

capital are extremely important in explaining and analyzing a wide variety of economic behavior. Michael Boskin argues that the effects of tax policy on various investments in human capital have received inadequate attention. He believes that if the current tax system does discourage such investments the consequences on productivity and growth are potentially enormous. Boskin points out some apparent asymmetries in the tax treatment of human capital and develops a framework of analysis for quantitative research on the incidence, efficiency, and growth aspects of taxing human capital.

The effects of Federal taxation on the distribution of wealth has long been a topic of major concern. James Smith's paper with Guy Orcutt and Stephen Franklin describes a quantitative analysis of intergenerational wealth transfers. Their simulations indicate that contrary to popular opinion the estate and gift taxes have not tended to increase the wealth equality. They examine the wealth distribution effects that would result from various tax reforms in this area. This model should also be useful for providing quantitative analysis in this area of tax policy.

Robert Spann presents an econometric analysis of a policy that would disallow the exclusion of interest income on State and local government debt. Most previous analyses of this issue have concentrated on the effects of this policy on State and local government activity. Spann describes a general equilibrium model that is capable of evaluating the policies' impact on an economy-wide basis. Spann's work indicates such a policy might significantly affect the composition of economic output. One major effect of the policy, according to Spann, would be a reduction in the marginal costs of Federal Government output and an increase in activity of this sector.

George S Tolley

GEORGE S. TOLLEY
Professor of Economics
University of Chicago

CONTENTS

Revisiting the "Austrian" Time Periods With Current Tax Policy in Mind

William Fellner

American Enterprise Institute
Yale University, Emeritus

Revisiting the "Austrian" Time Periods With Current Tax Policy in Mind

William Fellner

American Enterprise Institute
Yale University, Emeritus

I will suggest that the conception underlying the "Austrian" capital theory is unhelpful on the level of analysis at which it has been usually appraised but that an adapted version is very helpful in the appraisal of a problem involving uncertainty. The problem bears on the distinction between accrual and realization, and thus on a well-known issue of tax policy.

Simplified Presentation of "Austrian" Time Periods and Their Critique[1]

In the beginning of year I, L units of labor are employed during a "very short interval" which we shall not break up into even shorter ones. During this interval a specific output is produced that could be sold promptly for a *shade less than $4* as a final product[2], but in our illustrations that output will be used as an intermediate product. As a result of physical accretion or of quality improvement, and without the addition of further hired inputs, the value of that output will grow *to $5* by the beginning of year II *if* it is sold at that time as final product; yet the output can be allowed to grow further to a product worth a *shade*

[1] Readers wishing to recall the essentials of the controversy about the "Austrian" capital theory will derive much benefit from Parts I and II of Friedrich A. Lutz, *The Theory of Interest*, The Aldine Publishing Co.,Chicago, 1966. For the present purpose the author found it useful also to refresh his own memory on the debate (with details of which he felt more familiar some time ago) by consulting again E. V. Böhm-Bawerk's *Capital and Interest* (in particular Part I of Volume II in the German original, also Volume II of G. D. Huncke's English translation, the Libertarian Press, South Holland, Ill., 1959), along with passages bearing on this problem in Knut Wicksell's *Lectures on Political Economy* (English translation by E. Classen, London, 1936); also J. B. Clark, *The Distribution of Wealth* (The Kelley and Millman Edition, New York, 1956); some of the writings by F. H. Knight referred to on the opening page of Friedrich A. v. Hayek, "The Mythology of Capital," *Quarterly Journal of Economics*, February, 1936, reprinted in the American Economic Association's *Readings in the Theory of Income Distribution*, 1961; and Hayek's *Quarterly Journal of Economics* article itself. A few additional references follow later in this essay.

[2] We say "a shade less" to avoid indifference at a point of the analysis that will follow soon.

less than $6 by the beginning of the year III;[2] and it can be allowed to grow even further *to $6.60* by the beginning of the year IV.

We shall limit ourselves to comparing completion of the process in one year with its completion in three years, that is, sale of a finished product in the beginning of year II with sale in the beginning of year IV. Further, we shall assume that if one-year processes are adopted, these are repeated year after year, and if three-year processes are employed, a new such process is started in *each* successive year.

The one-year process implies valuing the final product at $5, and valuing the intermediate product that develops in the beginning of year I by a discounting operation "back from $5." At a compound interest rate of 22.3 percent, corresponding to 25 percent interest for a whole year, such discounting yields a value of $4 for the intermediate product when it comes into existence. Hence at this interest (r = 0.223) it pays to use the one-year process rather than to sell the initial output promptly. At a higher r it would not pay to employ the initial output of L units of labor as an intermediate product (capital) even in a process of merely one-year length, but we shall not consider a higher r.

The other value of r that we *shall* consider is 0.095, corresponding to 10 percent interest for a whole year. Whereas in the one-year process discounting at the lowered r would yield $4.54 for the intermediate product developing in the beginning of year I, the same r yields in the three-year process as much as $4.96 for that intermediate product.

The conclusion here is that, at r = 0.223, the one-year process will be adopted but that if additional savings reduce r to 0.095, then the three-year process will be adopted. Assuming that all nonlabor income other than pure interest becomes eliminated through competition, L units of labor will receive wages in the amount of $4.00 when the one-year process is chosen at 0.223, while the three-year process at r = 0.095 will bring a wage level of $496. This implies that through analogous adjustments to changes in interest rates, wages are changed all over the economy to the corresponding levels. In other words, a general equilibrium adjustment process has been "cut short" in this simplified exposition.

To avoid further discounting operations beyond the ending of these sequences, it is advisable to regard the final product as a *nondurable consumer good*. Our processes are described in terms reminiscent of the "Austrian" wine-maturing and tree-growing illustrations, though we have disregarded "land." With very little stretching, a story of this kind can be interpreted as relating to nearly-automatic machinery with service lives of alternative duration, scrapped just after the time when the equipment brings forth most of its output (and with practically no wear and tear meanwhile). The following Table 1 summarizes the properties of the processes we have described.

Even if we consider the "steady state" rather than "single process" versions of the Table, a finite postponement may be said to take place each year at the start of each new process, in the sense that the same

TABLE 1.—*Comparison of processes discussed*

	Single one-year process (For "Steady State" see Note)		Single three-year process				"Steady State": One three-year process started each year
	Year I[1]	Year II	Year I[1]	Year II	Year III	Year IV	
Value of intermediate product, beginning of year (discounted value of final product, based on $r = 0.223$ for one-year process and on $r = 0.095$ for three-year process).	4.00		4.96	5.46	6.00		16.42
Same midyear	4.47		5.20	5.73	6.29		17.22[2]
Final Product, beginning of year		5				6.60	6.60

[1] Year I is assumed to "begin" after the very short interval described in the text.

[2] The difference between this entry and that in the preceding line results merely from the simplifying assumption that the replacement of the Final Product with new inputs takes place once a year (in the beginning of the year). This explains also the quotes around the words Steady State in the title of the Column. There is no steadiness in this regard *during* the year.

NOTE.—A "Steady State," with the same *one-year process* repeating itself each year, is described the same way *except* that the entry for year II would appear in the same column as the other two entries, and all entries would describe any of the successive years equally well.

postponement is *repeating itself* as compared to more prompt sale of a final product. Whenever the number of processes in operation is increased, the observable characteristics of the single-process versions come to the fore more directly because then we have *additional* postponement.

Regardless of which version we consider, we conclude that the process adopted by producers when additional savings reduce r involves employing more capital per unit of labor. At a given level of technology it involves also employing more capital per unit of output. The increased capital intensity results *not merely* from deriving the value of the intermediate goods by discounting the final product at a lower r. There occurs also a "physical" increase in the sense that the process adopted at the lower r would be associated with holding more capital per unit of labor and of final product even if merely the physical accretion and/or quality improvement of the intermediate goods were taken into account. These are, of course, conclusions that have been derived also from theories making no use of the "Austrian" periods.

The concentration of the labor input into a "very short initial interval" eliminates from our particular sequences a number of complications of which account would have to be taken if the analysis were to focus on special aspects of capital theory. But these aspects will not concern us in this essay. The reswitching complications belong among those here deliberately disregarded because their significance —in contrast to their theoretical possibility—is very dubious.[3]

A simplification that has been more bothersome all along expresses itself in the implied nonexistence of initial capital inputs, that is, in the exclusive concern with inputs of "original factors" at the start of each process. It is an essential property of the real world that all processes start with capital inputs as well as others, and it is clearly unhelpful to

---

[3] The theoretical possibility of a reswitching complication can be illustrated in Austrian-type models as well as in others. To do so in a simple "Austrian" framework we should compare two processes, one of which requires significantly more natural input units late (rather than early) during the *first part* of the total time span we are considering, but not so in the *second part* of the total span, and the other of which requires significantly more natural input units late (rather than early) during the *second part* of the total span considered, but not so during the *first part* of the span. A simple illustration used by Samuelson describes the first of two such processes by the input sequence 0, 7, 0, and the second process by 2, 0, 6. Both processes result in the same final output.

Some economists have placed considerable emphasis on the difficulties that would develop if the effects of interest-rate reductions consisted of moving producers from one such process to another (from the first to the second process), with a tendency to be moved back by further rate-reductions. For instance, the processes specified in this footnote have switchpoints at $r = 0.693$ and at $r = 0.405$. There would be no assurance that the savings resulting in the interest-rate reductions would lead to additional output, nor could it be concluded that additional savings and interest-rate reductions lead to greater capital intensity in the "physical" sense described in the text for the processes of our Table. But no one has so far made a convincing case for interpreting observable phenomena in these terms. See Paul A. Samuelson: "A Summing Up," *Quarterly Journal of Economics*, November 1966; and his *Collected Scientific Papers*, vol. 3, edited by Robert C. Merton, pp. 230–245.

suggest that with reliance on additional "Austrian" sequences we should trace back, further and further, the capital goods from which the initial capital inputs derive. Once initial capital inputs are recognized, it becomes obvious that capital intensity can be increased not only by lengthening the kind of process we have spelled out but also by using at the start of such a process more capital per unit of labor, and also per unit of output, and conceivably in shorter processes. When r declines, the producer may decide to increase his reliance on mechanical devices of possibly small durability instead of selling "wine of older vintage," or he may decide to do both.

Yet even if we give up, as we should, on tracing back a producer's initial capital inputs to earlier processes, it is possible to argue that as long as the ratio of the capital stock to the output flow (consumption flow) is increased by *any* method, the effect of this change on the structure of production shows in the lengthening of a period of finite duration. The reason is that the mathematical dimension of a stock-flow ratio is *time*. The ratio of the capital stock to yearly consumption expresses the number of years of consumption (at the present level) whose equivalent is stored in the form of capital. This *is* a noteworthy formal characteristic of any economic system employing capital, even if it needs to be added that providing additional consumer goods for x years by gradually disinvesting the accumulated capital would bring changes in the valuation of the now existing capital goods throughout the x years.

The Basic Difficulty

However, what the present essay intends to make clear is that these arguments pro and con have not focused on the ultimate difficulty arising in connection with the pure capital theory sketched in our table. It is a crucial property of that analytical construction that all such processes involve a periodicity in the *completion* of processes, and also in the decision to *repeat them* in an economy in which on balance there is no net disinvestment. But in this regard something remains unexplained. To appreciate the importance of the unexplained element we must pay attention to the following.

The periodicities shown by the table are periodicities in *production plans*, not in consumption and saving plans nor, of course, in aggregate saving and consumption or in aggregate net investment. Individual savers may postpone their consumption by finite periods and then renew their postponement instead of using up their savings, *or* other savers may take their place at that time, *but there is no presumption that this would happen with the same periodicities as those underlying our Table and the production plans there sketched.* One may perhaps suspect that the life cycle hypothesis qualified this negative proposition to some extent, but in the next section it will be demonstrated that this is not the case. The saver either trades present consumption for a future

consumption flow over an indefinite span, or has his own periods of postponing consumption which are unrelated to the production periods we were considering. In particular, it would be a mistake to infer that given a three-year process of our Table at $r = 0.095$, rather than a one-year process at $r = 0.223$, the savers of year I are deprived of a consumption pay-off for two years longer. This conclusion would be wrong because, barring misjudgment on the part of the producers, the three-year process, implying $r = 0.095$, establishes itself as a result of an enlarged volume of voluntary savings for the entire 3-year period, including the second and the third year. There is no implication here of a longer waiting period for any of the year I savers who want to increase their consumption in year II.

Because the periodicities of our Table are periodicities in production decisions—and only in those—the question remains unanswered in what sense a producer has "completed" a process, as seen from his own point of view, when he sells finished consumer goods, and in what sense he is in a state of noncompletion when value-accruals develop *continuously*—with *no* "periodicity"—within his enterprise. After all, the producer is no ideologue deriving particular satisfaction from serving consumers rather than from experiencing value-accruals in other ways. The "Austrian" sequences imply a very sharp distinction between values realized through the sale of "finished products" (essentially of consumer goods) on the one hand, and continuous value accruals of "intermediate goods" on the other, but the protagonists of the theory have not made it clear that from the producer's viewpoint this distinction hinges on liquidity and hence on uncertainty, on marked imperfections and on credit-availability. Prominent "Austrians" (including Böhm-Bawerk and Hayek) have to be sure, expressed reluctance to go along with critics who had suggested the complete elimination of uncertainty from the kind of pure theory about which the neoclassical debate seemed to be conducted, but this is a far cry from clarifying the question with which the present essay is concerned. Neither individual savers nor savers in the aggregate complete and renew anything in the 1-year or the 3-year interval of our table, and what the *producer* does in fact complete and renew is a process defined as ending with "realization," pretty much in the particular sense in which the tax codes interpret that term.

If, say, 3 years after having started a process the producer arrives at the stage of "realization" by selling consumer goods, then this "realization" happens to coincide with the arrival of consumer goods in the market but this is not what interests the producer, and as concerns the saver it does not happen at the end of any 3-year postponement on his part. If, on the other hand, we include into our sequence producers of capital goods—which never came very easily in the "Austrian" illustrations—and if we need to piece together 1½ years for each of two successive producers before we get to the consumer, each of these producers will have arrived at "realization" after his 1½ years.

Producers will not be aware of the 3-year periodicity which the "Austrian" theory reads into such a course of events, and the irrelevance of either a 1½ or a 3-year periodicity for saving behavior will follow from the same reasoning as was presented above. In such cases the analysis of realization periods would have to focus on the shorter of the two spans, but we shall avoid this complication by considering processes ending with the sale of consumer goods, as the basic "Austrian" illustrations mostly do.

Accommodating the concept of "realization" in a framework conventionally regarded as one of "pure theory" would have required being quite explicit about how this is done. What we are saying amounts to a negative appraisal of the significance of the Austrian time-sequences on the level of discourse on which they have traditionally been appraised, with a hint (to be followed up presently) that an adapted or "transposed" version of the theory does possess significance.

Does the Life-Cycle Hypothesis Qualify the Conclusion?

The life-cycle hypothesis establishes a link between the saver's period of postponing consumption and the period of postponement expressed by the economy's ratio of stock to consumption-flow. The link between the postponement of consumption and the stock-to-flow ratio is especially close if we simplify the Modigliani-Brumberg analysis further than is usual, and we shall presently do so.[4] Were the particular stock-to-flow ratio, which is thereby related to the postponement of consumption, identical with the ratio of the *capital* stock to the consumption flow, then this would call for modifying our negative conclusion that saving—consumption periodicities are unrelated to the consumption-storing periods expressed by the technological ratio of capital to consumption. But the stock to be considered in the framework of the life-cycle hypothesis is *not* the capital stock used in productive processes.

A population acting according to the life-cycle hypothesis postpones its consumption from its working period to its retirement period. On the appropriate simplifying assumptions, the extent of this postponement determines not only the working population's average propensity to consume but also the ratio of the entire population's aggregate accumulated savings to the consumption flow. Consider a steady state, and place the average member of the working population at the age at

---

[4] Indeed, we shall simplify the analysis in a rather extreme fashion. For the original, see Franco Modigliani and Richard Brumberg, "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in *Post Keynesian Economics*, edited by Kenneth K. Kurihara, Rutgers University Press, New Brunswick, New Jersey, 1954, pp. 388–436. References to further contributions to the subject by Modigliani and Ando are found in Tibor Scitovsky, *Money and the Balance of Payments*, Rand McNally, Chicago, 1969, p. 19. For Scitovsky's own analysis of the demand for assets along similar lines, see *ibid.*, pp. 16–23.

which it has accumulated one-half of the stock of savings (claims) it will ever accumulate. Place the retired population at the age at which it has dissaved one-half of the savings it had ever acquired. Then, if all individuals will dissave all they had saved during their working years, it is easy to construct a model that yields a very simple relationship between the finite postponement of consumption by individuals and the ratio of the stock of aggregate savings to consumption.[5]

However, while the assumptions underlying such an oversimplified illustration can be relaxed to some extent without loss of the link between the stock of savings and the consumption flow, we cannot substitute the capital stock for the "stock of savings" in this context. What, for example, our Social Security system gives the taxed participants for their "forced savings" is mainly *not* a claim on the yield

---

[5] We assume that y is the yearly income earned by all members of the population during m years of their earning span and that each person consumes during this interval cy per year. They all will consume cy a year also in their n years of retirement, and will do so by dissaving their accumulated savings which at the beginning of their retirement period amount to $m\ (1-c)\ y$ per person. This implies disregarding interest. For each person thus postponing the consumption of his savings to his n years of retirement, we obtain during his life-time: $m(1-c)y = ncy$, hence

$$\frac{1-c}{c} = \frac{n}{m} \tag{1}$$

We consider a steady state with no further increase in the stock of savings, with a constant population and with no uncertainty. The active (income-earning) part of the population has on the average just saved one-half of what it will have saved by the time it retires, and the retired part of the population has on the average just dissaved one-half of what it will have dissaved at the end of their lives (i.e., the "representative" retired person has still $(n/2)$ cy left for consumption).

Using the subscripts a and r for denoting active and retired, we conclude that the aggregate *accumulated* savings of the active population—i.e., of the fraction $(m/m+n)$ of the total population P—are at any time:

$$S_a = \frac{m}{m+n} P \frac{m}{2} (1-c)\, y, \tag{2}$$

and that the aggregate *accumulated* savings of the retired population are at any time:

$$S_r = \frac{n}{m+n} P \frac{n}{2} cy. \tag{3}$$

Considering that the aggregate yearly consumption of the active population is:

$$C_a = \frac{m}{m+n} Pcy, \tag{4}$$

and that of the retired population is:

$$C_r = \frac{n}{m+n} Pcy, \tag{5}$$

the (S/C) ratio is the same for the two parts of the population. In view of (1) this ratio is:

$$\frac{S_a}{C_a} = \frac{S_r}{C_r} = \frac{n}{2} \tag{6}$$

of existing capital but a claim on tax revenue to be collected from the next generation. Disregarding the redistributive features of the system, the Social Security tax on employees is essentially a legally enforced consumption loan going from workers to the retired, and to be rapaid to the presently working population when *it* retires by those who will still be working. Thus, at least in the United States, the link between the postponement of consumption from working years to retirement on the one hand, and a stock-flow ratio on the other, is largely a link involving a *stock of consumption loans* rather than a stock of *capital.* With significantly different sociological implications (and with some differences in terminology) a very similar statement applies also to societies in which the young may be depended upon to take care of the old in their own families.

What we have described here as a stock of consumer loans nevertheless enters into the private "wealth" of those whom the legal system forces to make these loans—or who in other societies make them by tradition—but that wealth is simultaneously dissaved by the recipients who dissave also the revenue from the social security tax levied on nonbeneficiaries. Given the definitions used in national income accounting, the dissavings of the recipients do not show up in our society as an excess of their consumption over their "disposable income," nor do tax revenues representing forced savings enter into disposable income. But the fact remains that the forced savings of the social-security contributors are offset by the consumption of beneficiaries who dissave in the fundamental sense of consuming without producing. There is no capital formation involved in this accrual of private wealth to the social-security contributors. The contributors' wealth increases by the claims they acquire on income which will be produced subsequently and which they will consume at that time.[6]

While the relationship between the life-cycle hypothesis and the "Austrian" time-sequences is worth pondering, we can hardly be said to have found justification for emphasis on the length of a technologically definable period in the explanation of the type of investment which would correspond to the willingness of savers to postpone consumption by specific periods.

Relevance of an Adapted Form of the Theory

Nevertheless, if we compare the shorter with the longer process of our Table (p. 5), we must conclude that the producer using the longer process is postponing a reasonably well-defined result of his efforts by a finite period. This postponement has had a bearing on policy, particu-

---

[6] Martin Feldstein has made important contributions to the analysis of this problem. See his Social Security, Induced Retirement and Aggregate Capital Accumulation in *Journal of Political Economy*, Sept./Oct. 1974, and his Toward a Reform of Social Security, *The Public Interest* Summer issue, 1975.

larly on fiscal policy. To what extent policy *should be* influenced by the considerations to which we now turn has remained controversial. Yet policymakers have in fact been concerned with the consequences of this postponement, and in struggling their way through the problem's complexities they have adopted various compromises.

As was said above, the postponement is that of "realization," a concept the significance of which has to do with uncertainty and with the difference between well-established (near-perfect) markets and highly imperfect ones.

What in our Table is described as the final product is usually sold in regular markets with which the producer is familiar, and it is sold after the technological outcome of the process under consideration has become known. On the other hand, a stock of goods the producer has started using or processing can usually not be sold in markets that would be free from very serious imperfections. Usually the buyer of such goods needs to form a rather shaky probability judgment about the quality of what he is acquiring and about the trustworthiness of the seller in general. Moreover, in those illustrations of the "Austrian" sequences in which the intermediate product could be described physically as a "finished product of lesser age" (such as wine of very recent vintage), it remains true that, once the costs already incurred imply a commitment to a longer process, promptly liquidating the stock for immediate consumption entails losses; and liquidation of the stock as one of intermediate products to be further processed by others shares even in these cases many of the essential disadvantages encountered by the sellers of equipment in second-hand markets. All this is known to the investor and also influences his potential creditors.

In a single long process of our Table the producer arrives at the stage of "realization" after 3 years, while in the short process he reaches that stage in one year. What is probably more revealing is, however, the fact that in the last column of the Table the units tied up all the time in unrealized and not easily realizable form bear a much higher ratio to each year's realization than is the case in the first and the second column. This fact does indeed lend itself to being expressed as "having to wait longer" for all inputs to become realized output, if future conditions should make it desirable partially or wholly to liquidate.

The conventional way of expressing this difference stresses the concept of liquidity which is dealt with mostly in the theory of money and credit. This is appropriate, provided we do not play down the significance of that concept by implying that "fundamentals" can be explored without regard to it. Behind the surface of the concept of liquidity there is the interaction of uncertainty with market imperfections. A theory of investment without regard to uncertainty cannot penetrate very far.

Policymakers have been influenced by these considerations. In general the fiscal authorities are reluctant to tax "unrealized" gains, and, consistent with this reluctance, they do not make unrealized losses

tax-deductible. This practice implies disregarding the kind of gain which in the longer process of our Table expresses itself in the successive value-accruals of intermediate products. In the illustration we were using these value-accruals as "interest," but this does not affect the analysis of tax implications because to the extent that the process is self-financed, interest on the capital invested is taxed as profit. Note that in the foregoing illustrations all value-accruals result from physical accretion or quality-improvement, not from "rising prices."

Assuming *full self-financing*, the point here is that in the single three-year process a taxable profit will show only in the beginning of year IV, and the amount of the profit will be the excess of the $6.60's worth of final output over the $4.96's worth of wages paid to labor in the "very short initial interval." The resulting taxable profits thus are $1.64. Note that if unrealized profits were taxable, $0.50 would have developed in the beginning of year II, a further taxable profit of $0.54 in the beginning of year II, and $0.60 in the beginning of year IV. The discounted value of the tax liability would have been larger—the difference being about 9 percent in this particular illustration given the here implied compound rate of 9.5 percent.

In the "steady state" described in the last column of our Table, taxable profits are the same *each year*, amounting to $1.64 (the difference between $6.60 and the $4.96 paid for the initial input). After three calendar years—from the time when the producer already has all three processes going but at different stages—the tax liability ceases to be affected by the limitation of taxation to the final stage, that is by nontaxation of the value-accruals at the intermediate stages. But the difference between the two conceptions of taxation nevertheless remains significant for producers making net additions to the processes they are already operating. This is a difference resulting from lags in "completing" additional processes and it is a difference requiring no further clarification here.

The main and relatively new problem which calls for further analysis is the problem developing in an environment of *rising prices*. Turning to this problem, we need no longer refer to lags in completing additional production processes. We may look at the continuous flow described in the last column of our Table, implying now that the sector with which we are concerned has already arrived into a "steady state" with rising prices. To construct a "strong case" we shall assume that all market prices and money wage rates in the economy under consideration are rising at the rate of 100 percent from year to year.[7] On this assumption a problem of considerable interest develops as to the *line of demarcation* between "realized" and "unrealized." In the case of a uniform rate of price increase and full self-financing, one way of drawing the line is

---

[7] Not to complicate the matter we continue to explore it at a given level of technological knowledge.

clearly the "right" way because that way of drawing it leads at the same time to the proper separation of "real" gains from merely nominal (inflationary) value-increments.

We must remember that for producers who have started a single process in each successive year, and are now in the position described in the Table's last column, the data were derived from the figures relating to single three-year processes. It will be assumed that the price increases started three years ago. The price of a physical unit of the final output was $1 at that time, $2 two years ago, $4 last year, and it is $8 "now." Thus, the final output is sold now at $52.80 (eight-times the $6.60 shown in the Table), and this sale takes place simultaneously with the employment of L units of labor for feeding a raw product worth $39.68 into the pipelines (eight times $4.96). The difference between $52.80 and $39.68 is $13.12, eight times the $1.64 profit that on the price-stability assumption of the Table resulted from setting the input-purchase at $4.96 against the final sale valued at $6.60.

Even given the decision to exempt unrealized gains from taxation, the question needs to be answered, whether the realized (and thus taxable) gains of the year will in fact be interpreted as amounting to the $13.12 derived above or to as much as $47.84. They amount to $13.12 if we deduct from the $52.80 final sales the $39.68 *now spent on the initial input;* on the other hand the taxable gains amount to $47.84 if from the $52.80 we deduct merely the $4.96 *spent three years ago on the input from which the now maturing final product results.*

Explaining the difference with reference to the procedural rules of tax codes, we may say that the amount of $13.12 is obtained by the last-in, first-out (LIFO) method of valuation, while the amount of $47.84 is obtained by the first-in, first-out (FIFO) method.[8] Yet we have here also an essential difference between alternative ways of interpreting the concept of "realization" in these two cases. The LIFO tax base of $13.12 implies that replacing a component of the capital stock initially valued at $4.96 with a physically identical component now valued at $39.68 involves *no realization* of gains. On the other hand, the conception underlying the FIFO tax base of $47.84 is that when replacement takes place there is *realization through sale coupled with reinvestment, even though the reinvestment represents merely the maintenance of a physically unchanging stock.*

Can economic theory be brought to bear on the question: Which of these ways of looking at the matter is correct? The answer, I suggest, is clearly in the affirmative when we consider price increases that reflect general inflation. The amount of $13.12 does in this case provide the "right" answer in the sense that it permits the tax-free recovery of the real value of the firm's physical assets. To put it differently, current

---

[8] The terminology used in the text applies literally only to inventory valuation, but historical-cost depreciation of fixed capital is essentially FIFO, and replacement-cost depreciation is essentially LIFO.

profits are at the LIFO level of $13.12 if the cost of the input that entered the pipelines three years ago is computed by discounting the current sales of $52.80 at the same interest-rate as prevailed prior to the inflation (9.5 percent)[9]; current profits are at the FIFO level of $47.84 if, to obtain the cost of the same input, the final sales of $52.80 are discounted over three years at a rate that includes, in addition to the pre-inflation rate, full allowance for the 100 percent year-to-year inflation (i.e., $r = 0.095 + 0.693$).[10] Discounting by the second of these two methods would mean including in the tax base the inflationary revaluation of the stock which represents a nominal value-accrual accompanying the mere recovery of used-up real capital. It is reasonable to call that "wrong procedure," so much so that in this case *we do not even have to postulate that the tax-exemption of the revaluation gain results from interpreting the accrual as unrealized. It is enough to recognize that the revaluation "gain" is merely a reflection of inflation.*

However, if instead of reflecting general inflation the price increase of our business sector's final product reflects wholly or partly changes in the *relative* price structure, then there is less reason for becoming strongly assertive about what the "right" decision is. In this case the "real" value of the stock is rising. The decision on whether $13.12 or $47.84 are taxable[11] depends on how in our appraisal—and in the presumptive appraisal of creditors—the "realization" difficulties compare with each other in the following two cases, respectively: (a) liquidation "if need be" of the firm's capital stock through direct sale of the goods of which that stock is made up; (b) gradual liquidation "if need be" through nonreplacement of the capital when successive components of the stock are used up in the production of final output. Fiscal authorities generally take account of the difficulties connected with (a), and, therefore, value-accruals—the realization of which would create these difficulties—are considered nontaxable. Whether the difficulties connected with (b)—or with borrowing for their avoidance—are essentially identical is a subtler question.

The LIFO rule, resulting in our case in $13.12 taxable profits, is based on the judgment that (a) and (b) describe similar difficulties, and that these difficulties overlap in the sense that (b) might well lead over into (a). This is an argument deserving serious consideration even when the

---

[9] In this case the resulting cost of the input is that obtained by the FIFO method, that is, $39.68 or eight times the pre-inflation cost of $4.96 (aside from rounding errors in the various discounting operations).

[10] In this case the resulting cost of the input is the pre-inflation amount of $4.96 (aside from rounding errors in the various discounting operations).

[11] In this simplified illustration of specific price increases, using the $13.12 figure (eight times $1.64) for the increased profit would imply that the specific wage rates in this sector have also risen relative to wages in general. But all that matters is that the cost of acquisition of the input should have risen eight-fold, regardless of whether that input consists of labor employed in the enterprise in question or is acquired from other enterprises.

capital revaluation gains are "real" rather than merely "nominal" (inflationary). Yet conclusions based on this argument are somewhat less definitive—or less unqualified—than is the argument that in the event of a *general* price increase (with no relative increase of the capital-goods prices in the process we are considering) the \$13.12 LIFO profits of a self-financed enterprise should be taxed and the remainder of the \$47.84 should be regarded as a *merely nominal* nontaxable value-accrual. If the latter argument, relating to general inflation, is subject to any qualification whatever, it is merely that in the United States and in many other countries inflationary distortions of the tax structure have been tolerated in the area of personal income taxation. However, it would be clearly more desirable to eliminate these distortions than to approve of business-tax distortions. On the other hand, in the case of specific price increases in the sector we are considering, the problem changes to one of deciding whether turnover should be regarded as realization of a "real" gain even if a nonshrinking enterprise needs to engage in replacement at the same prices at which it is selling.

The American statutes relating to our problem date back to a "relatively noninflationary era." As long as the prices implied in our Table remain constant at \$1 for the final output, limiting taxation to realized gains involves taxing \$1.64—the difference between \$6.60 and \$4.96—this being the practice that was actually followed in such cases. In a noninflationary era the dilemma to tax the \$13.12 LIFO profits or, alternatively, the \$47.84 FIFO profits could arise only in the event of specific price increases in the processes under consideration. For that event the statutes give the enterprise an option to use the FIFO or the LIFO method as concern *inventories* but they hold the enterprise unconditionally to the equivalent of FIFO as concerns the write-off of *fixed capital*, that is, they hold them to "historical-cost depreciation." The enterprise must, of course, adhere consistently to the method chosen. If for inventories it opts for LIFO—which most enterprises have not done, though recently there has been a shift in that direction—then they must treat the corresponding inventory losses as nondeductible. These tax provisions reflect a compromise between interpreting sale-plus-replacement as having involved "realization" and interpreting that joint act as resulting in "unrealized" gains or losses.

It is understandable that in a relatively noninflationary period this kind of compromise should have been sought by policy makers. There is growing awareness now, however, that a pronounced upward trend of the price level calls for modifications. Along the lines developed in the present essay, the conclusion suggests itself that the LIFO rule would clearly be appropriate to inventory valuation if all investment were self-financed and if all increases of the goods used in production reflected the general inflation rate. Furthermore, in this case the analogous rule for depreciation—i.e., the rule of replacement-cost rather than historical cost depreciation—would be generally appropriate to the computation of deductible capital-consumption allowances.

We have already commented on the fact that the nature of the problem changes in some respects if we are faced with specific price changes in a sector rather than with a uniform rate of price increase. The complexities introduced into the problem by debt-financing will not be discussed here in any detail.[12] But a word should be added about the reason why these complexities develop. The reason is that exempting debt-financed revaluation gains because of their merely nominal (inflationary) character has clear implications for a carry-over of the exemption to creditors, while exempting debt-financed revaluation gains with the justification that the debtor has not realized them has no similarly clear implications in that regard. Behind this ambiguity there is the fact that the distinction between "realized" and "unrealized" leads us into analytical constructs that stress the role of uncertainty, and in such constructs the simple profit-maximization propositions need to be modified. One kind of expected profit may not be equivalent to another kind, even if the "expected values" are the same. This can be expressed by stressing the difference between the maximization of expected utility and that of expected monetary return, though not all economists concerned with the role of uncertainty find the assumption of utility maximization the most convenient alternative to that of profit maximization.

Conclusions

Once uncertainty and market imperfections are recognized, the distinction between "realized" and "unrealized" value-accruals acquires considerable significance, and an exploration of that difference leads into the analysis of postponement periods analogous to those of the "Austrians." It would be difficult to make sense of existing tax statutes, and of the arguments used in the policymaking process for modifying them, without recourse to an adapted version of "Austrian"-type time-sequences. At present this deserves attention particularly because of the desirability to adjust the tax system to inflationary trends.

On the other hand, without regard to the "realization" problem—thus without regard to uncertainty, market imperfections, and credit availability—the "Austrian" periods were not found revealing, not even when account was taken of the life-cycle hypothesis. The statement that, when using more capital-intensive processes at lower interest rates, producers are postponing the attainment of objectives in order to attain them more fully implies a distinction between "realization" and "accrual." The specific periodicities that can thus be read into the behavior of producers are not periodicities in saving-consumption behavior.

---

[12] On this see, W. Fellner, K. W. Clarkson, and John H. Moore, *Correcting Taxes For Inflation*, American Enterprise Institute, 1975.

The Income Tax and Charitable Contributions
Part I—Aggregate and Distributional Effects

Part II—The Impact on Religious, Educational and Other Organizations

Martin Feldstein

Harvard University

The Income Tax and Charitable Contributions Part I—Aggregate and Distributional Effects

Martin Feldstein*

Harvard University

(Reprinted by permission of National Tax Journal, March 1975, Vol. XXVIII, No. 1 and June 1975, Volume XXVIII. No. 2)

"If charity cost nothing, the world would be full of philanthropists."

—quoted in Leo Rosten's Treasury of Jewish Quotations

Abstract

Because charitable contributions are deductible in defining taxable income, the "price" of such gifts is less than the price of other consumption. This paper assesses the importance of this price effect by using a pooled time series of cross sections of charitable contributions by income class for the period 1948 through 1968 to estimate price and income elasticities. Alternative estimates of the price elasticity are generally greater than one and they cluster around 1.1. These results indicate that charitable contributions are increased substantially by the current provision of deductibility.

Private nonprofit organizations play a central role in the provision of a wide variety of public services. Higher education, research, health care, the visual and performing arts, welfare services, and community activities rely heavily on voluntary institutions. In 1972, American families contributed $17 billion to support these philanthropic and religious organizations.[1] The volume and distribution of these contributions is affected by the personal income tax and by the special provisions with respect to the deduction of charitable contributions.

---

* Professor of Economics, Harvard University. I am grateful to Charles Clotfelter and Daniel Frisch for assistance with this research and to W. Andrews, M. Bailey, J. Brittain, R. Freeman, R. Musgrave, J. Pechman, J. Schwartz, H. Smith, S. Surrey, and W. Vickrey for useful discussions and comments on a previous draft. This paper is part of a larger study of the effects of fiscal policies on capital formation and income distribution. This paper was previously published in the National Tax Journal, 1975.

[1] American Association of Fund-Raising Counsel (1973). Philanthropic organizations also received $2.7 billion from bequests, $0.8 billion from corporations and $2.2 billion from foundations.

The current paper provides new estimates of the effects of the income tax provisions on individual philanthropy.[2]

The income tax affects charitable contributions in two important ways. First, by decreasing disposable income the tax reduces all forms of philanthropy. Since effective average rates are higher for upper income families, the reduction in disposable income falls more heavily on education, health, the arts and other nonreligious charities.[3] Second, because contributions are deductible in determining taxable income, the tax makes the "price" of charitable contributions less than the price of other goods and services. More specifically, an individual with a marginal tax rate of 40 percent can give $100 to charity by forgoing $60 of personal consumption; for him the net price of charitable contributions is only 0.6.[4] In 1970, approximately 90 percent of individual contributions were itemized as tax return deductions; these contributions had an average net price of less than 0.74.[5] This price effect increases charitable contributions. Moreover, since marginal rates are higher in upper income groups, the induced increase in giving favors the same charities that lose most by the reduction of disposable income. The net impact of the tax on the total amount and distribution of contributions depends on the relative magnitudes of the income and price effects.

There are today a number of widely discussed proposals for changing the tax treatment of charitable contributions. These include the complete abolition of the deduction, the substitution of a system of tax credits, the introduction of a "floor" with a deduction or credit only for contributions above that level, and various modifications of the treatment of appreciated assets.[6] The issues raised by these proposals are complex and wide ranging. They involve the appropriate definition of income, problems of horizontal and vertical equity, the desirability of

[2] Earlier studies of this subject were reported by Kahn (1960), Schwartz (1970), Taussig (1967), and Vickrey (1962); see section 5 below.

[3] The most recent information on the distribution of contributions among types of charities in each income class is the Internal Revenue Service analysis of 1962 tax returns (Internal Revenue Service, 1965). In 1962, religious organizations received 61.0 percent of total itemized contributions but only 31.3 percent of the contributions of individuals with adjusted gross income over $25,000 and only 19.6 percent of the individuals with adjusted gross income over $50,000. (Internal Revenue Service, 1962, p. 6).

[4] The implied price is lower and more complicated to compute when the contribution includes a gift of appreciated property; this is considered in sections 1 and 3 below.

[5] Total individual giving in 1970 was $14.4 billion (American Association of Fund-Raising Counsel, 1973) while itemized deductions for contributions were $12.9 billion (Internal Revenue Service, 1972). The average net price was calculated by applying the marginal tax rate for joint returns to the contributions in each taxable income class. Since gifts of appreciated assets and state income taxes are ignored, this overstates the average net price of charitable contributions.

[6] See, for example, the discussions in Brannon (1973), Goode (1964), Kahn (1960), McDaniel (1972a, 1972b), Pechman (1971), Rabin (1966), Surrey (1972), Weidenbaum (1973) and U.S. Treasury Department (1969). These proposals were considered in the 1969 Hearings of the House Ways and Means Committee and of the Senate Finance Committee, and in the 1973 Hearings of the House Ways and Means Committee.

decentralized finance of public and quasi-public services, and the effects of the tax provisions on the level of contributions.[7] The current paper will not attempt to deal with this full range of analytic and philosophical questions. The focus is rather on the empirical issue of the income and price effects of the tax structure. With estimates of these effects it will be possible to evaluate the "efficiency" of the current tax treatment as a stimulus to charitable deductions, i.e., the amount of additional contributions received by charities per dollar of potential tax revenue forgone by the Treasury.[8] The price and income elasticities can also be used to assess the potential impact of any proposed tax change. Section 4 presents estimates of the effect that abolishing the charitable deduction would have on the distribution of charitable contributions, of tax payments and of net disposable income (income net of tax and charitable contributions).

The results presented in this paper indicate that charitable contributions are increased substantially by the current provision of deductibility. The alternative estimates of the price elasticity are generally greater than one and cluster around 1.1. This implies that the "efficiency" of the deduction as a stimulant to giving exceeds 100 percent; the deduction increases the amount received by charities by more than it reduces the revenue collected by the Treasury. These results stand in sharp contrast to Taussig's (1967) widely cited conclusion that the price effect is very small, that charities receive only five cents for each dollar of revenue forgone by the Treasury. They are closer to the estimates presented by Schwartz (1970) but indicate somewhat greater sensitivity to the deduction at all income levels. Possible reasons for these differences are discussed in section 5.

Since the present study is based on a richer sample of the same type of data used by Taussig and Schwartz, I believe that the current results should be given more weight in evaluating the evidence. Moreover, since this study was completed, Charles Clotfelter, Amy Taylor and I have used a variety of other microeconomic data sources to estimate the

---

[7] For thoughtful discussions of these issues, see the references cited in the previous footnote and papers by Andrews (1972), Bittker (1972), Vickrey (1962, 1973) and White (1959). None of these authors gives attention to the question of whether the charitable deduction is justified as a method of offsetting the income effect of the tax on charitable contributions. It is interesting in this context that the income tax law was amended to allow the charitable deduction in 1917 when tax rates were sharply increased to finance the war; the introduction of the deduction was intended to prevent the higher tax rates from substantially reducing philanthropy.

[8] This measure of the "efficiency" of the current tax rules has been central to much of the previous analysis. Taussig's (1967) widely cited study concluded that the "efficiency" was very low, approximately 5 percent. Several writers have argued that such low efficiency in stimulating contributions is a sufficient reason to abolish the current deduction or to modify it very substantially; see, e.g., McDaniel (1972a), Taussig (1967) and Surrey (1972). In contrast, others have argued that the efficiency is irrelevant because the charitable deduction should not be regarded as a "tax subsidy" but as a necessary correction in the calculation of an appropriate taxable income; see Andrews (1972) and Bittker (1972).

basic price and income elasticities of charitable giving. The results, presented in Feldstein and Clotfelter (1974) and Feldstein and Taylor (1975), are remarkably similar to those described in the current paper.

There are a number of problems that cannot be investigated adequately with the data used in this or previous studies. These limitations are discussed in section 5. Most of these shortcomings can be overcome with the microeconomic data that I have studied with Clotfelter and Taylor. It is reassuring that explicitly incorporating such things as the individual's wealth or demographic characteristics does not alter any of the conclusions of the current study.

1. Data and Specifications

Every second year the Internal Revenue Service publishes the value of itemized charitable contributions in each adjusted gross income (AGI) class.[9] The current study uses a time series of these cross-sections for the even years from 1948 through 1968. With 17 AGI classes,[10] the sample has 187 potential aggregate observations. By pooling data in this way it is possible to obtain substantial variation in real income and in the price of charitable contributions without the collinearity between these variables that exists within a single year.

It is inevitable in empirical research that the available data does not correspond exactly to the relevant theoretical quantities. Fortunately, the current data provides some scope for testing the sensitivity of the results to alternative measures of particular variables. When this is possible, the different estimates generally support the same conclusions. The substantial variation in prices and incomes imply that any bias that might be introduced by certain stochastic measurement problems (e.g., errors or transitory components in measured income) will be small. There are however other potentially serious problems, e.g., the lack of data on wealth and the aggregation of charitable contributions to all donees, that cannot be remedied until new sources of data are examined.

A variety of functional specifications relating charitable giving (G) to income (Y) and price (P) have been investigated. The most basic specification is the constant elasticity equation:

$$\log G_{it} = \alpha + \beta \log Y_{it} + \gamma \log P_{it} + \epsilon_{it}. \qquad (1)$$

The subscript i denotes the AGI class and the subscript t denotes the year. The variable ϵ_{it} is an unobservable residual that reflects random disturbances and specification errors. The more general specifications described below allow the income and price elasticities to vary with the levels of income and price.

[9] See, for example, Internal Revenue Service (1968), p. 65.

[10] The AGI class limits are $1000; $2000; $3000; $4000; $5000; $6000; $7000; $8000; $9000; $10,000; $15,000; $20,000; $50,000; $100,000; $500,000; $1,000,000; $1,000,000+.

The variable G_{it} is the average charitable contribution per return in AGI class i and year t. The contribution is defined as the *gross* amount given by the individual to the charity and not as the *net* cost of that contribution to the individual. These amounts include the value of donated assets as well as gifts of money. Contributions are measured in constant 1967 dollars by deflating with the consumer price index. Of course, only those taxpayers with itemized returns are included in the sample.[11]

An ideal measure of economic income cannot be obtained from the data provided in the tax return. Nontaxable income, accrued capital gains, and accounting losses make the reported values different from the appropriate theoretical variable. Two alternative definitions of disposable income have been used in this study: (1) adjusted gross income minus the tax that would have been paid if no contributions had been made, and (2) taxable income plus charitable contributions minus the tax that would have been paid if no contributions had been made.[12] The value of Y_{it} is the average real income per return in AGI class i and year t, measured in constant 1967 dollars. In some of the equations reported in section 3, this real income variable is supplemented or replaced by a measure of relative income; the specific definition of relative income will be described at that point. Analyzing data that is grouped by income class reduces the potential bias that arises from using current income instead of permanent income. If the income groups correctly classify individuals by permanent income, the parameter estimates are consistent even if individual current incomes differ from permanent income.[13] More generally, the very great variance in permanent incomes in the population of taxpayers relative to the average transitory variance implies that the bias from this source would be quite small.

The price variable (P) measures the individual's opportunity cost per dollar of charitable contribution in terms of forgone personal consumption or saving. An individual whose marginal tax rate is m can choose between (1) contributing one dollar to charity and (2) having $1 - m$ dollars for additional personal consumption or saving. We therefore define that individual's price of charitable giving by $P = 1 - m$. In

---

[11] In 1970, 90 percent of all individual contributions were deducted on itemized returns; see footnote 5 above. While only 47.7 percent of all taxpayers itemized their deductions, 91.4 percent of taxpayers with AGI over $15,000 itemized their deductions.

[12] Subtracting the tax that would have been paid if no contributions had been made is preferable to subtracting actual taxes paid because the latter depends on the contributions themselves. The results presented in an earlier version of this paper (Harvard Institute of Economic Research Discussion Paper No. 337, January 1974) were based on adjusted gross income minus tax actually paid.

[13] It is well known that the use of current income instead of permanent income is an example of the classical errors in variables problem. This use of grouped data is a generalization of Wald's (1940) method of instrumental variable estimation.

practice, P_{it} is measured by using the marginal tax rate for a joint return with the average taxable income in class i and year t.[14]

Contributions of appreciated assets create a special problem for measuring the price of charitable giving. When an asset is given away, its full value can be deducted from the donor's taxable income but there is no constructive realization and therefore no tax to be paid by the donor on the capital gain.[15] The opportunity cost (price) of a gift that is given in the form of an appreciated asset therefore depends not only on the individual's marginal tax rate but also on the fraction of the asset's value that is accrued capital gain and on the alternative disposition of the asset. An example will clarify the way in which these variables determine the relevant price. Consider an individual whose marginal rate is 40 percent and who contemplates donating an asset that is now worth \$100 and for which he originally paid \$30. If he gives the asset away he reduces his taxable income by \$100; he therefore reduces his tax liability by \$40 and thus increases his after tax income by \$40. If he instead sells the asset, he pays a tax of \$14 (half of his marginal rate on the capital gain of \$70) and increases his after tax income by \$86. For this individual, the opportunity cost of the \$100 contribution is therefore \$46 of foregone consumption. If the price is defined in terms of forgone consumption, the price of the gift is $P = 0.46$. This price clearly depends on the ratio of the asset's original cost (or basis) to its current value: an original cost of \$1 implies $P = 0.40$ while an original cost of \$100 implies $P = 0.60$. More generally, $P = 1 - mc(1 - B/A) - m$ where A is the current value of the asset, B is its basis or original cost, m is the marginal tax rate on income and mc is the marginal tax rate on capital gains; during the sample period, $mc = 0.5m$ with a maximum of 0.25.

The preceding calculation defined the opportunity cost of a donated asset in terms of forgone immediate consumption, i.e., it assumed that if the asset were not given away it would be sold in the current year. The price is higher and the calculation is more complex if the opportunity cost is defined in terms of forgone saving or wealth, i.e., if it is assumed that the asset would not otherwise be sold in the current year. The

[14] The marginal rate is actually calculated for taxable income plus charitable contributions, i.e., it is the marginal rate for the first dollar of contribution. With the current aggregate data, the choice between the first dollar price and the last dollar price has little effect. When appropriate, the marginal rate is modified for the existence of a tax surcharge. To allow for the effect of using the alternative tax computation, average taxable income in class i and year t is adjusted by subtracting one-half of the net capital gains reported on returns using the alternative tax. No attempt is made to allow for income averaging. A more exact method of evaluating P_{it} would be to (1) cross-clarify returns in each AGI class according to taxable income class; (2) find the marginal tax rate at the average taxable income in each subclass; and (3) find the weighted average of these for the AGI class using the distribution of total taxable income among the taxable income subclasses. This calculation was performed for 1968, the only year for which such data are available. Fortunately, the correlation between these P_{it}'s and the more easily calculated P_{it}'s described in the text is very high: $r = 0.99$.

[15] Since income of the donee organization is not taxable, it can sell the appreciated asset without paying any tax.

individual in the preceding example could retain the $100 asset or he could give it away and add the $40 tax saving to his wealth. Viewed in this way, his opportunity cost price is 0.60, the same as for contributions of money; moreover, this price is independent of the ratio of the capital gain to the present asset value. Since the individual who does not give away the asset also has a future tax liability, this tends to overstate the opportunity cost of a prospective contribution. However, by postponing the sale of the asset the individual can substantially lower the present value of the tax and, if the asset is never sold during the individual's lifetime, the capital gains tax liability is completely eliminated when the asset passes at death.[16]

It has not been possible to reflect the full complexity of appreciated asset gifts in the current study. Although the fraction of total contributions in the form of assets is known for each income class, there is no reliable data on the ratio of original cost to current value for such assets.[17] There is of course no information on what would have been done with the assets if they had not been contributed. In practice, I have used the information about the share of contributions in the form of appreciated assets and examined the implications of different assumptions about the ratio of basis to current value. These results are reported in section 3.

Before 1952, the deduction of charitable contributions was limited to no more than 15 percent of the taxpayer's adjusted gross income. An individual who contributed more than 15 percent of his income would face a price of one for marginal giving.[18] The limit was increased to 20 percent in 1952 and then to 30 percent in 1954. Since a significant number of high income taxpayers had previously been contributing at the maximum rate, these increases constituted reductions in their price of charitable contributions. The effective magnitude of these reductions depends on the number of taxpayers at each income level who had previously given the maximum and on the extent to which the effect of the limit was reduced by the carryover provision. The impact of these limits is examined in section 3.

Table 1 presents the values of G_{it}, Y_{it} and P_{it} for each AGI class for

---

[16] If the individual gives the asset away to another person, there is no constructive realization and the tax is postponed until the recipient sells the asset. The original owner can also consume most of the value of the asset by using it as collateral to borrow funds which he then consumes, thus enjoying the consumption while postponing or avoiding the capital gains tax. See Bailey (1969) for evidence that a very large share of accrued capital gains are never subject to capital gains taxation.

[17] The Treasury published "estimates" of the ratio of cost to current value for charitable contributions deducted in tax returns for 1962. (Internal Revenue Service, 1962, p. 8). These "estimates" imply that most assets are worth exactly their original cost. It is clear that this data is without value. I inquired directly at the Treasury and was advised that these "estimates" were meaningless and should be disregarded.

[18] The special provision for individuals whose contributions plus taxes exceeded 90 percent of their taxable income in eight out of the last ten years affected very few individuals and does not alter the basic point of this paragraph.

TABLE 1.—*Charitable contributions by income class, 1968*

AGI class ($1000)	Average contribution (G)	Average income* (Y)	Average price** (P)	Contribution ratio (G/Y)	Cumulative percentage of contribution
0–1	$90	$724	0.86	0.124	0.1
1–2	109	1,570	.86	.069	.8
2–3	145	2,439	.85	.059	2.3
3–4	164	3,329	.84	.049	4.6
4–5	178	4,216	.83	.042	7.5
5–6	183	5,507	.83	.033	11.2
6–7	207	5,968	.82	.035	15.6
7–8	220	6,825	.82	.032	20.4
8–9	232	7,694	.80	.030	26.0
9–10	258	8,533	.80	.030	31.7
10–15	305	10,710	.80	.028	55.6
15–20	428	14,542	.76	.029	67.7
20–50	761	22,541	.66	.033	82.9
50–100	2,267	45,745	.43	.050	88.9
100–500	9,695	96,689	.31	.100	95.6
500–1000	68,749	366,594	.25	.188	97.2
1000+	287,651	1,111,360	.25	.259	100.0

* Income is adjusted gross income minus tax paid.
** Price is based on gifts of money; $P = 1 - m$.
All amounts in 1968 dollars.

1968, the most recent year in the sample.[19] The income variable is adjusted gross income minus taxes. The price variable is based on gifts of money. For each income class, the table also shows the ratio of contributions to net income after tax and the cumulative proportion of total contributions.

Preliminary analysis indicated that the information in the current data is not sufficient for studying the behavior of taxpayers in the lowest and highest income groups. Low income individuals who file itemized returns are an unusual group with a disproportionately high fraction of aged persons and those with substantial negative transitory income. At the other extreme, adjusted gross income is an inadequate measure of economic income and no information is available about wealth. Moreover, the special features of private foundations and charitable trust make it extremely difficult to measure price for the highest income groups. The analysis of this paper focuses on AGI classes with mean real net income between $4000 and $100,000.[20] Table 1 shows that in 1968 this group accounted for 91 percent of all itemized contributions. Although the parameter estimates for this group are very similar to the results obtained when all 187 observations are used, restricting the sample provides more reliable estimates. Additional information on contributions of nonitemizers and on the income and assets of the wealthy is required to extend the current analysis to cover all individuals in a satisfactory way.

[19] Although data for 1970 is now available, a variety of changes in the tax treatment of charitable contributions in the Tax Reform Act of 1969 suggests that it would be unwise to pool 1970 with previous years without additional study.

[20] More specifically, an observation is included in the sample if the mean of AGI minus tax in 1967 dollars in that class and year is between $4000 and $100,000. This reduces the sample from 187 potential observations to 117 observations.

Each of the observations represents a different number of individual tax returns. However, the published values of total contributions and incomes are themselves estimates prepared by the Internal Revenue Service on the basis of a very large stratified sample of returns. The number of returns in each AGI class is selected to yield approximately the same sampling error in the resulting estimates. This suggests that relatively little gain in the efficiency of the parameter estimates could be obtained by using a weighted generalized least squares estimator.[21] The procedure of giving equal weight to all of the observations is therefore used in this study.

2. The Basic Estimates

For the estimates of this section, income (Y) is defined as the average real value per return of adjusted gross income minus taxes. The price of giving (P) is the opportunity cost of contributions of money, one minus the marginal rate of tax. Equation 2 presents the estimated equation with constant income and price elasticities:[22]

$$\ln G_{it} = -1.922 + \underset{(0.032)}{0.822} \ln Y_{it} - \underset{(0.101)}{1.238} \ln P_{it}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \begin{aligned}\bar{R}^2 &= 0.98\\ SSR &= 1.772\\ N &= 117\end{aligned} \qquad (2)$$

The income elasticity is 0.822 and the price elasticity is -1.238. The equation provides a very good explanation of the overall variation in the volume of contributions ($\bar{R}^2 = 0.98$). Despite the potential problem of collinearity between income and price, the standard errors of the estimated elasticities are quite small.

Several modifications of this basic specification are presented below. In general, these have elasticities of approximately the same size as equation 2. Before studying the additional estimates, it is therefore useful to consider the implications of these elasticity values. Since a full analysis is presented in section 4, only some individual examples are now examined. In 1968, taxpayers with adjusted gross income between \$10,000 and \$15,000 contributed an average of \$305.[23] The average

[21] The weighting would be complicated not only by the IRS sampling procedure but also by the fact that a log-linear specification is used. Only for returns with incomes below \$6000 did the relative error of the estimate of giving exceed 4 percent; above \$10,000 the relative error was less than 1 percent. See Internal Revenue Service, 1968, pp. 65 and 189.

[22] An earlier version of this paper (Harvard Institute of Economic Research Discussion Paper No. 337, January 1974) reported a price elasticity of -1.18 with P_{it} defined in terms of actual taxable income and T_{it} defined as AGI minus actual tax. The income elasticity was very similar (0.828). The sum of squared residuals was lower (1.730) but this reflects the spurious simultaneity of giving and the explanatory variables.

[23] These amounts are all in 1968 dollars.

marginal rate for these taxpayers was 0.20, implying an average price of 0.80. If contributions were not deductible, the price would rise by 25 percent (from 0.80 to 1.00) and, therefore, given a price elasticity of −1.24, contributions would fall by about 24 percent or $74.[24] The amount is not implausible nor contrary to the common assertion that the deductibility of contributions is likely to have only a "small" effect on the amount given by lower income households.[25]

For taxpayers with adjusted gross incomes between $50,000 and $100,000, the average contribution was $2,267 and the average price of giving was 0.43. Most of the difference in average contributions between the $10,000 and $15,000 class and the $50,000 to $100,000 class is obviously due to the difference in income rather than the difference in price; lowering the price from 0.80 to 0.43 for the $10,000 to $15,000 AGI class would only raise their average giving to $659 per taxpayer. The low average price in the $50,000 to $100,000 class implies that the deductibility of charitable contributions has a substantially greater effect than in the lower AGI class. Eliminating the deductibility of contributions would raise the price by 133 percent (from 0.43 to 1.00) and would therefore lower contributions by about 65 percent or $1,473.[26]

During the 20-year sample period, there have been a great many gradual changes in economic and social factors that may influence the rate of charitable giving. The rise in college attendance, the increase in government activities in areas previously dominated by philanthropic organizations, the changing role of religion and the growth of the suburbs are all likely to have different and countervailing impacts. To test whether these trends had any *net* effect on giving or on the previously estimated elasticities, an exponential time trend is added to the specification of equation 2:

$$\ln G_{it} = \underset{}{-1.649} + \underset{(0.023)}{0.806}\ \ln Y_{it} \underset{(0.071)}{-1.272}\ \ln P_{it} + \underset{(0.001)}{-0.014}\ \text{TIME} \tag{3}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \begin{aligned}\bar{R}^2 &= 0.99\\ SSR &= 0.88\\ N &= 117\end{aligned}$$

The coefficient of the time variable implies a moderate negative trend

[24] More exactly, $(1.25)^{-1.24} = 0.76$ implying that contributions are decreased by 24 percent or $73.72. These calculations assume that an additional small change is made in tax rates to leave total taxes paid (and therefore net income) unchanged.

[25] This point has been stressed by Aaron (1972), Kahn (1960), McDaniel (1972a) and Vickrey (1962) among others. In 1968, 55 percent of the total itemized deduction for charitable gifts was on returns with AGI below $15,000 and 31 percent on returns with AGI below $10,000. Although the implied effect on the average individual gift is small, the aggregate effect is substantial. I return to this in section 4.

[26] The price increases imply $(2.33)^{-1.24} = 0.35$ or a 65 percent decrease in charitable giving.

in relative contributions; the income and price elasticities are essentially unchanged from equation 2.[27]

Although constant income and price elasticities are convenient simplifications, the log-linear form is an unnecessary restriction on the analysis. As a more general specification, the price elasticity is allowed to vary linearly with the level of the price and the income elasticity is allowed to vary linearly with the logarithm of the level of income. The estimated equation

$$\ln G_{it} = \underset{(0.702)}{3.647 + (-0.404} + \underset{(0.039)}{0.069} \ln Y_{it}) \ln Y_{it} - (\underset{(0.220)}{0.981} + \underset{(0.578)}{0.545}\ P_{it}) \ln P_{it}, \tag{4}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \bar{R}^2 = 0.98 \quad SSR = 1.709 \quad N = 117$$

shows that the income elasticity increases with the level of income but that the variation in the price elasticity is not significantly different from zero.[28] If the income elasticity is allowed to vary but a constant price elasticity is assumed, the estimated price elasticity is -0.910 (S.E., 0.207), slightly lower than the result in the basic specification of equation 2. But such differences must be regarded with great caution. It is always difficult to assess second order properties with any precision. It is therefore interesting to note that two quite different specifications with varying income and price elasticities also support the basic result of equation 2.

The first alternative method of generalizing the constant price elasticity specification is to reestimate the basic equation with different price elasticities in different parts of the price range. For this purpose, the observations are grouped into those for which price exceeds 0.70, those for which price is between 0.30 and 0.70, and those for which price is less than 0.30. Estimating an equation with three price elasticities is equivalent to estimating three separate equations for the three groups of observations while constraining the income coefficients and constant term to be the same; i.e., three separate price variables appear in the equation but only one is non-zero for each observation. The estimates in

[27] This may partly reflect the fact that the *relative* income of this group is declining slightly with time; when the entire sample is used, the coefficient of TIME is much smaller, positive and insignificant.

[28] The *logarithm* of the level of income is used so that the variable is not dominated by the top income classes. However, very similar results are obtained when the income elasticity is allowed to vary linearly with income and the price elasticity with price. The income elasticity is an increasing function while the variation in the price elasticity is not significant. There is no statistical basis for choosing between the equations; SSR = 1.707.

equation 5 indicate a slightly lower price elasticity for the high of the price range (low income individuals) and a slightly higher price elasticity for the high end of the price range,

$$\ln G_{it} = \underset{(0.731)}{6.752 + (-1.121} + \underset{(0.041)}{0.109} \cdot \ln Y_{it}) \ln Y_{it}$$

$$- \underset{(0.206)}{0.865} \ln PL3_{it} - \underset{(0.217)}{0.775} \ln P37_{it} - \underset{(0.268)}{1.173} \ln PG7_{it} \tag{5}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \bar{R}^2 = 0.98, \quad SSR = 1.616, \quad N = 117$$

where ln $PL3$ is the logarithm of the price if the price is less than or equal to 0.30 but is zero otherwise; similarly, ln $PL37$ refers to the price if it is between 0.30 and 0.70 while l*n* $PG7$ is the logarithm of the price when greater than 0.70. The differences, however, are small and not significantly different from each other. The large standard errors emphasize the difficulty of assessing variations in price elasticity with this data but again show that allowing for the possibility of such variation provides no indication that the simpler specification distorts the price elasticity.

The second alternative generalization is to reestimate the basic equation separately in several income classes without any constraints on the coefficients. The limits of the income classes were defined by mean real adjusted gross income. Equation 6 reports the result with adjusted gross incomes of less than $10,000:[29]

$$\ln G_{it} = -0.803 + \underset{(0.060)}{0.679} \ln Y_{it} - \underset{(0.564)}{1.796} \ln P_{it}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$10{,}000 \qquad \bar{R}^2 = 0.75, \quad SSR = 0.774, \quad N = 64 \tag{6}$$

The income elasticity is below the overall value and the price elasticity is above the overall value. But the relatively large standard errors show the difficulty of estimating when the variation in income and price is substantially limited. Among taxpayers with real incomes between $10,000 and $20,000, the price and income elasticities are very similar to the basic equation:

$$\ln G_{it} = -2.053 + \underset{(0.225)}{0.846} \ln Y_{it} - \underset{(0.757)}{1.035} \ln P_{it}$$

$$\$10{,}000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$20{,}000 \qquad \bar{R}^2 = 0.66, \quad SSR = 0.514, \quad N = 27 \tag{7}$$

[29] More specifically, an income class observation is included in this subsample if the real value in 1967 dollars of the mean AGI minus tax in the class is below $10,000.

Because of the limited range of variation and the very small number of observations, the standard errors are again quite large. It is reassuring therefore that very similar results are obtained for the next income class, from \$20,000 to \$100,000:

$$\ln G_{it} = \underset{}{-2.734} + \underset{(0.169)}{0.906} \ln Y_{it} - \underset{(0.250)}{1.132} \ln P_{it} \tag{8}$$

$$\$20{,}000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \begin{aligned}\bar{R}^2 &= 0.97\\ SSR &= 0.355\\ N &= 26\end{aligned}$$

In spite of the small number of observations, there is sufficient independent variation in both income and price to permit estimates with relatively small standard errors. Comparing the SSR value of equation 2 with the sum of the SSR values for equations 6, 7 and 8 shows that the disaggregation does not significantly increase explanatory power; the SSR is reduced by only 0.129 and the corresponding F statistic of 1.5 is not significantly different from zero.

Only in the highest income group (taxpayers with net income above \$100,000) is the price elasticity substantially lower than the basic estimate:

$$\ln G_{it} = -6.772 + \underset{(0.063)}{1.377} \ln Y_{it} - \underset{(0.106)}{0.290} \ln P_{it} \tag{9}$$

$$\begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} > \$100{,}000 \qquad \begin{aligned}\bar{R}^2 &= 0.97\\ SSR &= 1.622\\ N &= 31\end{aligned}$$

This low price elasticity is very suprising in view of the widely held opinion that the high income taxpayers are likely to be most sensitive to changes in the price of charitable giving. It is clear that this low estimate of the price elasticity is associated with an estimated income elasticity that is higher than the value obtained in other equations. For taxpayers with incomes over \$100,000, the ratio of contributions to income increases rapidly as income rises and price falls; equation 9 attributes this increase primarily to the higher income rather than to the lower price. The standard error of the income elasticity in equation 9 is quite small and the standard error of the price elasticity, although large relative to the coefficient, is small enough to imply that the estimated price elasticity is very much less than the average price elasticity of equation 2. However, these formal sampling properties of the parameter estimates are misleading; problems of measurement and specification are more important potential sources of error in this equation than the random sampling variability. At these very high income levels, adjusted gross income is a less adequate measure of economic income and wealth is a more important influence on giving. The measurement of price is also more clouded by the tax treatment of gifts of appreciated assets, by the limits on deductible contributions,

and by the use of trusts and other indirect methods of giving. The next section deals briefly with some of these problems but the issues cannot be fully resolved with the current data. It is for this reason that the current study has been restricted to the sample of observations under $100,000.[30]

If these difficulties are ignored and all of the 187 possible observations are used, the resulting estimates are quite similar to the basic results of equation 2:

$$\ln G_{it} = -\ 1.784 + \underset{(0.027)}{0.811} \ln Y_{it} - \underset{(0.077)}{1.455} \ln P_{it}$$

$$\text{All observations} \qquad \bar{R}^2 = 0.98 \quad SSR = 16.19 \quad N = 187 \tag{10}$$

At the present, however, it is best to remain agnostic about the income and price elasticities of individuals with incomes over $100,000 and under $4000.[31]

Each of the equations of this section has been reestimated with the alternative definition of disposable income: taxable income plus charitable contributions minus the tax that would have been paid if no contribution were made. In each equation the estimated income elasticity is lower and the price elasticity is greater than in the corresponding equation with income measured by AGI minus tax. Comparing the sums of squared residuals for the corresponding equations shows that the AGI variable (Y) explains the variation in giving substantially better than the taxable income variable (YT). For example, equation 11 should be compared with equation 2 in which the estimated price elasticity is -1.24 and the sum of squared residuals is only 1.772.

---

[30] After this study was complete, I was able to use the Treasury Tax Files for 1962 and 1970 to calculate the average of the individual prices in each AGI class rather than the price for the average taxable income in that class. The values agree quite closely below $100,000 but are substantially higher above $500,000. This biases down the estimated price elasticity of equation 9.

[31] After this paper was accepted for publication, Joe Pechman and John Brittain suggested adding the term $\ln Y \cdot \ln P$ to the basic equation as a further test of the varying price elasticity. This variable is significant and implies that the price elasticity is an increasing function of income; the specific point estimates imply a *positive* price elasticity for income below $8,300, a price elasticity of -0.98 at $50,000 and a price elasticity of -1.36 at $100,000. Although a specification that implies a positive price elasticity is clearly unacceptable, the evidence does strongly suggest that the absolute price elasticity increases with income. Some preliminary analysis with a rich body of microeconomic data (the 1970 Treasury Tax file) supports this conclusion and indicates that the price elasticity is relatively constant and below one for low and moderate incomes but then rises rapidly with income. These results will be discussed in detail in Feldstein and Taylor (1974).

$$\ln G_{it} = 1.69 + \underset{(0.031)}{0.445} \ln YT_{it} - \underset{(0.128)}{2.044} \ln P_{it}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \bar{R}^2 = 0.95 \quad SSR = 4.354 \quad N = 117 \tag{11}$$

Although an after tax measure of income seems more appropriate, as a further test of the robustness of the estimated price elasticity the basic specification was reestimated using real AGI (not net of tax) to measure income. The parameter estimates are similar to the original specification but the estimates of equation 2 are preferable because net AGI is a theoretically better measure of income:

$$\ln G_{it} = -1.617 + \underset{(0.030)}{0.787} \ln Y_{it} - \underset{(0.112)}{0.903} \ln P_{it}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \bar{R}^2 = 0.98 \quad SSR = 1.772 \quad N = 117 \tag{12}$$

These alternative estimates lend some weak support to the relative high price elasticities reported in equations 1 through 9. They also suggest the possibility of substantial bias from using an inappropriate measure of income. If a broader definition of income than AGI is the true determinant of charitable giving, the use of AGI might bias the estimated price elasticity. To evaluate the likelihood that this would cause an upward bias in the absolute price elasticity, it is useful to examine the way in which the bias occurs. Let the true specification be given by:

$$\ln G = \alpha + \beta \ln I + \gamma \ln P + \epsilon \tag{13}$$

where I is the "true" measure of income. Consider the effect of using adjusted gross income (y) as the measure of income and estimating

$$\ln G = \alpha + \beta \ln y + \gamma \ln P + u. \tag{14}$$

The residual u in equation 12 is equivalent to $\epsilon + \beta \ln I - \beta \ln y = \epsilon + \beta \ln (I/y)$. From the usual formula for the analysis of specification bias (Theil, 1966), it follows that the expected value of the estimate of γ in equation 12 would be:

$$E(\hat{\gamma}) = \gamma + \beta E[\text{reg}\,(\ln (I/Y), \ln P |\ \ln y)\,] \tag{15}$$

where reg $(\ln (I/y), \ln P|\ \ln y)$ is the coefficient of $\ln P$ in the regression of $\ln (I/y)$ on $\ln P$ and $\ln y$. If this auxiliary regression coefficient is negative, the expected value of γ will be less than the true value γ, i.e., the absolute value of the price elasticity will be biased upwards. The auxiliary regression coefficient will be negative if at each level of

adjusted gross income (y), those taxpayers with higher marginal tax rates (i.e., lower value of P) have higher ratios of "true" income to adjusted gross income.

It is not clear whether this is more likely than the opposite. There are two countervailing effects. First, at each level of adjusted gross income, those with the highest marginal tax rates have the greatest incentive to reduce their taxable income through such things as the holding of tax exempt bonds, home ownership and the substitution of accrued capital gains for realized income. All of these things would increase the ratio of total economic income to AGI. Such a positive association between marginal tax and the ratio of "true" income to AGI would cause an upward bias in the absolute value of the estimated price elasticity. Against this reason for an upward bias one must balance a reason for a downward bias. It follows from the definitions of AGI and taxable income that, at each level of AGI, those with the highest marginal tax rates have the least deductions for interest, taxes and charitable contributions. These smaller deductions are likely to indicate smaller amounts of "other income" not included in AGI: imputed income on residences and accrued gains on assets used to secure loans. This would imply a negative correlation at each level of AGI between the marginal tax rate and the ratio of true income to AGI. This in turn would imply that the absolute price elasticities of this section are actually biased downwards rather than upwards. Unfortunately, only when estimates have been made with more comprehensive data will it be possible to know whether the use of AGI imparts any substantial bias.

3. Additional Specifications

This section presents several alternative modifications of the basic model. The use of relative income instead of real absolute income is examined first. The implication of the special tax treatment of gifts of appreciated assets is then studied. Finally, the effects of the limits on deductible gifts are examined.

Relative Income. Charitable contributions support activities that produce positive externalities. A philanthropic activity generally benefits not only those who are the direct recipients of its service but also those who, like the individual donor, believe that the service should be provided. Thus, an alumnus who contributes to his college's scholarship fund benefits not only the scholarship student but also the other alumni who enjoy seeing their college support students in this way. In deciding how much to contribute, an alumnus may consider how his own income compares with the other alumni who are also potential contributors and "indirect beneficiaries." Similarly, a member of a church congregation may apply a relative "ability to pay" criterion in deciding what he believes to be his "fair share" of his church's expenses. Such considerations suggest that some measure of relative income should be added to

the basic specification examined above.[32] An extreme form of this hypothesis would use relative income instead of real absolute income.

The examples of college and church donations indicate the difficulty of developing an appropriate measure of relative income. Moreover, the options are severely limited by the aggregate form of the current data. Only the most obvious possibility has been examined in this study: the ratio of donor's income (AGI minus tax) to average per capita income for that year. This is denoted YR.

Equation 16 shows that when the relative income variable is added to the basic specification, its coefficient is highly significant but the price elasticity is essentially unchanged. The result is similar

$$\ln G_{it} = \underset{}{2.882} + \underset{(0.064)}{0.199} \ln Y_{it} - \underset{(0.072)}{1.255} \ln P_{it} + \underset{(0.059)}{0.613} \ln YR_{it}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net Income}\end{matrix} < \$100{,}000 \qquad \begin{aligned} \bar{R}^2 &= 0.99 \\ SSR &= 0.904 \\ N &= 117 \end{aligned} \tag{16}$$

if YR is added to the specification with varying price and income elasticities.

The more extreme assumption that contributions depend only on relative income and price does not explain the variation in contributions as well as the basic model. Equation 17 shows that substituting YR for Y slightly increases the price elasticity and reduces the sum of squared residuals from 1.772 to 0.980.

$$\ln G_{it} = 4.428 + \underset{(0.022)}{0.784} \ln YR_{it} - \underset{(0.071)}{1.329} \ln P_{it} \qquad \begin{aligned} \bar{R}^2 &= 0.99 \\ SSR &= 0.980 \\ N &= 117 \end{aligned} \tag{17}$$

Appreciated Assets. The special problems raised by gifts of appreciated assets have already been discussed. Gifts of appreciated property lower the effective price of giving. Since such gifts are more common in higher income classes,[33] the basic price series used above does not decrease rapidly enough as marginal tax rates increase. The result is likely to be an overestimate of the absolute price elasticity.

The available data severely limits the possibility of dealing adequately with this problem. There is information on the value of contributions in each AGI class that are in the form of assets but no information on the original basis of those assets or the fraction of those assets that would have been sold if they had not been given away. Separate calculations have been made using different assumptions

[32] Note that this reason for including relative income is quite different from Schwartz (1970) emphasis on the relative incomes of donors and recipients.

[33] In 1966 the fraction of contributions in the form of assets rose from 3.8 percent for adjusted gross incomes between $10,000 and $15,000 to 47 percent for adjusted gross incomes over $100,000.

about the ratio of appreciation to asset value. In each calculation, the ratio of appreciation to value is assumed to be the same for all taxpayers. It is further assumed that all assets that are donated would otherwise be sold, an assumption that biases downward the price associated with each ratio of appreciation to value. The resulting estimates must therefore be regarded as a very imperfect attempt to deal with gifts of appreciated assets.

Equation 18 shows the result of assuming that 50 percent of the value of donated assets is the original basis while the remaining 50 percent is appreciation. The estimated price elasticity (-1.11) is only slightly smaller than in the basic equation while the estimated income elasticity is unchanged.[34] Comparing the sum of squared

$$\ln G_{it} = -\ 1.934 + \underset{(0.031)}{0.825} \ln Y_{it} - \underset{(0.094)}{1.166} \ln P50_{it}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \begin{aligned}\bar{R}^2 &= 0.98\\ SSR &= 1.754\\ N &= 117\end{aligned} \tag{18}$$

residuals with that for the original specification (1.772) suggests that the current assumption is barely preferable.[35] In interpreting these results, the statement that "an average of X percent of the value of donated assets is appreciation" should be interpreted a shorthand for the more correct statement that "taxpayers respond to both the actual appreciation ratio and the opportunities to postpone realization by acting as if the assets had to be realized immediately if not donated but that the appreciation ratio is only X percent." This implies that the ratio of appreciation to value implied by the estimate will be appropriately lower than the actual (unknown) appreciation ratio of donated assets.

Alternative assumptions about the ratio of appreciation to value have only very slight effects on the estimated elasticity and the sum of squared residuals. If the ratio of appreciation to value is 0.25, the price elasticity is -1.202 and the SSR is 1.762. With an appreciation ratio of 0.75, the price elasticity is -1.128 and the SSR is 1.749. It is clear that there is too little information in the data to estimate the appreciation ratio. Fortunately, the choice of appreciation ratio does not affect the estimated price elasticity.

Deduction Limits. Raising the limit on the maximum charitable deductions increased the amount of giving by high income taxpayers.

[34] The variable $P50_{it}$ is defined as the weighted average of $(1 - m_{it})$ and $1 - m_{it} - .50mc_{it}$ where m_{it} is the marginal rate on income and mc_{it} is the marginal tax rate on capital gains; the weights are the fractions of donations in money and in assets in income class i.

[35] Comparing the sums of squared residuals is equivalent to a likelihood criterion in the context of the current specification. The assumed ratio of appreciation to asset value with the lowest sum of squared residuals yields the maximum likelihood estimator of that ratio and of the other regression parameters.

The ceiling was raised from 15 percent of adjusted gross income to 20 percent in 1952 and then to 30 percent in 1954. Internal Revenue Service data show that the early limits were reached by a significant fraction of taxpayers with adjusted gross incomes over $50,000 but by almost no taxpayers with lower incomes (Kahn, 1960, p. 79). A natural way to express the effect of these changes in deduction limits is as proportional reductions in contributions by high income taxpayers in the years before 1954. In equation 19, the variable DL1 is equal to 1 for 1948 and 1950 in income brackets over $50,000 and equal to zero otherwise; DL2 is 1 for 1952 in those income brackets and zero otherwise.[36] The coefficients of these dummy variables are estimates of the proportional reductions in giving due to the limits in those years and should therefore be negative.[37]

The estimates of equation 19 imply that the limits on deductions before 1954 reduced total contributions in the specified income

$$\begin{aligned} \ln G_{it} = &- 1.857 + \underset{(0.034)}{0.812} \ln Y_{it} - \underset{(0.123)}{1.332} \ln P_{it} \\ &- \underset{(0.093)}{0.163}\, DL1 - \underset{(0.114)}{0.145}\, DL2 \end{aligned} \tag{19}$$

$$\$4000 < \begin{matrix}\text{Mean Real}\\ \text{Net AGI}\end{matrix} < \$100{,}000 \qquad \begin{aligned} \bar{R}^2 &= 0.98 \\ SSR &= 1.711 \\ N &= 117 \end{aligned}$$

groups. The income and price elasticities are essentially unchanged from equation 2.

Because the sample is restricted to observations with mean real net AGI below $100,000, equation 19 does not provide any estimate of the overall effect of the deduction limit on all high income donors. Equation 20 uses the full sample of 187 observations to obtain some very tentative values of this effect for the three high AGI groups:

$$\begin{aligned} \ln G_{it} = &-1.731 + \underset{(0.027)}{0.803} \ln Y_{it} - \underset{(0.078)}{1.533} \ln P_{it} \\ &- \underset{(0.111)}{0.176}\, DL1 - \underset{(0.158)}{0.511}\, DL2 \end{aligned} \tag{20}$$

$$\text{All Observations} \qquad \begin{aligned} \bar{R}^2 &= 0.98 \\ SSR &= 15.19 \\ N &= 187 \end{aligned}$$

4. Aggregate and Distributional Effects

The current parameter estimates are clearly preliminary and may be subject to serious error. Some possible sources of bias are discussed in

---

[36] Here the income bracket is defined by the *current* dollar AGI *before* tax.

[37] They will, of course, also reflect other specific factors that caused the behavior of those years to depart from the remainder of the period.

the next section. It is nevertheless interesting to examine what these estimates imply about the effects of the current tax treatment on the volume and distribution of charitable contributions, tax payments and net personal income. More specifically, this section examines the effects of eliminating the deduction for charitable contributions and reducing all tax rates (on itemized returns) proportionately to keep government revenue constant. The elimination of the deduction reduces giving while the reduction in the tax rates increases giving. However, since income after tax remains unchanged while the price of giving rises, the net effect is a fall in charitable contributions.

To develop estimates of the full aggregate and distributional effects requires estimates of the income and price elasticities for all income classes. The basic method used in this section is to assume that the values obtained for incomes between $4000 and $100,000 hold for other incomes as well. Although this group contains about 90 percent of the itemized contributions, the dangers of such an extrapolation are obvious. With this method, the calculations show that the reduction in total contributions is large, probably about 35 percent of itemized giving and therefore about 30 percent of all individual contributions. Since the reductions are particularly large in high income groups, religious organizations are affected relatively less than educational, cultural and other nonreligious organizations.

TABLE 2.—*Basic predicted effects of eliminating the charitable deduction 1968**

AGI class ($1000) (1)	Number of itemized returns (1000's) (2)	Average charitable contributions G_{it} (3)	G'_{it} (4)	G'_{it}/G_{it} (5)	Tax ratio† (6)	Net disposable income ratio (7)
0–5	5,328	156	126	0.810	1.076	1.005
5–10	12,233	221	172	.778	1.000	1.007
10–15	8,731	305	233	.764	0.982	1.009
15–20	3,132	428	307	.718	.982	1.012
20–50	2,232	761	460	.605	.988	1.017
50–100	294	2,267	816	.360	1.002	1.032
100–500	77	9,695	2,380	.245	1.037	1.056
500–1000	2.6	68,749	12,827	.187	1.101	1.083
1000+	1.1	287,651	54,117	.188	1.152	1.104
Average		$348	$238	.657	1.0	
Total	32,030	$11,139 million	$7,316 million			

* Based on parameter values of equation 2. Total government revenue remains constant.
† Biased upward by the presence of nontaxable returns. See text.
Totals may not agree because of rounding.

Table 2 presents detailed results for 1968. These illustrative predictions use the basic specification of equation 2 with constant income and price elasticities. Eliminating the deduction would raise the price of giving to 1 in all income classes. The additional tax revenues that would result are redistributed in this calculation by a proportional reduction

in the effective tax rate in every income class.[38] The resulting change in contributions in each income class is then calculated from the equation:

$$\ln G'_{it} - \ln G_{it} = 0.822\,(\ln Y'_{it} - \ln Y_{it} + 1.238 \ln P_{it} \qquad (21)$$

where G'_{it} is the predicted average contribution after the tax change and Y'_{it} is the average adjusted gross income minus the new tax on that income. Since eliminating the deduction raises the price of giving to 1, $\ln P'_{it} = 0$ and therefore does not appear in equation 21.

The average contribution in 1968 is given for broad income classes in column 3 and the corresponding predicted contribution if the deduction is eliminated appears in column 4. Total 1968 giving falls from $11.1 billion to $7.3 billion.[39] The ratios of predicted contributions to actual contributions that are presented in column 5 show that the relative reduction in giving is much greater among high income individuals than in lower income groups. While taxpayers with adjusted gross incomes of $10,000 to $15,000 would cut contributions by 24 percent (from $305 to $233), a reduction of 75 percent is predicted for taxpayers in the $100,000 to $500,000 class (from $9,695 to $2,380).

Eliminating the charitable deduction and returning the additional revenue by a common proportional tax reduction would raise the taxes paid by high income individuals and lower the taxes paid by low income individuals. Column 6 shows the ratios of the tax payments if the deduction were eliminated and tax rates cut to maintain the actual total tax payments in 1968. Middle income individuals pay reduced taxes while those with incomes above $50,000 would pay increased taxes.[40] The differences are quite substantial. Although average taxes fall by only two percent in the $10,000 to $15,000 class, taxes rise by 10 percent in the class of taxpayers with incomes of $500,000 to $1,000,000.

---

[38] The new tax at each income level in 1968 is calculated as follows: (1) The additional tax revenue due to eliminating the deductible is calculated for each income class as the product of the 1968 deduction and the corresponding marginal rate. (2) The sum of these additional tax revenues is added to total 1968 tax collections. (3) The ratio of actual tax collections to the new sum is the factor by which all tax liabilities are scaled down. The value of this was 0.943 reflecting additional revenues of $3.3 billion and a 1968 total collection from itemized returns of $56.9. (4) This factor is then applied in each income class to the sum of the 1968 tax and the additional revenue from eliminating the charitable contribution deduction. All dollar amounts are in current 1968 dollars.

[39] Two things should be remembered in interpreting these numbers. First, these totals refer only to itemized giving; all individual giving in 1968 was estimated to be $12.6 billion. Second, although the reduction reflects the redistribution to taxpayers of the additional tax revenues, this has very little effect on total contributions; if the additional revenues were retained by the government, predicted giving would fall by $3.9 billion.

[40] These are of course only averages for each income class. The tax ratio falls below one at an AGI of $7000. Since no distinction is made between taxable and nontaxable returns, the increased taxes are overstated for the lowest income classes. Many of those returns are nontaxable and would remain so even if the charitable deduction were excluded. The amounts involved are so small that the resulting misestimate of additional revenue would have no significant effect on higher income classes.

The distributional effect of eliminating the deduction is quite different if we focus on the change in net disposable income rather than the change in tax payments. Net disposable income available for personal consumption or saving is defined as adjusted gross income minus taxes and charitable contributions. Because charitable contributions fall sharply in higher income groups, their predicted personal consumption and savings increase despite the greater taxes that they pay. Column 7 presents the ratio of predicted net disposable income to actual 1968 net disposable income. Net disposable income rises at every income level, with the increase ranging from less than two percent for incomes under $50,000 to more than 8 percent over $500,000.

Although the effect of eliminating the charitable deduction is of course greater if government revenues are not constrained to remain constant, the difference is quite small. Eliminating the deduction would yield an additional $3.3 billion in tax revenues in 1968.[41] If this revenue is not returned to the taxpayers through a general tax cut, total charitable contributions would fall by $3.8 billion. The gross "efficiency" of the deduction as measured by the ratio of additional contributions received by charities per dollar of potential tax revenue forgone is 1.15.

Generally similar results are obtained from calculations with other equations for charitable contributions. When gifts of appreciated assets are distinguished and an effective appreciation ratio of 0.5 is used (based on equation 18), charitable contributions in the absence of the deduction are estimated to be $7.4 billion. Finally, equation 10 (which uses the entire sample of 187 observations) implies contributions of $6.9 billion. Although there are some differences in the distributional impacts, in each case eliminating the deduction would reduce giving proportionately more in high income groups and would result in greater increases in their net disposable income than that of lower income groups.

5. Conclusions and Caveats

The empirical findings of this study are clear. The aggregate Internal Revenue Service data for 1948 through 1968 imply that the volume of charitable contributions is quite sensitive to the price of giving that is implied by the tax treatment. Almost all of the estimates of the price elasticity are greater than one. Eliminating the current deduction of charitable contributions would reduce total itemized giving by approximately 28 to 56 percent,[42] depending on the particular equation specification. The loss of contributions would be relatively greatest for

[41] This ignores the additional revenue that would result if some of the donated appreciated assets were sold instead.

[42] Since itemized giving accounts for approximately 90 percent of total individual giving, these reductions in itemized giving correspond to between 25 and 50 percent reduction of total individual giving.

educational, medical and cultural organizations. Philanthropies would lose more in the contributions they receive than the government would gain in additional tax revenues. Net disposable income after tax and charitable contributions would rise in all income groups with the highest percentage increase in the highest income groups.

These empirical results must however be regarded with substantial caution. Those who wish to assess the impact of our tax system on charitable giving must balance the current results against the conclusions of previous research on this subject and must consider the important factors that have been neglected in all of this work. It is appropriate to conclude this paper by reviewing these problems.

Although a number of writers have discussed the impact of the tax treatment of charitable contributions,[43] only two studies have used explicit statistical models to separate the income and price effects. The most frequently cited of these studies is the research of Michael Taussig (1967). Taussig examined a sample of 47,678 itemized individual tax returns for 1962. He found extremely low price elasticities (absolute elasticities not greater than 0.10) and concluded that the current tax deductibility of charitable contributions therefore does little to stimulate charitable giving.[44] Taussig's own paper is full of warnings about the shortcomings and potential biases of his results; these need not be repeated here.[45] However, three basic problems with Taussig's method should be emphasized. First, he used the marginal rate for actual taxable income, i.e., net of the individual's charitable contribution. An individual who gives more to charity therefore has, ceteris paribus, a lower marginal rate and a higher price. This introduces a spurious positive association of price and giving and therefore biases the negative price elasticity towards zero. Although this is relatively insignificant for aggregate data, it is quite important for microeconomic data.[46] Second, income was also measured net of taxes actually paid rather than of the taxes that would have been paid with no charitable contribution. This introduces a spurious simultaneity between income and contributions since the relevant budget constraint is defined by disposable income before any contributions are made. Third, because Taussig's sample is limited to only one year, the marginal tax rate and the price of charitable giving is an exact function of the individual's taxable income. Although relating charitable contributions to adjusted gross

[43] See the works cited on page 22.

[44] Taussig's estimates are based on a specification like the current equation 2 except that the logarithm of the marginal tax rate is used instead of the logarithm of the price. The corresponding price elasticities were derived from these marginal rate elasticities by Schwartz (1970, p. 1280).

[45] See also the discussion of Taussig's work in Schwartz (1970), pp. 1280–82.

[46] After this study was complete, I was able to reanalyze the original 1962 microeconomic data that was studied by Taussig. The results of this reanalysis, presented in Feldstein and Taylor (1975), indicate the importance of the bias due to Taussig's endogenous price variable.

income net of tax avoids the existence of an exact functional relation, the problem of collinearity between income and price is exacerbated by Taussig's procedure of dividing his sample into five income classes. Taussig notes that within each class "the main source of variation in the tax rate facing the taxpayer still remained the tax schedule used by the filer of the return" (Taussig, 1967, p. 8). Since these different types of tax schedules (i.e., married couples, single individuals and heads of households) represent demographic differences that would be expected to have substantial effects on giving, the primary source of variation in the tax price in Taussig's sample is itself mainly a reflection of other important influences.[47]

The study by Schwartz (1970) is methodologically closer to the current research. Schwartz used aggregate time series data based on the summaries of tax returns that are published by the Internal Revenue Service. Instead of developing a time series of cross sections as in the current study, Schwartz aggregated the data into only three time series and estimated separate equations for each time series.[48] For the period from 1929 through 1966, this produced 31 observations for each regression. Since the introduction of the standard deduction in 1941 and its extension to incomes over $3000 in 1944 had a very substantial effect on the extent of itemizing, Schwartz also estimated his equations for the subsamples 1929 through 1943 and 1944 through 1966.[49] The recent sample contained only 16 observations. With this data, Schwartz estimated equations like equation 3 of the current study (the basic constant elasticity equation with a time trend).[50] For each annual observation, the income variable was the average disposable income for the entire income class (e.g., $10,000 to $100,000) and the price variable was the average price of money gifts for that income class.

The relatively small number of observations and the use of separate samples by income groups preclude precise estimation; in more than half of the cases, the estimated price elasticity is less than twice its standard error. These difficulties are compounded by the use of single annual averages to represent the very wide range of incomes and prices *within* each of the three groups.[51] In spite of these problems, the

[47] Single individuals have a higher marginal rate and therefore lower price than married couples. Since single individuals tend for other reasons to make smaller contributions. Taussig's procedure introduces a further spurious positive association between price and giving.

[48] The three time series corresponded to taxpayers grouped by current income into those with less than $10,000 of adjusted gross income, those between $10,000 and $100,000, and those with more than $100,000. The use of current dollar limits to define these groups implies that the real income limits change substantially over time.

[49] When the complete sample was employed, a dummy variable was used to represent the shift in giving after 1943. No allowance was made for the effect of the change in deduction limits in 1952 and 1954.

[50] A more general equation with a relative income variable was also estimated; see above, footnote 34.

[51] If the log-linear model is appropriate at the individual level, an aggregate log-linear specification should use *geometric* means for the income, price and contributions

evidence does indicate the existence of considerable price elasticities. For the interval 1929 through 1966, Schwartz found a price elasticity of −0.69 for incomes below $10,000, −0.76 for incomes of $10,000 to $100,000, and −0.41 for incomes over $100,000. The corresponding standard errors are 0.49, 0.20, and 0.10. For the period after 1943, the elasticities in the groups with incomes below $10,000 and above $100,000 are almost identical to the value for the entire period. In the middle range ($10,000 to $100,000), the estimate is substantially less (−0.17) and has a large standard error (0.32), reflecting the very narrow range of price variation (except for one year, the price remained between 0.558 and 0.671). In short, Schwartz' estimates are imprecise but generally imply a substantially higher price elasticity than that found by Taussig and a lower elasticity than that found in the current study.

The current study as well as the research of Taussig and Schwartz suffers from the limits imposed by the use of the official tax return data. Perhaps the most serious problem is the lack of information on permanent economic income and wealth. Adjusted gross income becomes a less adequate measure as income rises. Similarly, the influence of wealth rather than current income is likely to be very important at the highest income levels. A second important shortcoming is restriction to analyzing the contributions of taxpayers with itemized returns. While this restriction is unimportant for high income individuals, it eliminates substantial information on the behavior of those with lower income. In addition, demographic characteristics, educational background, religious affiliation and other factors that influence charitable giving[52] may be correlated with the income and price variables in a way that biases the estimates of the structural parameters and the derived predictions of the effects of tax changes. Feldstein and Clotfelter (1974) have analyzed survey data on households[53] which contains better measures of income and wealth, income on demographic characteristics, and the contribution of households that did not itemize. The estimates obtained with this data strongly support the current conclusions.

Explaining aggregate charitable contributions to all types of organizations by a single equation may hide important differences in the relations governing gifts to different philanthropies. The different effects of prospective tax changes on the major types of philanthropies is at least as interesting as the total effect on all charitable contributions. The substantial differences in the distribution of religious and

---

variables. The error involved in using arithmetic mean increases with the size of the interval and therefore represents a more serious problem in Schwartz work than in the current study.

[52] On the importance of such factors, see Morgan *et al.* (1962) and Barlow and Morgan (1966).

[53] The data is the Federal Reserve Board Survey of Consumer Finances (Projector and Weiss, 1966).

nonreligious giving suggests the potential importance of such decomposition. An analysis of the differences in the impact of alternative tax policies on religious, educational and other charitable organizations is presented in the second part of this article and will appear in the next issue of the *National Tax Journal.*

References

Aaron, Henry, "Federal Encouragement of Private Giving," in *Tax Impacts on Philanthropy.* Symposium conducted by the Tax Institute of America, December 2–3, 1971, Washington, D.C. Princeton: Tax Institute of America, 1972, pp. 210–213.

American Association of Fund-Raising Counsel, Inc., *Giving USA.* New York: American Association of Fund Raising Counsel, Inc., 1973.

Andrews, William D., "Personal Deductions in an Ideal Income Tax," *Harvard Law Review,* 86:2 (December 1972), pp. 309–385.

Barlow, Robin *et al., Economic Behavior of the Affluent.* Washington, D.C.: The Brookings Institution, 1966.

Bittker, Boris I., "The Propriety and Vitality of a Federal Income Tax Deduction for Private Philanthropy," in *Tax Impacts on Philanthropy.* Princeton, N.J.: Tax Institute of America, 1972, pp. 145–170.

Brannon, Gerard M., "The Effect of Tax Deductibility on the Level of Charitable Contributions and Variations on the Theme," Mimeograph, 1973.

Feldstein, Martin and Charles Clotfelter, "Tax Incentives and Charitable Contributions in the United States: A Microeconometric Analysis," *Journal of Public Economics,* 5 (1976), pp. 1–26.

Feldstein, Martin and Amy Taylor, "Taxation and Charitable Contributions: An Analysis of Individual Tax Return Data for 1962 and 1970," *Econometrica* (November 1976).

Goode, Richard, *The Individual Income Tax.* Washington, D.C.: The Brookings Institution, 1964.

Harberger, Arnold C. and Martin J. Bailey, *The Taxation of Income from Capital.* Washington, D.C.: The Brookings Institution, 1969.

Internal Revenue Service, *Statistics of Income 1962: Individual Income Tax Returns.* Washington, D.C.: U.S. Government Printing Office, 1965.

———, *Statistics of Income 1966: Individual Income Tax Returns.* Washington, D.C.: U.S. Government Printing Office, 1969.

———, *Statistics of Income 1968: Individual Income Tax Returns.* Washington, D.C.: U.S. Government Printing Office, 1970.

———, *Statistics of Income 1970: Individual Income Tax Returns.* Washington, D.C.: U.S. Government Printing Office, 1972.

Kahn, C. Harry, *Personal Deductions in the Federal Income Tax.* Princeton: Princeton University, 1960.

McDaniel, Paul R., "An Alternative to the Federal Income Tax Deduction in Support of Private Philanthropy," in *Tax Impacts on Philanthropy.* Princeton, N.J.: Tax Institute of America, 1972, pp. 141–209.

———, "Federal Matching Grants for Charitable Contributions: A Substitute for the Income Tax Deduction," *Tax Law Review,* 27:3 (Spring 1972), pp. 377–413.

Morgan, James N. *et al., Income and Welfare in the United States.* New York: McGraw-Hill Book Company, Inc., 1962.

Projector, Dorothy S. and Gertrude S. Weiss, *Survey of Financial Characteristics of Consumers.* Washington, D.C.: Federal Reserve Technical Papers, 1966.

Pechman, Joseph A., *Federal Tax Policy.* New York: W. W. Norton and Company, Inc., 1971.

Rabin, A. "Charitable Trusts and Charitable Deductions," *New York University Law Review,* 41 (1966), pp. 912–925.

Schwartz, Robert A., "Personal Philanthropic Contributions," *Journal of Political Economy*, 78:6 (Nov/Dec 1970), pp. 1264–1291.

Surrey, Stanley S. *et al.*, *Federal Income Taxation*. Mineola, N.Y.: The Foundation Press, Inc., 1972.

Taussig, Michael K., "Economic Aspects of the Personal Income Tax Treatment of Charitable Contributions," *National Tax Journal*, 20:1 (March 1967), pp. 1–19.

Theil, H., *Economic Forecasts and Policy*. Amsterdam: North-Holland Publishing Company, 1961.

U.S. Treasury Department Tax Reforms Studies and Proposals, in U.S. Congress, House Ways and Means Committee and Senate Finance Committee. 91st Congress, 1st Session, 1969.

Vickrey, William S., "One Economist's View of Philanthropy," in Frank Dickinson (ed.), *Philanthropy and Public Policy*. New York: National Bureau of Economic Research, 1962, pp. 31–56.

———, "Private Philanthropy and Public Finance," mimeo, 1973.

Wald, A., "The Approximate Determination of Indifference Surfaces by Means of Engel Curves," *Econometrica*, 8 (1940), pp. 144–175.

Weidenbaum, Murray L., "A Modest Proposal for Tax Reform," *The Wall Street Journal*, April 4, 1973, p. 18.

White, Melvin I., "Proper Income Tax Treatment of Deductions for Personal Expense," *Tax Revision Compendium*, Compendium of Papers on Broadening the Tax Base Submitted to the Committee on Ways and Means, House of Representatives, U.S. Congress. Washington: U.S. Government Printing Office, 1959, vol. 1, pp. 370–71.

The Income Tax and Charitable Contributions Part II—The Impact on Religious, Educational and Other Organizations

Martin Feldstein*

Abstract

Several studies have shown that the volume of gifts to all charities combined is quite sensitive to the reduced cost of giving that is implied by the income tax. The sensitivity of charitable giving to potential tax changes differs substantially among the major types of donees. Gifts to educational institutions and hospitals are very sensitive to the cost of giving while religious organizations are much less sensitive than the others. Eliminating the charitable deduction would reduce total individual giving by an estimated 20 percent, but gifts to educational institutions and hospitals would be cut approximately in half. Although replacing the current deductible by a 30 percent tax credit would increase total giving by some 15 percent, educational institutions and hospitals would still lose about 20 percent of current gifts.

The current deductibility of charitable gifts in the calculation of taxable income has been both sharply criticized and strongly defended. The supporters argue that the deduction is needed to stimulate private expenditures on worthwhile activities that produce substantial benefits that do not accrue to the individual donors. The critics reply that the current system is unfair because it makes the donor's net personal cost per dollar of gift much lower for high income individuals than for individuals with lower incomes.[1]

---

* Professor of Economics, Harvard University. I am grateful to Daniel Frisch for assistance with this research and to the Commission on Private Philanthropy and Public Needs for financial support. This paper was published in the *National Tax Journal*, 1975.

[1] Although some of the critics have also argued that the current system is inefficient because the Treasury forgoes substantially more revenue than the charities gain, substantial evidence is now accumulating that points in the other direction; see pages 21 to 47 and Feldstein and Clotfelter (1974). Andrews (1972) has also argued that the effect of the subsidy is irrelevant because, following Simons (1938), taxable income should be defined as the sum of personal consumption and net accumulation; since charitable gifts are neither consumed nor accumulated, they should not be part of taxable income. The pros and cons of the current system and various alternatives are discussed extensively in Bittker (1972), Goode (1964), Kahn (1960), McDaniel (1972a, 1972b), Pechman (1971), Pifer (1972), Rabin (1966), Surrey (1972), Vickrey (1962, 1973), Weidenbaum (1973), White (1959) and U.S. Treasury Department (1969).

Many of the proposals for changing the current tax treatment of charitable gifts have favored substituting a tax credit or matching grant for the current deduction. If charitable gifts are deductible in calculating taxable income, a one dollar gift costs 78 cents for a family with taxable income of $10,000 but only 38 cents for a family with a taxable income of $100,000.[2] For a family that does not itemize its deductions, the cost is a full dollar. In contrast, with tax credit of 30 percent, the cost of a one dollar gift would be 30 cents for all taxpayers regardless of their income and of whether they itemize. Instead of a tax credit, the same effect could be achieved with a matching grant to the charities themselves at a corresponding rate; a 30 percent tax credit to donors is equivalent to a matching grant to donees at 43 percent.[3]

By selecting an appropriate rate for the tax credit (or matching grant), the government can induce the same aggregate contribution that would have been made if contributions were deductible. Since the econometric evidence indicates that the price elasticity of giving is approximately minus one and does not differ significantly among income groups,[4] the total cost in terms of forgone tax revenues is essentially independent of the method used to stimulate contributions. The change from the current system of deductions to a tax credit or matching grant would therefore require no change in tax rates. In principle, therefore, if the current deduction were replaced by a tax credit, the worthwhile activities of the charities would still be achieved, the taxes of high income individuals would generally rise and the taxes of lower income individuals would fall.

In evaluating any such proposal,[5] it is important to consider its potential effects on the distribution of gifts to different types of charities. The current study shows that replacing the current system of deductions with a 30 percent tax credit would raise total giving by about 15 percent but would drastically alter the amounts received by different charities. More specifically, gifts to educational institutions and hospitals would fall sharply while gifts to religious organizations would rise. A similar pattern of large differences is also estimated for the other possible tax proposals examined in this study, complete elimination of the deduction with no tax credit and replacement of the deduction by a 50 percent tax credit.

Section 1 of this paper describes the data and defines the basic

---

[2] These figures are for gifts of cash. For gifts of appreciated assets, the relative price is even lower for high income households.

[3] With a 30 percent tax credit one dollar received by a donee costs the donor 70 cents; with a 43 percent matching grant, a 70 cent net contribution induces an additional matching grant of 30 cents.

[4] This price elasticity is the elasticity of giving with respect to the net cost to the donor (net of deductions or credits) per dollar received by the donee. See pages 21 to 47 and Feldstein and Clotfelter (1974) for evidence that this elasticity is actually slightly greater than one.

[5] Other tax credit proposals have suggested a credit only for contributions above some minimum percentage of income or, more generally, a rate of credit that varied with the ratio of contribution to income; see McDaniel (1972a, 1972b).

variables. The second section discusses the problems of specification and estimation. The parameter estimates are presented in section 3. The simulated effects of alternative tax policies are analyzed in section 4. Section 5 discusses some general implications and directions for future work.

1. Data and Definitions

Every second year, the Internal Revenue Service publishes the value of itemized charitable contributions in each of 17 adjusted gross income (AGI) classes. For 1962, the published report divided these contributions among five major types of charities: (1) religious organizations, (2) educational institutions, (3) hospitals, (4) health and social welfare organizations (including United Funds, the Red Cross and specific disease associations), and (5) a residual group including libraries, museums, zoos, musical organizations, and literary, educational and scientific foundations. This is the only source of data on the distribution among different types of charities of the contributions of high income households. The current study is based primarily on this disaggregated 1962 data with some use of a pooled sample of aggregate data by income class for each of the reporting years from 1948 through 1968.[6]

The availability of disaggregated data for only 1962 raises two problems. First, it is difficult to know the extent to which the behavior of donors has altered during the past twelve years. The possibility of such change must be borne in mind in considering the parameter estimates and the simulations of alternative future tax policies. Although there is no subsequent disaggregated information on contributions to different donees, there is some evidence to suggest that the estimates based on 1962 data are still relevant. In a study using the Treasury Tax Files (large stratified samples of individual tax returns) for 1962 and 1970, Feldstein and Taylor (1974) found that the price and income elasticities for 1962 were almost identical to those for 1970. The second problem is that, with a single year's cross-section sample of aggregate data, the price is functionally related to taxable income. Several methods of dealing with this problem are discussed below. Using all of these methods provides a range of estimates but does not alter the basic conclusions about the relative impacts of alternative policies on different charities.

Each of the estimation methods relates charitable giving (G_{ji}) to income (Y_i) and price (P_i):

$$G_{ji} = f_j\,(Y_i,P_i) + \epsilon_{ji.} \tag{1}$$

The subscript i denotes the adjusted gross income (AGI) class and the

[6] The pooled sample stops with 1968 because the current paper makes use of estimates derived in an earlier study (Feldstein, 1974). Final data for 1970 was not available when the previous study began.

subscript j denotes the type of charity. The variable ϵ_{ji} is a random error that reflects random disturbances and specification errors.

More specifically, the variable G_{ji} is the average charitable contribution to charities of type j per itemized return in AGI class i in 1962. The contribution is defined as the gross amount given by the individual to the charity and not as the net cost of that contribution to the individual. These amounts include the value of donated assets as well as gifts of money.

The value of Y_i is the average income per itemized return in income class i. Income is defined as adjusted gross income minus the tax that would have been due if the individuals made no charitable contributions. This is more appropriate than income minus the taxes actually paid since that would make the net income depend on the size of the charitable gift. Analyzing data that are grouped by income class reduces the potential bias that arises from using current income instead of permanent income. If the income groups correctly classify individuals by permanent income, the parameter estimates are consistent even if individual current incomes differ from permanent income.[7] More generally, the very great variance in permanent incomes in the population of taxpayers relative to the average transitory variance implies that the bias from this source would be quite small. Although the use of adjusted gross income instead of a more general measure of total economic income would seem to be a more serious potential problem, the parameter estimates for total giving based on AGI are very similar to the estimates based on a broader measure of economic income obtained in a household survey (Feldstein and Clotfelter, 1974).

The price variable (P) measures the individual's opportunity cost per dollar of charitable contribution in terms of forgone personal consumption or wealth. An individual whose marginal tax rate is m can choose between (1) contributing one dollar to charity and (2) having $1-m$ dollars for additional personal consumption or saving. We therefore define that individual's price of charitable giving by $P = 1-m$. In practice, P_i is measured by using the 1962 marginal tax rate for a joint return with the average taxable income in class i.[8]

Contributions of appreciated assets create a special problem for measuring the price of charitable giving. When an asset is given away, its full value can be deducted from the donor's taxable income but there is no constructive realization and therefore no tax to be paid by the donor on the capital gain.[9] The opportunity cost (price) of a gift that is

---

[7] This use of grouped data is a generalization of Wald's (1940) method of instrumental variable estimation.

[8] To allow for the effect of using the alternative tax computation, average taxable income in class i is adjusted by subtracting one-half of the net capital gains reported on returns using the alternative tax. No attempt is made to allow for income averaging or state income taxes.

[9] Since income of the donee organization is not taxable, it can sell the appreciated asset without paying any tax.

given in the form of an appreciated asset therefore depends not only on the individual's marginal tax rate but also on the fraction of the asset's value that is accrued capital gain and on the alternative disposition of the asset. An example will clarify the way in which these variables determine the relevant price. Consider an individual whose marginal rate is 40 percent and who contemplates donating an asset that is now worth $100 and for which he orginally paid $30. If he gives the asset away, he reduces his taxable income by $100; he therefore reduces his tax liability by $40 and thus increases his after tax income by $40. If he instead sells the asset, he pays a tax of $14 (half of his marginal rate on the capital gain of $70) and increases his after tax income by $86. For this individual, the opportunity cost of the $100 contribution is therefore $46 of forgone consumption. If the price is defined in terms of forgone consumption, the price of the gift is $P = 0.46$. This price clearly depends on the ratio of the asset's original cost (or basis) to its current value: an original cost of $1 implies $P = 0.40$ while an original cost of $100 implies $P = 0.60$. More generally, $P = 1 - mc\,(1-B/V) - m$ where V is the current value of the asset, B is its basis or original cost, m is the marginal tax rate on income and mc is the marginal tax rate on capital gains; during the sample period, $mc = 0.5m$ with a maximum of 0.25.

The preceding calculation defined the opportunity cost of a donated asset in terms of forgone immediate consumption, i.e., it assumed that if the asset were not given away it would be sold in the current year. The price is higher and the calculation is more complex if the opportunity cost is defined in terms of forgone saving or wealth, i.e., if it is assumed that the asset would not otherwise be sold in the current year. The individual in the preceding example could retain the $100 asset or he could give it away and add the $40 tax saving to his wealth. Viewed in this way, his opportunity cost price is 0.60, the same as for contributions of money; moreover, this price is independent of the ratio of the capital gain to the present asset value. Since the individual who does not give away the asset also has a future tax liability, this tends to overstate the opportunity cost of a prospective contribution. However, by postponing the sale of the asset the individual can substantially lower the present value of the tax and, if the asset is never sold during the individual's lifetime, the capital gains tax liability is completely eliminated when the asset passes at death.[10]

This paper makes no attempt to reflect the special treatment of gifts of appreciated assets. All contributions are treated as if they are gifts of cash. This has relatively little effect for middle and low income households; for households with income less $50,000, gifts of assets

[10] If the individual gives the asset away to another person, there is no constructive realization and the tax is postponed until the recipient sells the asset. The original owner can also consume most of the value of the asset by using it as collateral to borrow funds which he then consumes, thus enjoying the consumption while postponing or avoiding the capital gains tax. See Bailey (1969) for evidence that a very large share of accrued capital gains are never subject to capital gains taxation.

account for less than 10 percent of total giving. In contrast, for households with incomes over $100,000, gifts of assets account for more than half of total giving. The primary effect of ignoring the lower price for gifts of appreciated assets is therefore to understate the incentive to giving that the current system provides to the highest income classes. As a result, the analysis will tend to understate the effects of tax changes on the amount of giving in these income groups. Since these groups account for a disproportionately large share of the support of educational institutions, hospitals, and other non-religious organizations, the analysis will understate the effect of tax changes on this type of philanthropic organization. The very large relative effects reported below should therefore be regarded as conservative estimates.

Table 1 presents the average contribution to each of the major types of philanthropic organization for each AGI class in 1962. These contributions of course include only gifts that are reported as itemized deductions. It is clear that the relative importance of different donees changes dramatically as income rises. Table 2 shows this in a different way by presenting the cumulative distributions of total contributions to each type of donee. While 81.6 percent of total itemized gifts to religious organizations come from households with AGI below $15,000, only 24.6 percent of gifts to educational institutions comes from this group. Similarly, only 1.1 percent of religious gifts come from taxpayers with incomes over $100,000, while this group provides 33.1 percent of gifts to educational institutions.

2. Specification and Estimation

In the earlier studies of total charitable giving, I found that a log-linear equation with constant price and income elasticities was an appropriate specification. The same specification has been used here for each of the individual types of donees:[11]

$$\ln G_{ij} = \alpha_j + \beta_j \ln Y_i + \gamma_j \ln P_i + \epsilon_{ij}. \tag{2}$$

There are two potential problems with this approach. First, the statutory relation between income and price many make the price and income elasticities underidentified. For a single year, price is an exact non-linear function of taxable income.[12] However, the income variable used here is based on adjusted gross income rather than taxable income so that there is no functional relation. The correlation between $\ln Y$ and $\ln P$ is actually low enough in the sample to permit quite precise estimation of the β_j's and γ_j's. If there is a problem of underidentification, it is that the current log-linear specification is an arbitrary

[11] It would clearly be desirable to extend the current type of study to include more general specifications with varying price and income elasticities. The current sample of a single year's aggregate data is not rich enough for such an analysis.

[12] In the previous study with aggregate data, changes in tax rates from 1948 through 1968 provided a source of independent variation.

TABLE 1.—*Itemized contributions to major types of philanthropic organization by donors's income class, 1962*

Adjusted gross income ($000)	Number of itemized returns (000)	Average contribution per return ($)					
		Religious organizations	Educational institutions	Hospitals	Health and welfare organizations	All others	All donees
0–5	8,268	103	1	0.5	17	24	145
5–10	12,803	159	3	1	33	38	234
10–15	3,526	240	6	2	54	55	357
15–20	860	323	19	8	86	97	533
20–50	850	481	66	34	158	210	949
50–100	118	886	370	164	488	900	2,808
100–500	25.5	1,816	2,458	848	1,643	5,817	12,577
500–1000	0.8	4,380	14,590	5,414	7,910	38,270	70,582
1000+	0.3	7.696	47,450	15,234	14,813	180,848	266,038
All Incomes	26,452	173	10	4.3	40	56	284

Gifts are itemized deductions for contributions in 1962. Discrepancies in totals reflect rounding of averages for individual donees.

TABLE 2.—*Cumulative percentages of itemized contributions to major types of philanthropic organizations by donor's income class, 1962*

Adjusted gross income ($000)	Number of itemized returns	Contributions					
		Religious organizations	Educational institutions	Hospitals	Health and welfare organizations	All others	All donees
0–5	31.3	18.6	2.9	3.9	13.1	13.5	16.0
5–10	79.7	63.1	16.9	16.0	52.2	50.0	55.8
10–15	93.2	81.6	24.6	23.5	70.0	59.0	72.5
15–20	96.5	87.7	30.5	29.5	77.0	64.6	78.6
20–50	99.7	96.6	51.0	55.2	89.6	76.7	89.5
50–100	100.0	98.9	66.9	72.4	95.0	83.8	93.8
100–500	100.0	99.9	89.8	91.6	98.9	93.8	98.0
500–1000	100.0	100.0	94.1	95.4	99.5	95.8	98.9
1000+	100.0	100.0	100.0	100.0	100.0	100.0	100.0

approximation to a more general true (but unknown) non-linear relation between contributions, price and income. The second problem is that the constant elasticity specification for each type of donee is inconsistent with a constant elasticity specification for giving to all donees combined. This is again a problem of using a particular approximation to an unknown non-linear function. Although such an approximation problem arises in all econometric work, the problem is more apparent here because there is both the aggregate and individual relations.

These two problems could in principle be alleviated by combining the 1962 disaggregated data with the data for all donees that are available for 1948 through 1968. The individual coefficients by donee and the aggregate coefficients could be estimated simultaneously subject to the constraint that the individual coefficients by donees imply the aggregate coefficients based on all the data. Although this procedure would be simple for a linear model, it is not tractable in a model with constant elasticties for the individual types of donees. The current study therefore uses two simpler alternative methods of combining information from the two sets of data. With both methods, the 1962 data alone do not determine both the income and price elasticity. Fortunately, the unconstrained estimates of equation 2 and these two alternative methods imply the same general pattern of effects of the proposed tax changes.

The first alternative specification (Specification II) assumes that the price elasticity is the same for all donees. The exact specification

$$\ln \frac{G_{ij}}{G_i} = \alpha'_j + \beta'_j Y_i + \epsilon'_{ji} \tag{3}$$

where total giving (G_i) follows

$$\ln G_i = \alpha_0 + \beta_0 Y_i + \gamma_0 P_i + \epsilon_{0i} \tag{4}$$

implies

$$\ln G_{ij} = (\alpha_0 + \alpha'_j) + (\beta_0 + \beta'_j)\ Y_i + \gamma_0\ P_i + (\epsilon_{0i} + \epsilon'_{ji}) \tag{5}$$

A change in P_i causes the same proportional change in all the G_{ij}, the actual amount depending on the individual's income level. Although the price elasticity is the same for all donees, tastes differ among the income classes. There is no compelling argument for such a specification but its investigation seems worth while. The common price elasticity (γ_0) is obtained by estimating equation 4 for the entire sample for 1948 through 1968: $\gamma_0 = -1.238$ ($S.E. = 0.101$).[13] Equation 5 is then estimated for each of the individual donees with the price elasticity fixed at this value. Note that with this use of "extraneous information" from the entire sample there can be no problem of collinearity between

[13] See Feldstein (1974), equation 2.

price and income and no problem of the inconsistency of the disaggregated specification and the aggregate equation.[14]

The second alternative specification (Specification III) assumes that the types of donee organizations for which individuals have relatively high income elasticities are also the types for which price elasticities are relatively high. For example, donations to religious organizations may be much less sensitive to both price incentives and income than gifts to educational and cultural institutions. The model is again far from compelling but also worth exploring as an alternative extreme to the specification in which all types of donees have the same price elasticity. This implies the specification:

$$\ln G_{ij} = \alpha''_j + \beta''_j \ln Y_i + k\,\beta''_j \ln P_i + \epsilon''_{ij} \tag{6}$$

where k is the same constant for all β_j.

Although the individual donee equations could be estimated simultaneously subject to this proportionality constraint, the aggregate relation of equation 4 suggests a simpler alternative. Since $G_i = \sum_j G_{ij}$, the partial elasticity of aggregate giving (G) with respect to income satisfies:

$$\frac{Y\partial G}{G\partial Y} = \frac{Y\partial \Sigma_j G_j}{G\partial Y} = \sum_j \frac{Y\partial G_j}{G\partial Y} = \sum_j \left(\frac{G_j}{G}\right)\left(\frac{Y\partial G_j}{G_j\partial Y}\right), \tag{7}$$

i.e., the aggregate elasticity is a weighted average of the individual donee elasticities weighting by the share of total giving to that donee. The same is true for the partial price elasticity. With the aggregate elasticities defined by equation 4, it follows from 7 that the aggregate income elasticity (β_0) is related to the individual elasticities (β_j's) by

$$\beta_0 = \sum_j \frac{G_j}{G} \beta_j \tag{8}$$

Similarly, for the price elasticities

$$Y_0 = \sum \frac{G_j}{G} k\beta_j = k\beta_0. \tag{9}$$

The individual elasticities of equation 6 will be consistent with the aggregate elasticities of equation 4 if k is estimated by the ratio of the aggregate price elasticity to the aggregate income elasticity: from Feldstein (1974, equation 2) $\hat{k} = \hat{\gamma}_0/\hat{\beta}_0 = 1.506$.[15] Note that with this extraneous estimation of k, equation 6 is estimated with a single composite explanatory variable. Just as in specification II, this avoids the problems of collinearity and inconsistency that could arise with equation 2.

[14] Equation 3 is an equation for the share of the total. Stated alternatively, the income elasticities of equation 5 only define the β'_j's up to a constant which can be selected as β_0.

[15] The aggregate income elasticity was 0.822 (S.E. = 0.030) and the price elasticity was −1.238 (S.E. = 0.101).

All three specifications have been estimated with the seventeen income class observations for 1962. Each of the observations represents a different number of individual tax returns. However, the published values of total contributions and incomes are themselves estimates prepared by the Internal Revenue Service on the basis of a very large stratified sample of returns. The number of returns in each AGI class is selected to yield approximately the same sampling error in the resulting estimates. This suggests that relatively little gain in the efficiency of the parameter estimates could be obtained by using a weighted generalized least squares estimator. The procedure of giving equal weight to all of the observations is therefore used in this study.[16]

3. The Parameter Estimates

The parameter estimates for all three specifications are presented in Table 3. The unconstrained estimates (Specification I) all have reasonable negative price elasticities and reasonable positive income elasticities. The absolute price elasticity for gifts to religious organizations is only 0.49 while all of the other elasticities are greater than one. The income elasticity of 0.63 for religious organizations is also lower than for other types of donees but the differences are not as great as with the price elasticities. By contrast, educational institutions and hospitals have income elasticities of approximately one and price elasticities greater than two.

Before studying the results of the other specifications, it is useful to comment on the implications of these price elasticities. Since a full analysis is provided by the simulations of section 4, only some brief remarks are presented here. A price elasticity of exactly one implies that the amount of giving responds to changes in price in such a way that the net cost to the individual donor is unaffected by the price. Donees receive an amount equal to the sum of this constant net cost to donors plus the revenue forgone by the Treasury. The "efficiency" of the incentive is thus 100 percent, i.e., 100 percent of the revenue forgone by the Treasury is a net addition to the receipts of the donee. With a price elasticity absolutely less than one, a decrease in the price of giving raises the gross gift by less than the net cost to the Treasury. For

---

[16] Only for returns with incomes below $6000 did the relative error of the estimate of giving exceed 4 percent; above $10,000 the relative error was less than 1 percent. Although the earlier study of total charitable gifts restricted the sample by excluding taxpayers with incomes below $4000 or above $100,000, there are too few degrees of freedom in the current analysis to further restrict the sample. After this study was completed, it was able to use the Treasury Tax File for 1962 to calculate the average of the individual prices in each AGI class rather than the price for the average taxable income in that class. The values agree quite closely between $4000 and $100,000 but diverge outside those limits. In particular, the microeconomic average price is substantially higher. When the regression equations were reestimated with this price series, all of the price elasticities were substantially increased. The results presented below are therefore biased in a conservative direction.

TABLE 3.—*Price and income elasticities by type of charity*

Type of charity	Specification I			Specification II			Specification III		
	Price	Income	SSR	Price	Income	SSR	Price	Income	SSR
Religious organizations	−0.49	0.63	0.107	−1.24	0.38	0.734	−0.78	0.52	0.209
	(0.08)	(0.03)			(0.03)		(0.02)	(0.01)	
Educational institutions	−2.23	1.22	4.495	−1.24	1.54	5.586	−1.97	1.31	4.573
	(0.54)	(0.19)			(0.08)		(0.08)	(0.05)	
Hospitals	−2.44	1.08	5.954	−1.24	1.87	12.51	−1.90	1.26	6.272
	(0.62)	(0.22)			(0.12)		(0.09)	(0.06)	
Health and welfare	−1.19	0.85	0.214	−1.24	0.83	0.217	−1.25	(0.83	0.218
organizations	(0.12)	(0.04)			(0.02)		(0.02)	(0.01)	
All others	−2.63	0.65	0.810	−1.24	1.10	2.941	−1.55	1.03	2.128
	(0.23)	(0.08)			(0.06)		(0.05)	(0.03)	

Specification I: Unconstrained price and income elasticities.
Specification II: All price elasticities equal.
Specification III: Ratio of price and income elasticities equal.
Standard errors are shown in parentheses.
SSR = Sum of squared residuals.

religious giving, the estimated price elasticity of Specification I implies an efficiency of only 49 percent for a small reduction in the price of giving.[17] In contrast, the high price elasticities for the other types of giving imply efficiencies greater than 100 percent. For gifts to educational institutions, the estimated price elasticity implies that small price reductions have an efficiency of 223 percent, i.e., for every dollar of revenue forgone by the Treasury, educational institutions receive an additional \$2.23.[18]

Specification II constrains all of the price elasticities to be −1.24, the value obtained with the pooled sample of 20 years of data on total gifts. The income elasticities that are obtained when this constraint is imposed are similar to the unconstrained estimates for educational institutions and for health and welfare organizations but differ substantially for the other three types of donees. Comparing the sums of squared residuals for the two specificiations also shows that substantially more explanatory power is lost by the constraint for these three types of donees; however, an approximate likelihood ratio test shows that the constraint is only binding at the 10 percent level for religious organizations.[19] The parameter estimates of Specification II will therefore be used along with the other estimates for the simulation of the next section.

Specification III constrains the ratio of the price elasticity to the income elasticity to be 1.506 for all types of donees. The resulting estimates are generally rather close to the unconstrained estimates of Specification I, the main exception being religious organizations and the composite group of "all others." The sums of squared residuals indicate that Specification III is generally substantially better than Specification II. The approximate likelihood ratio test implies that the constraint of Specification III is not binding at the 10 percent level for any type of donee.

4. Effects of Alternative Tax Changes

The individual parameter values can be used to estimate how various proposed changes in the tax law would affect contributions to the five major types of charitable organizations. The current section will examine the implications of both the complete elimination of the charitable deduction and, alternatively, the replacement of the deducton by a 30 percent tax credit. Estimates will be presented for all three specifications to show that the parameter differences among the

[17] More specifically, the efficiency is 100 $(P^{-.49} - 1)/(1 - P)P^{-.49}$. The efficiency is slightly greater for larger price reductions, but even with $P = 0.5$, the efficiency is less than 60 percent.

[18] With an elasticity greater than one, the efficiency falls for larger price reductions. With $P = 0.5$, the efficiency is 157 percent; with $P = 0.2$, it is 122 percent.

[19] An asymptotic likelihood ratio test is based on the statistic $-2 \ln(SSR_I/SSR_{II})$ which is distributed as chi-square with one degree of freedom.

specifications are small in comparison to the differences among types of charities.

Elimination of the deduction would increase tax revenues by several billion dollars while the introduction of the tax credit would involve losses of several billion dollars. To make the estimated effects of alternative tax rules as comparable as possible, I assume that all tax rates are changed proportionately to keep total tax revenues constant. Thus the elimination of the deduction is accompanied by a proportional reduction of all tax rates while the tax credit plan involves a tax increase.[20]

The method of estimating the effects of a tax change is described most easily for the complete elimination of the deduction; the modifications in this method for the tax credit will then be described. Consider first how this calculation would be done if all households filed itemized returns.[21] Since all of the specifications are of the form:

$$\ln G_{ij} = \alpha_j + \beta_j \ln Y_i + \gamma_j \ln P_i, \tag{10}$$

the predicted change in contributions by individuals in class i to charities of type j is given by:

$$\ln G'_{ij} - \ln G_{ij} = \beta_j (\ln Y'_i - \ln Y_i) - \gamma \ln P_i \tag{11}$$

where G'_{ij} is the amount given after the change in the tax law (i.e., after the elimination of the deduction), Y'_i is the income net of tax after the change in the tax law, and $\ln P'_i = 0$ since elimination of the deduction implies $P'_i = 1$ for all individuals. Since the actual giving to each type of charity (G_{ij}) is known, equation 11 implies a value for G'_{ij} and therefore for total giving to charities of type j: $\sum_i N_i G'_{ij} = G'_j$ where N_i is the number of itemizing taxpayers in income class i.

The difference between Y'_i and Y_i reflects the change in tax rates that the government makes to keep total tax revenue unchanged after the deduction has been eliminated: thus $Y'_i > Y_i$. The value of the percentage reduction in all tax rates (and therefore in all individual tax liabilities) is simply the ratio of the tax revenue currently forgone because of the charitable deduction ($2.14 billion in 1962) to the total taxes paid by all those who file itemized returns ($29 billion in 1962); it is completely independent of the price and income elasticities.

To extend the calculation to taxpayers who do not itemize, it is necessary to estimate the amounts of the contributions that are currently made by these individuals.[22] Let $\hat{G}_{ij}$ be the estimated average

---

[20] Although the tax rate adjustments are required for complete comparability, their effects on total giving to each type of charity are quite small.

[21] Although eliminating the deduction does not change the price of giving faced by those who do not itemize their deductions, the general reduction in tax rates to offset the additional tax revenues raises their disposable incomes and therefore their charitable gifts. This effect is obviously small and was ignored in Feldstein (1974).

[22] Although this is relatively unimportant for studying the elimination of the deduction, it is quite important for the other proposed changes since nonitemizers would also receive the tax credit.

gift in 1962 to charities of type j by individuals in income class i who used the standard deduction. Similarly, let $\hat{Y}_{ij}$ be the average income in this group (as reported by the Internal Revenue Service in the annual Statistics of Income). Finally, let $\hat{G}'_{ij}$ and $\hat{Y}'_{ij}$ be the corresponding variables after the tax change. Since $\hat{G}_{ij}$ is unknown, the value of $\hat{G}'_{ij}$ cannot be estimated from the expected change in giving as it was in equation 11 for taxpayers who itemize. Instead, we now estimate $\hat{G}_{ij}$ and $\hat{G}'_{ij}$ separately on the assumption that the only relevant difference between itemizers and nonitemizers with the same income is the different prices that they currently face.[23]

Because the estimated equations for itemizing taxpayers do not explain their giving perfectly, there is a residual difference between actual giving and the giving predicted for current income and price:

$$u_{ij} = \ln G_{ij} - \alpha_j - \beta_j \ln Y_i - \gamma_j \ln P_i. \tag{12}$$

The value of the residual u_{ij} may be positive or negative and has an average of zero for all income classes combined. Each residual reflects both the use of a log-linear approximation and the omission of factors other than income and price that influence charitable giving. The method of estimating $\hat{G}'_{ij}$ for taxpayers who itemize their deductions implicitly includes u_{ij} in the calculated value of $\hat{G}'_{ij}$, i.e., equation 11 is identically equivalent to:

$$\hat{G}'_{ij} = \alpha_j + \beta_j \ln Y'_i + \gamma_j \ln P_i + u_{ij}. \tag{13}$$

The assumption that giving by itemizers and nonitemizers differs only because of the price difference therefore implies that u_{ij} should also be included in the estimated values of $\hat{G}_{ij}$ and $\hat{G}'_{ij}$.

Since $P_i = P'_i = 1$ for nonitemizers, $\ln P_i = \ln P'_i = 0$ and the calculated values of giving by nonitemizers are:

$$\ln \hat{G}_{ij} = \alpha_j + \beta_j \ln \hat{Y}_i + u_{ij} \tag{14}$$

and

$$\ln \hat{G}'_{ij} = \alpha_j + \beta_j \ln \hat{Y}'_i + u_{ij} \tag{15}$$

The previous calculation of Y'_i for itemizers only must be modified to reflect the reduced tax rate for nonitemizing taxpayers. The percentage reduction in all tax rates (for both itemizers and nonitemizers) is the ratio of the extra revenue generated by eliminating the deduction ($2.14 billion in 1962) to the total income tax receipts for that year (45 billion) or 0.047.

The simulation method is more complex if the deduction is replaced by a tax credit. The substitution of a 30 percent tax credit for the deduction makes the price of giving $P = 0.7$ for all taxpayers, including

[23] Feldstein and Clotfelter (1974), using survey data on giving by itemizers and nonitemizers, show that there is little difference in their income and price elasticities and that a variety of economic and demographic factors (including homeownership and employment status) have no effect on giving when income and price are taken into account.

both itemizers and nonitemizers. The implied contributions are then:

$$\ln G'_{ij} = \alpha_i + \beta_j \ln Y'_i + \gamma_j \ln (0.7) + u_{ij} \tag{16}$$

for itemizers and

$$\ln \hat{G}'_{ij} = \alpha_j + \beta_j \ln \hat{Y}'_i + \gamma_j \ln (0.7) + u_{ij} \tag{17}$$

for nonitemizers. The values of Y'_i and $\hat{Y}'_i$ are now more complicated to calculate than they were for the elimination of the deductible because the change in the tax rate depends on the cost of the tax credit which in turn depends on the amount of charitable giving. Since the amount of charitable giving itself depends on the change in the tax rate, an iterative procedure must be used to solve the simultaneous system of interdependent relations.

The new procedure begins by calculating the net change in tax revenue that would result from substituting the tax credit for the deduction if all giving remained unchanged: $\sum_i N_i (1-P_i) \sum_i G_{ij} - 0.3[\sum_i N_i \sum_j G_{ij} + \sum_i \hat{N}_i \sum_j \hat{G}_{ij}]$ where N_i is the number of itemizing taxpayers in income class i and $\hat{N}_i$ is the number of nonitemizing taxpayers in that class. This difference represents a small net loss in revenue. The ratio of this change in tax revenue to the total tax receipts provides a "trial" value of the factor by which all tax rates can be changed to keep government revenue constant. The corresponding "trial" values of income, Y_i^T and $\hat{Y}_i^T$, are then used in equations 16 and 17 to estimate the corresponding trial values of giving G_{ij}^T and $\hat{G}_{ij}^T$. These new values of giving imply a different change in tax revenue: $\sum_i N_i(1 - P_i)\sum_j G_{ij} - 0.3\ [\sum_i N_i \sum_j G_i{}^T{}_j + \sum_i N_i \sum_j G_i{}^T{}_j\ ]$. The ratio of this revenue change to the total tax revenue provides a new and better estimate of the appropriate factor by which to change all tax rates. Although this process might be repeated again, the additional accuracy that could be gained at this stage is very small and too small to warrant the additional computations. This factor is therefore used to evaluate Y'_i and $\hat{Y}_i$. Equations 16 and 17 then imply the values of G'_{ij} and $\hat{G}'_{ij}$. Finally, total giving to charities of type j is $\sum_i (N_i G'_{ij} + \hat{N}_i \hat{G}'_{ij})$.

TABLE 4.—*Effects of eliminating the charitable deduction on total contributions by type of charity*

Type of charity	Total 1962 gifts[1] ($ million) With deductions	Deduction eliminated (I)	(II)	(III)	Percentage change (I)	(II)	(III)
Religious organizations	7,942	7,383	6,283	6,920	−7	−21	−13
Educational institutions	305	108	159	118	−65	−48	−61
Hospitals	113	43	67	52	−62	−41	−54
Health and welfare organizations	1,572	1,233	1,218	1,216	−22	−23	−23
All others	1,969	1,064	1,485	1,368	−46	−25	−31
Total	11,901	9,830	9,214	9,674	−17	−23	−19

[1] Total 1962 gifts by itemizers and nonitemizers. The estimated gift with the deduction reflects the actual total gifts of itemizers and the estimated 1962 gifts of nonitemizers. See text for methods and definitions of Specifications I, II, and III.

Table 4 presents the effects on the total contribution to each type of charity of eliminating the charitable deduction. The results are presented separately for each of the three specifications. Although there are some substantial differences in the implications of the alternative specifications, the basic pattern is the same for all three specifications; the differences among the specifications are small in comparison to the differences in impact among the types of charities. Total giving is calculated to fall by approximately \$2.3 billion or 20 percent of total 1962 contributions.[24] Gifts to religious organizations are predicted to fall by approximately 14 percent (if all three specifications are given equal weight). The unconstrained specification I implies a decrease of only 7 percent. In contrast, educational institutions are predicted to lose between 48 and 65 percent. Hospitals are also predicted to suffer dramatic reductions in contributions of between 41 and 62 percent.

Substituting a 30 percent tax credit for the current charitable deduction would increase total charitable giving by approximately 17 percent, from \$11.9 billion in 1962 with the deduction to \$13.9 with the tax credit. The total cost of the tax credit would be \$4.17 billion, nearly twice the \$2.14 billion revenue loss with the charitable deduction. The additional loss in revenue by the Treasury is equal to the additional

TABLE 5.—*Effects of substituting a 30 percent tax credit for the charitable deduction on total contributions by type of charity*

Type of charity	Total 1962 gifts[1] ($ million) (Deduction)	Tax credits (I)	Tax credits (II)	Tax credits (III)	Percentage change (I)	Percentage change (II)	Percentage change (III)
Religious organizations	7,942	8,783	9,776	9,163	+11	+23	+15
Educational institutions	305	238	246	237	−22	−19	−22
Hospitals	113	103	104	103	−9	−8	−9
Health and welfare organizations	1,572	1,885	1,895	1,897	+20	+21	+21
All others	1,969	2,716	2,310	2,374	+38	+17	+21
Total	11,901	13,725	14,331	13,775	+15	+20	+16

[1] Total 1962 gifts by itemizers and nonitemizers. The estimated gift with the deduction reflects the actual total gifts of itemizers and the estimated 1962 gifts of nonitemizers. See text for methods and definitions of Specifications I, II and III.

gain in revenue by the donees. Table 5 shows that despite this \$2.1 billion increase in total charitable giving, educational institutions are predicted to suffer a fall of some 20 percent in their receipts. In contrast, religious organizations, health and welfare organizations, and the residual category are all predicted to gain.

---

[24] This is a substantially smaller reduction than the 34 percent decrease for itemizing taxpayers only that was reported in Feldstein (1974a). This is not at all surprising since eliminating the deduction actually causes an increase in giving by nonitemizers. It is quite close to the 26 percent reduction estimated for all taxpayers in Feldstein and Clotfelter (1974). The higher value in that study probably reflects the allowance made there for gifts of appreciated assets; see page 9 above.

To avoid an increase in tax rates, the tax credit would have to be approximately 20 percent. The resulting increase in price would still leave religious organizations with a predicted gain but would imply an even larger fall in gifts to educational institutions and hospitals and might turn the effect on the residual "all others" category to a loss.

To understand the differential effects on different types of donees, it is useful to decompose the aggregate estimates of Tables 4 and 5 by income class. Table 6 shows the predicted effects of the two tax proposals on religious organizations, educational institutions and all remaining types of donees (i.e., hospitals, health and welfare organizations and "all others"). The results are presented only for the unconstrained Specification I. Of the 62 million taxpayers in 1962, some 56 million had incomes below $10,000. Only 38 percent of this group filed itemized returns and those who did had very low marginal rates. The elimination of the deduction therefore would reduce giving in this group by only 9 percent (from an average gift of $146 per taxpayer to $133) while a 30 percent tax credit would increase giving by nearly 25 percent (to an average gift of $183). This group contributed an average of $106 to religious organizations and accounted for some 74 percent of total religious giving. This is the basic reason for the insensitivity of total religious gifts to the charitable deduction and for the increase in such gifts when the deduction is replaced by a 30 percent tax credit.

The low income groups make extremely small average contributions to educational institutions. Although the tax credit causes a large proportional increase in educational gifts (because of the high price elasticity), the average gift per taxpayer still remains less than three dollars. In contrast, the wealthiest 150,000 taxpayers (with incomes over $50,000) contributed an average of $908 and accounted for 44 percent of total educational gifts. The substitution of the 30 percent tax credit for the current deduction substantially increases the price of giving for this group and reduces their education gifts by a predicted 87 percent to an average of only $116.

5. Conclusions

The conclusions of this analysis are clear and easy to summarize. Several studies have now shown that the volume of gifts to all charities combined is quite sensitive to the cost of giving that is implied by the income tax. This is confirmed by the price elasticities for individual types of donees that were estimated in the current study. These individual equations imply that eliminating the charitable deduction would cause charities to lose slightly more revenue than the Treasury would gain in additional taxes, also a basic conclusion of the previous studies.

The sensitivity of charitable giving to potential tax changes differs substantially among the major types of donees. Gifts to educational

TABLE 6.—*Effects of alternative tax treatments on average contributions to educational, religious and other donees: Specification I*

[Average gifts by all taxpayers[1]]

Adjusted gross income ($000)	Total number of taxpayers (000)	Religious Organizations			Educational Institutions			All Remaining Donees		
		Actual 1962	Deduction eliminated	Tax credit 30%	Actual 1962	Deduction eliminated	Tax credit 30%	Actual 1962	Deduction eliminated	Tax credit 30%
0–5	33,699	$ 79.5	$ 76.1	90.9	$ 0.5	$ 0.4	$ 0.9	$ 24.7	$ 21.7	$ 41.6
5–10	21,527	150.1	140.6	167.2	2.4	1.7	3.9	60:0	46.3	88.4
10–15	4,930	230.6	212.1	251.7	5.2	3.4	7.6	98.4	70.6	134.4
15–20	1,045	313.4	279.0	330.8	17.3	9.8	21.5	175.3	111.1	211.2
20–50	941	468.4	381.8	451.2	61.5	22.8	49.9	376.2	169.4	320.7
50–100	121	875.5	561.8	659.3	360.7	43.9	94.9	1,514.0	274.0	513.6
100–500	26	1,803.1	892.6	1,048.0	2,427.7	87.9	190.1	8,206.3	553.5	1,037.4
500–1000	0.8	4,365.8	1,682.6	1,982.2	14,518.1	164.1	357.3	51,341.1	1,342.9	2,528.7
1000+	0.3	7,673.5	2,963.8	3,522.2	47,312.0	537.9	1,191.0	210,279.8	5,520.3	10,529.2
All incomes	62,290	127.5	118.5	141.0	4.9	1.7	3.8	58.7	38.6	73.6

[1] Average 1962 gifts by itemizers and nonitemizers. The estimated gift with the deduction reflects the actual average gifts of itemizers and the estimated average gifts of nonitemizers. See text for methods and definition of Specification I.

institutions and hospitals are very sensitive to the cost of giving while religious organizations are much less sensitive than the others. Eliminating the charitable deduction would reduce total individual giving by an estimated 20 percent, but religious gifts would fall by only some 14 percent while gifts to educational institutions and hospitals would be cut approximately in half. Although replacing the current deductible by a 30 percent tax credit would increase total giving by some 15 percent, educational institutions and hospitals would still lose about 20 percent of current gifts.

These empirical results have important implications for tax policy in this area. Earlier proposals to eliminate the charitable deduction were based on the false presumption that doing so would have little or no effect on charitable giving. The more recent suggestions to replace the charitable deduction with a tax credit or matching grant need not reduce overall giving but would substantially change the distribution of giving among different types of charities. A tax credit or matching grant that either maintained the current total giving or the current total cost to the Treasury would sharply reduce the gifts received by educational institutions, hospitals and other donees currently favored by high income donors.

It is appropriate to conclude by noting that the current analysis of giving to different types of charities is based on a small number of aggregate observations by income class for 1962. The empirical results are therefore much less compelling than the conclusions about total giving that have been supported by a wide range of both microeconomic and aggregate data. It is reassuring, however, that the results are quite insensitive to the choice among three quite different specifications, that the aggregate implications are very similar to the results obtained with much richer sets of microeconomic and aggregate data, and that previous microeconomic studies have shown no change in price and income elasticities between 1962 and 1970. It would nevertheless obviously be very useful to repeat the current study with newer and better data.

Until such future research is done, the current analysis is the only available evidence of the likely effects on different types of charities of the proposed tax changes. This evidence indicates that there seems to be no practical way to maintain the current levels of private support for higher education, hospitals and community cultural activities without a method like the charitable deduction that makes the price of giving decline with income.[25] If the deduction is eliminated or replaced by a

---

[25] McDaniel (1972a, 1972b) has suggested a variable rate tax credit that would make the rate of credit an increasing function of the ratio of giving to income. Higher income taxpayers now contribute a substantially greater portion of their incomes. If they continued to do so after this tax credit was substituted for the deduction, they would receive relatively higher tax credits and thus have relatively lower prices for charitable giving than lower income households. In principle, the resulting distribution of giving would more closely approximate the current pattern. However, the observed correlation of

tax credit, the levels of activity in these areas will decline unless new sources of finance are found. It is not clear that this can be done in a way that maintains the current diversity and pluralism in the provision of these services.

References

1. Andrews, William D., "Personal Deductions in an Ideal Income Tax," Harvard Law Review, 86:2 (December 1972), pp. 309–385.
2. Bailey, Martin J., "Capital Gains and Income Taxation," in A. Harberger and M. Bailey (eds.), The Taxation of Income from Capital. Washington: The Brookings Institution, 1969.
3. Bittker, Boris I., "The Propriety and Vitality of a Federal Income Tax Deduction for Private Philanthropy," in Tax Impacts on Philanthropy. Princeton, N.J.: Tax Institute of America, 1972.
4. Feldstein, Martin S., "The Income Tax Charitable Contributions: Part I—Aggregate & Distributional Effect," National Tax Journal, March, 1975.
5. Feldstein, Martin S. and Charles Clotfelter, "Tax Incentives and Charitable Contributions in the United States: A Microeconometric Analysis," Harvard Institute of Economic Research Discussion Paper No. 381, September 1974.
6. Feldstein, Martin S. and Amy K. Taylor, "Taxation and Charitable Contributions: An Analysis of Individual Tax Return Data for 1962 and 1970," *Econometrica* (November 1976).
7. Goode, R., The Individual Income Tax. Washington: The Brookings Institution, 1964.
8. Kahn, C. Harry, Personal Deductions in the Federal Income Tax. Princeton, N.J.: Princeton University Press, 1960.
9. McDaniel, Paul R., "An Alternative to the Federal Income Tax Deduction in Support of Private Philanthropy," in Tax Impacts on Philanthropy. Princeton: Tax Institute of America, 1972a, pp. 171–209.
10. ———, "Federal Matching Grants for Charitable Contributions: A Substitute for the Income Tax Deduction," Tax Law Review, 27:3 (Spring 1972b), pp. 377–413.
11. Pechman, Joseph A., Federal Tax Policy. New York: W. W. Norton & Co., Inc., 1971.
12. Rabin, "Charitable Trusts and Charitable Deductions," New York University Law Review, 41 (1966), pp. 912–25.
13. Simons, Henry C., Personal Income Taxation. Chicago: University of Chicago Press, 1938.
14. Surrey, Stanley S. et al., Federal Income Taxation. Mineola, N.Y.: The Foundation Press, Inc., 1972.

---

income and the share of income given to charity reflects the current system of deductions rather than an inherent tendency of high income taxpayers to donate a higher portion of their incomes. The econometric evidence indicates that the income elasticity of charitable giving is less than one, i.e., if all households faced the same cost of giving, lower income households would contribute a large portion of their incomes (although obviously smaller absolute amounts). These lower income households would then, under the McDaniel proposal, receive a higher rate of tax credit. Unless the high price elasticity of giving to educational institutions and other nonreligious organizations dominated the basic preference of low income households for religious and community welfare organizations, the McDaniel proposals would only make the share going to religious and community welfare organizations larger than it would be with a flat rate tax credit or matching plan. An alternative proposal, suggested to me in correspondence by Nicholas Tideman, would provide differential rates of tax credit for different types of charities. Although this could in principle achieve any desired level of contribution to each type of charity, I suspect that achieving the current pattern would require either tax credits that are nearly equivalent to the current deductions or a much larger cost to the Treasury.

15. Vickrey, William S., "One Economist's View of Philanthropy," in Frank Dickinson (ed.), Philanthropy and Public Policy, 1962, pp. 31–56.
16. ———, "Private Philanthropy and Public Finance," Mimeograph, 1973.
17. Wald, A., "The Approximate Determination of Indifference Surfaces by Means of Engel Curves," Econometrica, 8 (April 1940), pp. 144–175.
18. Weidenbaum, Murray L., "A Modest Proposal for Tax Reform," The Wall Street Journal, April 4, 1973, p. 18.
19. White, Melvin I., "Proper Income Tax Treatment of Deductions for Personal Expense," in Tax Revision Compendium, Compendium of Papers on Broadening the Tax Base submitted to the Committee on Ways and Means, House of Representatives, U.S. Congress. Washington: U.S. Government Printing Office, 1959, Vol. 1, pp. 370–71.
20. U.S. Treasury, Tax Reforms Studies and Proposals, U.S. Congress, House Ways and Means Committee and Senate Finance Committee, 91st Congress, First Session, 1969.

The Impact of a Tax Cut on Interest Rates and Investment: Crowding Out, Pulling In and All That

Patric H. Hendershott

Purdue University

The Impact of a Tax Cut on Interest Rates and Investment: Crowding Out, Pulling In and All That[1]

Patric H. Hendershott

Purdue University

The possibility of the 1975 Federal tax cut crowding out private investment has been debated vigorously, largely on the pages of the *Wall Street Journal* (*WSJ*). In its most extreme form, the *WSJ* argument is that private investment will be crowded out dollar-for-dollar by consumption induced by the tax cut, unless the deficit is financed by the Federal Reserve. A less extreme position is that some investment will be crowded out, thus slowing future economic growth. To use *WSJ* terms, the economy will be consuming its "seed corn."

While the extreme form of the argument may have considerable validity during a period of close to full employment of resources, it would seem to be utterly without merit when substantial excess capacity exists. Moreover, under the latter circumstances even the modified crowding-out argument may be incorrect. It is true that the deficit will raise interest rates, and investment is negatively related to rates, but the deficit may not raise the rates (long term) to which investment is negatively related. Also, the argument fails to allow for an investment accelerator. When enough excess capacity exists, firms will not invest at any level of interest rates. What is needed to induce investment is the expectation that the goods produced with the new capital will be profitably marketed. To the extent that the tax cut raises consumption outlays and expectations of greater sales, investment will be "pulled in."

The net effect of increased interest rates crowding out investment and of income increases pulling it in would seem to depend on a number of factors in addition to the form of the investment functions (the interest rate and income responses). These include the method of financing the deficit (long-term financing will drive up the long-term interest rates upon which investment depends) and the strength of the

[1] This paper is written for the Office of Tax Analysis of the Department of the Treasury under contract 05-1109. The assistance of Harvey Galper, Ralph Bristol Jr. and James Winder is gratefully acknowledged. An extended version of this paper appears in the December 1976 issue of *Journal of Finance.*

consumption response (a smaller response means a smaller accelerator effect).[2]

The present paper attempts to shed some light on these issues by simulating an econometric model under various assumptions regarding the financing of and expenditure responses to a tax cut. The financial sector discussed in Section I is a quite detailed 47 stochastic-equation flow-of-funds model.[3] The real sector is an extremely small 8 equation aggregate demand model (infinite elasticity of supply is assumed) put together in conjunction with the Office of Tax Analysis of the United States Department of the Treasury. The latter consists of saving functions for households, businesses, the Federal government (tax receipts) and the rest of the world (imports) and investment relations for housing, plant and equipment, inventories, and State and local Government construction. These equations will be introduced as needed.

The simulation results are presented in sections II and III. Section II describes "pure" multiplier experiments assuming different propensities to consume the tax cut (0.0, 0.4, and 0.8) and different methods of financing (all short term, all long term, and 50/50). Experiments with endogenous investment functions, both with and without accelerator responses, are reported in Section III. The pure multiplier simulations are particularly useful to understanding the working of the financial model and to illustrating some basic propositions. The experiments with investment responses are, of course, of greatest interest.

I. The Flow of Funds Model

The model is novel in both its scope (number of sectors and markets analyzed) and its internal consistency. While this is hardly the place for a detailed discussion of the model, a few paragraphs about the form and structure of the model would be useful.

Twelve sectors and markets/securities are delineated. The nine endogenous sectors are households, nonfinancial businesses, state and local governments, federally sponsored credit agencies, commercial banks, savings and loan associations, mutual savings banks, credit unions, and other finance. The rest of the world, Treasury, and Federal Reserve are treated as exogenous. Primary securities in the model are divided into three all-inclusive categories; short term (open market paper, Treasury bills, consumer credit and short-term bank loans), long term other than home mortgages (corporate, foreign, Treasury and

---

[2] The willingness of the monetary authority to finance the deficit, thereby preventing a rise in real interest rates (and thus crowding out) and stimulating a larger rise in real income (and thus pulling in), is probably of the greatest importance. Since this seems to be well recognized, we will not dwell on it.

[3] The model equations are presented in "A Brief Description of a Flow of Funds Model," Krannert Institute Paper No. 509, Purdue University, April 1975 and "Revised Estimates for the Nonbank Finance Sectors," mimeo, June 1975.

state and local bonds, nonhome mortgages, term loans and equities), and home mortgages. The primary security rates associated with the markets for the two aggregate security groupings are the 4–6 month prime commercial paper rate and the new issue corporate bond yield. Secondary or intermediary financial markets include money, life insurance policy loans, eurodollar loans, bank savings accounts, savings and loan shares, mutual savings bank deposits, credit union shares, contractual saving (insurance and pension fund reserves), and trade credit. The yields on the first seven intermediary instruments are treated as exogenous (the first two are constant), and the last two markets are taken to be exogenous in their entirety.

The model contains 47 behavioral equations, including 31 sectoral demand equations and fourteen sectoral supply equations. The primary determinants of sectoral issues and purchases are interest rates and nonfinancial investment and saving (for the nonfinancial sectors) or increments to deposits/contractual saving (for the financial sectors).

The internal consistency of the model reflects the two fundamental identities underlying the flow-of-funds accounting structure. First, every economic unit or sector must spend on goods or financial assets all of its receipts, or, put another way, the sector must finance all of its expenditures. Second, for every seller there is a buyer, or financial instruments that are issued must be purchased. The first identity is the sources-equal-uses or balance-sheet constraint for a unit or sector; the second is the supply-equals-demand or sales-equal-buys constraint for a market or instrument. The sources-equals-uses constraint has been imposed by employing a simultaneous procedure in the estimation of each sector's financial behavior. The issues-equal-purchases constraints are used to determine the three primary security rates in the model. Not only is this a more straightforward approach to interest rate determination, but, in contrast to the standard bank-reserves financial model, it allows for the operation of all "preferred habitats" on interest rate spreads and the analysis of debt management types of operations such as the Federal Reserve's "Operation Twist" and Federal agency support of the home mortgage market (i.e., the issuance of short-term securities to finance purchases of home mortgages). Since the entire financial behavior of each participant in every market is explained, the impact of any policy action on every market and on each sector is completely specified.

Before turning to the simulations, a few words about the empirical parameters of the model are in order. In particular, the interest rate substitution in the model is probably not as large as some might expect. There is significant substitution between longs and home mortgages on the part of mutual savings banks and other finance (life insurance companies) and between both longs and shorts and savings accounts (shorts for bank accounts and longs for nonbank) by households. Also, shorts and mortgages are effective substitutes due to the willingness of Federally sponsored credit agencies to issue shorts to purchase mort-

gages when the yield on the latter rises. While there is some short-run substitution between longs and shorts on the supply side due to regressive interest rate expectational responses, there is little long run direct substitution.

In spite of the relatively low interest rate elasticities, the model is able to explain the strong comovement in interest rates during the estimation period.[4] This may be caused by the tendency of security supply schedules to shift simultaneously in the same direction due to either joint security supply responses to a given investment change (e.g., an increase in consumer durable investment is financed by issues of consumer credit and home mortgages) or to a tendency for various investment components to rise and fall together. We might also take some comfort in recent data suggesting that interest rates may not move as closely as we thought, and thus that securities may not be such close substitutes. While short-term interest rates always decline by greater magnitudes than long-term rates during recessions, the 5.57 percentage point decline in the commercial paper rate from its third quarter peak relative to the 71 basis point drop in the corporate bond rate is astounding.[5] Nonetheless, we would be less than candid if we did not acknowledge that the model probably understates the interest rate substitution between longs and shorts.

II. Multiplier Simulations

The period chosen for the simulations of a tax cut is the period in the last recession during which the NIA deficit was greatest. After an $8.1 billion surplus in 1969, a deficit appeared in the first quarter of 1970 and rose, almost monotonically, through the fourth quarter of 1971, after which it receded. The deficit and personal disposable income are both increased by $5 billion (a *quarter* rate) in the third and fourth quarters of 1970 and the first and second quarters of 1971, for an annual total of $20 billion. The model is simulated first without the tax cut (the control simulation) and then with it. Commercial bank reserves are held constant throughout (the Federal Reserve finances none of the deficit).

The results of a number of simulation experiments are reported in this section. In each of these investment is constant. The experiments differ with respect to the assumptions made regarding households' consumption response to the tax cut and the maturity of the securities issued to finance the tax cut. The three consumption assumptions are: no increase—$MPC = 0.0$, a small increase—long-run MPC of 0.4, and a large increase—long-run MPC of 0.8. The alternative financing as-

[4] See P. H. Hendershott and F. S. Orlando, "Interest Rate Behavior of Flow-of-Funds and Bank-Reserves Financial Models," *Journal of Money Credit and Banking*, November 1976.

[5] Commercial paper rates were 11.72% in July 1974, 6.15% in April 1975; corporate (FRB new issue series) rates were 10.38% in September 1974, 9.67% in April 1975.

sumptions are all short issues, all long issues, and half and half. The results will be discussed largely in terms of the three primary security rates in the model. The six quarters of simulation will generally be divided into three half-year periods—the first half-year of tax cut, the second half-year, and the half-year after the tax cut is removed.

A. Complete saving of the tax cut

We begin with the extreme case of the entire tax cut being saved by households. Before examining the simulated impact on interest rates, it is instructive to consider several possible outcomes. If households purchase only the type of securities the Treasury issues, then the tax cut would have no impact on interest rates. The demand and supply for this type of security would rise equally, while the demand and supply for other securities would be unchanged. If, on the other hand, households bought some primary securities of maturity other than those issued by the Treasury, we would expect the yields on these other securities (say long terms) to decline and the yields on securities of the maturity issued by the Treasury (say short term) to rise. The tax cut would in this case be equivalent to a continuing debt-management operation. A third possibility is that households will tend to direct some of the saving to commercial bank deposits. Since commercial banks hold reserves against these deposits and since the Federal Reserve is assumed *not* to supply additional reserves, this increase in deposits causes a shortfall in security demand, raising interest rates. In fact, open market (short term in our model and in most other financial sectors) interest rates will rise sufficiently to restrain the demand for required reserves (deposits) to equal the unchanged supply. To the extent that substitution between short term and other securities exists, on either the demand or supply side, other (bond and mortgage) rates will tend to rise sympathetically.

Households will, of course, purchase a wide variety of financial assets with the increased saving; if interest rates are unchanged, over half is ultimately (taking into account the portfolio preferences of financial intermediaries) directed at long-term securities, with the rest divided about equally between shorts and home mortgages (via savings accounts). Thus, the impact on interest rates could be either a rise in the yields on the type (maturity) of securities the Treasury issues and a decline in other yields (the dominance of the "debt-management" effect) or a rise in all rates (the dominance of the "intermediation" effect). Table 1 suggests that the result depends on the maturity of the Treasury issues. When the Treasury is assumed to issue short-term securities only, the commercial paper rises markedly (1 to 1½ percentage points) and the corporate bond and mortgage rates decline modestly (10 to 20 basis points). When the Treasury issues only long terms, all interest rates rise, the paper and bond rates by about 40 basis points (during the year of financing) and the home mortgage rate by 10

TABLE 1.—*Impact of a \$5 billion (per quarter for 4 quarters) tax cut under different (short versus long) financing assumptions, where all of the tax cut is saved*
[In basis points]

	Commercial paper rate		Corporate bond rate		Home mortgage rate	
	Short	Long	Short	Long	Short	Long
1st half-year	94	37	−2	34	−5	3
2nd half-year	145	44	−10	48	−17	9
3rd half-year	108	24	−12	28	−21	24

basis points. The paper rate rises largely for the reason noted above—the need to restrain the tendency for a rising demand for required reserves. The home mortgage rate rises (ever so slightly) because mortgages and bonds are relatively close substitutes in the portfolios of mutual savings banks and life insurance companies. In the third half-year, when the financing and increasing deposit demand cease, the corporate and paper rates decline toward their initial levels. The mortage rate, in contrast, continues to be "pulled up" by the higher bond rate.

Before moving on to simulations with positive consumption responses, it is probably useful to emphasize an important proposition that follows from our model and to contrast our results with those that bank-reserves financial models would provide. The proposition is: *a tax could lower long-term interest rates if it is financed by short-term security issues*. Regarding bank reserves models, if the demand for money (required reserves) depends on GNP only, then the tax cut would not affect it or anything else. If the demand for money depends on disposable income and/or wealth, then short-term interest rates would rise directly and long-term rates would be pulled up through various term-structure and risk-structure relations. Since a debt-management operation, even a continuing one, has no impact in existing bank-reserves models, there is no mechanism by which any interest rate could decline.

B. Positive consumption responses

We now turn to an analysis of the impact on interest rates of alternative assumptions about the response of consumption to the tax cut. To do this, a multiplier matrix must be specified. Equations (1H) and (1L) are alternative high and low incremental consumption (ΔCON) functions:

$$\Delta CON = .5\Delta Yh + .1\Delta Yh_{-1} + .1\Delta Yh_{-2} + .1\Delta Yh_{-3} \qquad (1H)$$

$$\Delta CON = .1\Delta Yh + .1\Delta Yh_{-1} + .1\Delta Yh_{-2} + .1\Delta Yh_{-3} \qquad (1L)$$

Household saving (SAV_h) is, of course, simply the difference between disposable income and consumption:

$$\Delta SAV_h \equiv \Delta Yh - \Delta CON. \qquad (2)$$

The remainder of the matrix consists of savings functions for business (SAV_b = cash flow), government (SAV_g = tax receipts less expenditures), and the rest of the world (SAV_r = imports less exports), the identity that the change in total savings is zero (because investment is assumed to be unchanged) and the definition of household disposable income Yh:

$$\Delta SAV_b = .1\Delta Y \tag{3}$$

$$\Delta SAV_g = .2\Delta Y - X \tag{4}$$

$$\Delta SAV_r = .1\Delta Yh \tag{5}$$

$$\Delta SAV_h + \Delta SAV_b + \Delta SAV_g + \Delta SAV_r \equiv 0 \tag{6}$$

$$\Delta Yh = \Delta Y - \Delta SAV_b - \Delta SAV_g = .7\Delta Y + X \tag{7}$$

Cash flow and imports are each assumed to be one-tenth of GNP and of disposable income, respectively, and Federal tax receipts are assumed to rise by one-fifth of any increase in GNP less any tax cut. The change in disposable income is simply the change in total income less the changes in business and government saving. The long-run tax multipliers implicit in the matrix are, for MPC = .4 and .8, 0.38 and 1.37.[6]

TABLE 2.—*Impact of a $5 billion (per quarter for 4 quarters) tax cut with low and high consumption responses, where financing is short-term and investment is constant*
[In basis points and billions of dollars]

	Commercial paper rate		Corporate bond rate		Home mortgage rate		GNP (quarterly rate)	
Period	Low MPC	High MPC	Low MPC	High MPC	Low MPC	High MPC	Low MPC	High MPC
1st half-year	93	89	0	2	−14	−12	0.7	3.7
2nd half-year	142	126	−9	−7	−27	−21	1.8	5.9
3rd half-year	98	73	−14	−14	−17	−7	1.5	3.1

Table 2 presents the impacts for small (low MPC) and large (high MPC) consumption responses, assuming the deficit is financed by short-term security issues. As can be seen, the impacts on interest rates differ little from each other (and—referring back to Table 1—from the impact when the tax cut is assumed to be saved in its entirety). The same result holds when long-term financing is employed. This will be surprising to those who view the demand for money as purely a transactions demand; the higher income level tends to raise the demand for money and thus bank reserves, necessitating a rise in short-term interest rates to reduce demand to the fixed supply. Offsetting the impact of higher income on the demand for money is

[6] Two additional relationships are used in the model: consumer durable outlays are assumed to be 16 percent of consumption and gross corporate product is assumed to be 56 percent of GNP.

lower household saving (money demand depends on income *and* wealth). In these simulations, total saving—that of households, businesses (retained earnings), foreigners (the trade deficit), and the government (the Federal surplus)—equals a fixed (by assumption) level of investment. As a higher portion of the change in household disposable income generated by the tax cut is spent, household saving decreases and retained earnings, imports and the surplus increase. The changes in the sectoral components of saving under the different consump-

TABLE 3.—*Changes in the saving matrix in the fourth quarter of a $5 billion (per quarter) tax cut under different assumptions regarding the consumption response*
[In billions of dollars]

	Zero MPC	Low MPC	High MPC
Personal saving	5.0	4.1	2.4
Business saving (Retained earnings)		0.2	0.6
Foreign saving (US imports less exports)		0.3	0.7
Federal saving (Surplus)	−5.0	−4.6	−3.7

tion assumptions are listed in Table 3. As the consumption response changes from a low to a high *MPC*, the lower household saving tends to lower the demand for bank deposits, while the greater increase in income raises the demand. The simulation results suggest that the effects are largely offsetting and lead us to another proposition: *a larger consumption and thus income response to a tax cut does not necessarily imply higher interest rates.*

Another set of simulations explores the impact of alternative financing methods in more detail. A high propensity to consume is assumed and investment is still constant. In addition to issuing all short-term or all long-term securities, I consider the case where the financing is

TABLE 4.—*Impact of a $5 billion (per quarter for 4 quarters) tax cut under different (short, half and half and long) financing assumptions, with a high propensity to consume and investment constant*
[In basis points]

	Commercial paper rate			Corporate bond rate			Home mortgage rate		
	Short	50/50	Long	Short	50/50	Long	Short	50/50	Long
1st half-year	89	67	42	2	16	32	−12	−5	3
2nd half-year	126	85	46	−7	16	40	−21	−5	13
3rd half-year	73	43	12	−14	2	16	−7	11	29

half short-term and half long-term. The results are reported in Table 4. To reiterate an earlier finding, long-term (including home mortgage) rates fall when only shorts are issued and rise when only longs are issued; short-term rates rise in either case. The 50/50 financing gives interest rate results that are very close to a simple average of the two extremes (the model seems to be quite linear in this respect). As a re-

sult, the bond rate still rises, but the home mortgage rate falls slightly during the period of deficit financing (this rate would be approximately unchanged if two-thirds long and one-third short financing were employed).

At this point we must decide which type of financing and magnitude of consumption response to use in the experiments with endogenous investment outlays. Two considerations seem relevant regarding the financing assumption. First, short-term financing will, in fact, largely be employed. Second, under short-term financing long-term interest rates decline regardless of the nature of the consumption response. Since investment outlays respond (negatively) to long-term interest rates and not to short-term rates, there could not be any "crowding out." While this is no great problem *per se* (it would, of course, shorten the present paper), the fact that the result is likely due to an understatement of the substitution between short- and long-term security demands in the model is a problem. Thus, 50/50 financing, while not likely to occur, yields results which seem most believable. However, even under 50/50 financing the home mortgage rate declines during the first year.[7] Since housing is the most interest-sensitive investment component (see below), crowding out will be negligible even in this case.[8] To us, this virtually destroys the case for crowding out in the absence of an abnormal proportion of long-term financing. What remains to be seen is whether or not the case has any substance even when long-term financing is employed.

The importance of the magnitude of the consumption response to the simulation results with investment functions is clear. A high response (and thus tax cut multiplier) means a large accelerator and thus a greater pulling in effect of the tax cut. The high (.8) propensity to consume seems appropriate for normal changes in disposable income during normal times. However, the low (.4) propensity might well be the expected response to a possibly temporary tax cut during a deep recession when households feel great pressure to reduce their debts. There is another reason for employing the low consumption assumption throughout. During a deep recession, when considerable excess capacity of plant exists, the accelerator itself is unlikely to be as powerful as it would be when the economy is closer to full employment. Firms could calculate that expected increases in sales could be met by operating existing plants more fully and, thus, that increases in plants were unnecessary. In order not to overstate the pulling-in effect of the accelerator, the low propensity to consume is employed in the investment simulations.

---

[7] That a continuing issue of $2½ billion per quarter of long-term securities would raise the long-term rate by 1/5 percentage point relative to the home mortgage rate does not seem at all unreasonable.

[8] The cumulative crowding out of investment does not reach $100 million until the sixth quarter of the simulation.

III. Investment Responses

In this section we introduce investment functions having negative partials with respect to interest rates (allowing crowding out) and positive partials with respect to changes in income (pulling in). The

TABLE 5.—*Interest rate and accelerator responses of investment components*

	P & E		HOUS	CONSTR	INV
	Rcor	Δ(GNP – P & E)	Rmor	Rcor	Δ(GNP – INV)
t	—	—	—	−45	0.06
t-1	−8	0.047	−448	−75	.24
t-2	−24	.100	−395	−98	.15
t-3	−46	.134	−237	−103	.11
t-4	−71	.137	−279	−98	
t-5	−98	.115	181	−75	
t-6	⋮	.060	⋮	−45	

assumed investment responses are listed in Table 5. There are negative interest rate responses with respect to the corporate bond rate (Rcor) for business plant and equipment (P & E) and for State and local construction (CONSTR) and with respect to the home mortgage rate (Rmor) for housing (HOUS); accelerator relationships are introduced for P & E and inventory investment (INV). Dots (⋮) indicate the distributed lags are longer than the five quarters listed, but only five lags are needed since the simulations are restricted to six quarters.[9]

The interest rate responses for the two investment simulations (with

TABLE 6.—*Impact on interest rates of a $5 billion (per quarter for four quarters) tax cut financed by long-term issues with a low consumption response, investment endogenous*
[In basis points]

	Rcp		Rcor		Rmor	
	No accelerator	Accel-erator	No accelerator	Accel-erator	No accelerator	Accel-erator
1st half-year	38	39	34	34	3	3
2nd half-year	47	50	42	42	13	13
3rd half-year	20	22	24	25	23	24

and without the accelerator) are reported in Table 6. The responses are quite similar to those of the no-investment, long-term financing results presented in Table 1 because the investment responses are relatively small. As in that earlier case, the paper and bond rates are up by about 35 basis points during the first half year and 40–50 points during the second. During the first half year after the tax cut ceases, they return toward their initial values, being only 20–25 basis points higher. The

[9] The medium-term (six quarter) interest rate elasticities of the investment equations are: −1.8 for housing, −0.6 for STL construction and −.07 for plant and equipment.

mortgage rate rises continuously, reaching a maximum of about one-quarter percentage point higher by the end of the simulation.

The impacts on GNP and the investment components are given, by

TABLE 7.—*Impact on income and investment of a $5 billion (per quarter for four quarters) tax cut financed by long-term issues with a low consumption response*
[In billions of dollars at *quarterly* rates]

	GNP		INVENTORIES		P & E	
	No accelerator	Accelerator	No accelerator	Accelerator	No accelerator	Accelerator
1	0.4	0.5	—	0.03	—	—
2	0.9	1.1	—	.14	—	0.02
3	1.4	1.8	—	.22	−.01	.06
4	1.9	2.4	—	.30	−.03	.12
5	1.4	1.9	—	.25	−.05	.18
6	0.8	1.2	—	.02	−.09	.16
Σ	6.8	8.9	0	.06	−.18	.54

	HOUSING		CONSTR		Total fixed investment	
	No accelerator	Accelerator	No accelerator	Accelerator	No accelerator	Accelerator
1	—	—	−.02	−.02	−.02	−.02
2	−.02	−.02	−.04	−.04	−.06	−.04
3	−.03	−.03	−.08	−.08	−.12	−.05
4	−.04	−.05	−.12	−.12	−.19	−.05
5	−.12	−.12	−.16	−.16	−.33	−.10
6	−.17	−.17	−.18	−.18	−.44	−.19
Σ	−.38	−.39	−.60	−.60	−1.16	−.45

quarter rather than half year, in Table 7. The response of total fixed investment–the sum of P & E, housing and State and local construction (CONSTR)—is also listed. The first point of interest is the existence of significant crowding out in the no-accelerator case. By the sixth quarter fixed investment is $440 million less than it otherwise would have been, and the cumulative crowding out is over a billion dollars ($1.16 billion.)[10] The larger interest rate responses of housing and construction imply that these outlays, rather than P & E, receive the brunt of the crowding. While the crowding out is significant, it is hardly a dollar-for-dollar offset to the consumption induced by the tax cut. The decline in fixed investment is one-sixth of the $7 billion increase in consumption, and only 6 percent of the deficits generated by the tax cut.[11]

---

[10] The reader is reminded that the type of financing (long term) that maximizes crowding out has been assumed.

[11] The cumulative tax cut was $20 billion, and the total loss of Treasury revenue—and thus the issue of Treasury securities—was $18.6 billion (household and business tax payments rise in response to the higher level of income and production).

When the high-consumption assumption is employed, crowding out is similar in magnitude (long-term interest rates are about the same). However, the decline in fixed

The existence of an accelerator will, of course, raise GNP. Accelerators work on both inventories and P & E. That for inventories operates quickly, supplying an extra direct lift to GNP of about a quarter of a billion dollars in the third through fifth quarters and then tapering off quickly as the cessation of the tax cut has a dampening impact on GNP. The P & E accelerator operates more slowly, reaching its peak after the inventory accelerator has started to fade. The higher cumulative GNP during the six quarters in the accelerator case relative to no accelerator is $2.3 billion. It follows directly from the extra $1½ billion of inventory and P & E investment and their multiplier effect on domestic consumption. As can be seen in Table 7, the other two investment components, which have not been specified as having accelerators, are virtually unchanged.

Because the concern about crowding out is usually expressed in terms of fixed investment, the increase in inventories will not be viewed as an offset to the crowding out of higher interest rates. The nearly ¾ billion dollar accelerator effect of P & E[12] is an offset, but it is a less than full offset. Total fixed investment is still almost $200 million lower in the sixth quarter, and cumulative fixed investment $½ billion less than it would have been in the absence of a tax cut. The crowding-out effect of the tax cut seems to outweigh the pulling-in effect, when the tax cut is financed in the most detrimental fashion (long-term issues).[13] The decline in investment is, however, less than 5 percent of the increase in consumption.

A fuller description of the impact of the tax cut under the low-consumption, full-investment and long-financing assumptions is given on pages 84–86. Changes in security issues and financial assets, as well as in investment and saving, for households, nonfinancial businesses, state and local governments, and the finance sectors are reported and interpreted.

A final simulation illustrates the importance of the extent to which the Federal Reserve finances the deficit. The Federal Reserve is assumed to finance 5 percent of $1 billion of the tax-cut induced deficit; this consists of $250 million purchases during each of the four quarters the tax cut is in effect. As a result, interest rates are substantially reduced relative to when none of the deficit is financed via money creation, and bank deposits rise markedly. Demand deposits are up over $4 billion and time deposits over $9 billion.

The impact of the tax cut on interest rates is summarized in Table 8. Not only do rates rise much less than in the case of a "pure" tax cut, but

---

investment is a much smaller percentage of the increase in consumption (which is a much greater $25 billion) and larger percentage of Treasury issues (which are now only $13 billion).

[12] The accelerator response is the total response ($540 million) less the interest rate response (–$180 million).

[13] This is not the case when the high MPC is assumed. The greater consumption results in a pulling in effect of $3 billion, triple the crowding out effect.

TABLE 8.—*Impact on interest rates of a $5 billion (per quarter for 4 quarters) tax cut, assuming long-term issues, a low consumption response, full investment response, and a purchase of 5 percent of the issues by the Federal Reserve*

	Rcp	Rcor	Rmor
1st half-year	−25	10	−10
2nd half-year	−60	2	−24
3rd half-year	−65	−17	−27

commercial paper and mortgage rates fall during the period of the cut, and all rates are lower after the tax cut. (Again we emphasize the assumption of excess capacity and thus no increase in inflationary expectations.) Given that housing responds to the decline in the mortgage rate more than P & E and STL construction do to the increase in the bond rate, no crowding out is possible. That is, even when the Treasury finances the deficit solely by long-term issues, a monetary purchase equal to one-twentieth of the issue eliminates any possibility of crowding out.

Summary

The following three conceptual propositions are necessary to an understanding of the simulation results.

(1) The effect of a tax cut on interest rates of securities that are significantly different from those being issued to finance the cut is uncertain. Both "debt management" and "intermediation" effects exist, and they work in offsetting directions. The former results from part of the proceeds of the tax cut being directed at securities other than those issued to finance the cut; the latter is due to the increased demand for bank deposits and thus required reserves and the resultant shortfall in security demand and thus rise in interest rates. If the tax cut is financed by short-term issues and the debt-management effect dominates, excess demand for the long-term securities will develop and their yields will decline. In such an event there cannot be any crowding out.

(2) Given a fixed level of bank reserves, an increased consumption and thus income response to the tax cut may result in either higher or lower interest rates. The greater increase in income raises the incremental demand for money, while the lower household saving (relative to the lower consumption case) dampens it. Thus it is uncertain as to whether the demand for required reserves rises, and interest rates must increase to choke off the demand, or whether demand falls, and rates must decline to increase demand.

(3) While any increase in long-term interest rates caused by the tax cut reduces (crowds out) fixed investment, increases in income raise (pull in) investment via the accelerator mechanism. Thus it is uncertain conceptually whether even a tax cut that raises long-term interest rates will depress or stimulate investment.

Four results of the simulations that bear on the ambiguities summarized above seem worthy of note.

(1) The simulation results suggest that the method of financing (short versus long) is of great importance regarding interest rate, and thus investment, responses. If short-term financing is employed, long-term rates (bond and mortgage) decline (i.e., the debt-management effect dominates). Under long-term financing, both the bond and mortgage rates rise (the short-term rate rises in either case). Since investment outlays are negatively related to long-term interest rates, crowding out can occur only if long-term financing is utilized. While one may not wish to accept this extreme version of the argument, due to a likely understatement of interest rate substitution in the model, it should not be difficult to agree that the method of financing matters. Even those who argue that a one-shot debt-management operation will not permanently affect the relationship between rates (and I would accept such an argument) acknowledge that a continuing operation will. And a continuing tax cut is analogous to a continuing debt-management operation.

(2) The magnitude of the consumption response to the tax cut appears to have little impact on the demand for money and thus on required reserves and interest rates. Empirically the income and wealth responses are almost equal and opposite in sign.

(3) When long-term financing is employed and accelerator relationships are absent, 16 cents of fixed investment is crowded out for every dollar increase in consumption outlays. This result does, of course, depend on the assumption of substantial excess capacity, an assumption that seems approximately correct for the size of tax cut being analyzed and the state of the economy during the first half of 1975.

(4) When accelerator responses are introduced for P & E and inventories, the crowding out effect still dominates, with fixed investment declining by about 5 percent of the increase in consumption.

These empirical results depend greatly on the investment functions employed (as well as on the underlying financial model). Greater interest rate sensitivity would obviously increase crowding out (assuming some long-term financing), while a smaller P & E accelerator—or a lower consumption response—would reduce pulling in. However, exclusive reliance on short-term financing or modest (5 percent) support by the Federal Reserve if the financing is long term would almost certainly prevent crowding out.

Appendix: Sectoral Sources-and-Uses Statements

In this appendix we present and discuss changes in the sources-and-uses statements and in the quantities of assets supplied and demanded for the final, nonmonetary financed, tax cut simulation reported in the paper. This provides a more detailed and complete picture of the impact of a tax cut and of the workings of the financial model. Recall that the $20 billion dollar tax cut was financed by long-term security

issues and that a low consumption response ($MPC = .4$) and both negative interest rate and positive accelerator investment responses were assumed.

The financial asset categories are money (MON), total bank and nonbank savings accounts (SV), home mortgages (MOR), other long-terms (L) and short-terms (S) which for households is subdivided into consumer credit (CC), security credit (SC) and policy loans from life insurance companies (PL). For simplicity, all finance sectors (commercial banks, savings and loans, mutual savings banks, credit unions, federally sponsored credit agencies, and other finance) are aggregated. First, we present the investment-saving nexus, where the investment categories are those presented in Table 7 in the text plus net consumer durables outlays. The subscripts on sectoral saving are h (households), b (nonfinancial businesses), s (State and local governments), f (Federal Government) and r (rest of the world). The data (in billions of dollars) include the total cumulation over the six-quarter period. As noted in the text, housing and STL construction get crowded out, while outlays on durables and business fixed and inventory capital rise. The \$18.2 billion decline in Federal saving is more than offset by an increase in household saving (defined broadly to include purchases of durables). Business and foreign saving (retained earnings and the negative of the U.S. trade balance) each rise by nearly a billion dollars, and state and local saving (including construction outlays) is assumed to be unchanged. The complete sectoral sources-and-uses statements are:

Investment and Saving
[Billions of dollars]

HOUS	−0.4	SAV_h + CDUR	18.4
CDUR	1.5	SAV_b	0.9
P & E	0.5	SAV_s + CONSTR	0.0
INV	1.0	SAV_f	−18.2
CONSTR	−0.6	SAV_r	0.9
	2.0		2.0

Households

ΔMON	−0.7	ΔS		0.1
ΔSV	4.1	ΔCC	1.0	
ΔS	1.3	ΔSC	−1.8	
ΔL	13.3	ΔPL	0.9	
HOUS	−0.4	ΔMOR		0.6
CDUR	1.5	SAV_h + CDUR		18.4
	19.1			19.1

Nonfinancial businesses

ΔMON	0.1	ΔS	0.8
ΔSV	−0.8	ΔL	−1.4
ΔS	−0.5	SAV_b	0.9
INV	1.0		
P & E	0.5		
	0.3		0.3

STL governments

ΔMON	0.2	ΔL	−1.4
ΔSV	−0.4		
ΔS	−0.6		
CONSTR	−0.6		
	−1.4		−1.4

Finance

ΔS	−0.2	ΔMON	−0.4
ΔL	2.1	ΔSV	2.9
ΔMOR	0.6		
	2.5		2.5

Federal Government				Rest of world			
		ΔL	18.2	ΔS	0.9	SAV_r	0.9
		SAV_r	−18.2				

We begin the sector-by-sector discussion with households. On the liability side, consumer credit issues rise to finance durable outlays, security credit issues decline sympathetically with the fall in the market value of stocks due to the increase in the bond rate, and policy loans rise due to the increase in the paper rate. Households also respond to equity capital losses by reconstructing their desired portfolio proportions over a period of time (5 years according to our estimates). This is achieved by purchasing more equities (long terms) at the expense of liquid assets and by issuing relatively more debt. The rise in mortgage issues in spite of the fall in housing outlays is partially attributable to this phenomenon. The rise is also partly due to the fact that some durable outlays are mortgage financed. The increase in household saving is largely invested in financial assets. Money holdings decline (the impact of the rise in the paper rate more than offsets the effect of the rise in saving), and savings accounts rise by less than their normal share of saving due to higher paper and bond rates. The large purchase of long-term securities (three-fourths of the new Treasury issues) is partially due to the aforementioned equity capital losses.

Long-term corporate issues decline in spite of the increase in corporate fixed investment because of the rise in saving and of the higher bond rate which causes a temporary shift toward greater short-term financing. The latter and the rise in inventories induce greater short-term issues. The combination of greater investment and smaller issues means that financial asset purchases must decline. The rise in bond rates and the reduction in construction outlays result in a similar decline in state and local government long-term issues and financial asset purchases. The issues and purchases of the aggregate finance sector balance the markets and close the system.

Research Tasks on the Economics of Tax and Other Policies Towards Petroleum

James C. Cox and Arthur W. Wright

University of Massachusetts

Research Tasks on the Economics of Tax and Other Policies Towards Petroleum

James C. Cox and Arthur W. Wright[1]

University of Massachusetts

I. Statement of the Topic and General Approach

The goal of the research discussed here is to understand the effects and interactions of a group of public policies towards the petroleum industry. The policies studied include Federal tax treatment of petroleum corporate income, state "market-demand prorationing" (MDP), the Federal oil import quota, and regulation of field prices for natural gas in interstate commerce by the Federal Power Commission (FPC). Our research on petroleum policies is an outgrowth of the framework for analyzing the economics of public policies which we first developed in our paper on the oil industry's "tax burden" [1]. That framework reveals the necessity of studying not only the effects of a given policy but also its relative effectiveness compared to alternative policies; and the study of relative effectiveness requires empirical estimates of the effects on target variables of the full range of policies studied. The policy target variable implicit in our research to date is "national security" or "energy independence"; proved reserves of crude oil and nonassociated natural gas are used as a measure of this target.

We reported in [2] estimates of the determinants of investment in petroleum reserves for the five states which practice MDP (Kansas, Louisiana, New Mexico, Oklahoma, and Texas). Using these estimates, based on 1959–71 time series data, we were able to conclude that: (1) Federal tax policy probably had a statistically significant positive impact on investment in reserves of oil and gas; (2) state MDP probably had an ambiguous impact on reserves; (3) the oil import quota had a statistically significant positive impact on reserves. In the cases of Federal tax policy and state MDP, the qualifier is necessary because policy changes would not only affect petroleum reserves directly, but would also displace equilibrium market price and quantity, thereby indirectly affecting reserves. If the price elasticity of demand for crude

---

[1] Associate Professors of Economics, University of Massachusetts, Amherst. The financial support of the U.S. Treasury, under Contract No. TOS-75-12, OS-771, is gratefully acknowledged; the views expressed in the paper are those of the two authors and do not represent the official position of the Treasury.

petroleum was greater in absolute value than 0.0659, the indirect effect of Federal tax policy had the same sign as the direct effect. In contrast, for the same value of the price elasticity of demand, the indirect impact of state MDP was opposite in sign to its direct impact. The effect of the oil import quota depends on two estimated coefficients (on price and domestic-quantity-supplied) which are positive in sign; since a change in the import quota would alter both price and quantity in the same direction, the effect of the quota was unambiguous and no qualifier is needed above.

The analysis in [2] has two limitations in terms of the ultimate goal of our research. The first limitation is that our evaluations of tax policy and MDP are quantitatively indeterminate because of the displacements of equilibrium price and quantity when the policy parameters change. The second limitation is that the analysis in [2] is oil-chauvinist because: (a) We do not expect that the market-demand factor is a determinant of nonassociated gas reserves, whereas the results in [2] assume it is; (b) Even though in [2] we allow for the jointness in both finding and producing oil and gas through the use of Divisia indexes of reserves, gross additions to reserves, output, and price, we are not able to deal with the important problem of the effects of the regulation of the field price of natural gas by the FPC. This is because, while MDP can be interpreted as state regulatory commissions picking points off demand curves by limiting output from given reserves, FPC price regulation must be interpreted as generating observations from industry supply curves, to the extent that regulation led to excess demand for gas in interstate markets. We made use of the above interpretation of MDP to justify a single-equation specification of the model in [2], with both price and quantity entering as independent variables—collinear, perhaps, but not simultaneously generated by another estimating equation. Since the FPC regulates field prices but not production of natural gas, we could not make a similar argument for natural gas.

In research supported by the U.S. Treasury, we have been attempting to overcome the two limitations just described. Thus we have sought first to estimate, alternatively, reduced-form equations for price and quantity, and structural demand and supply equations, in order to permit the calculation of the displacements of equilibrium price and quantity when the policy parameters change. Second, we have sought to separate oil (and associated-and-dissolved natural gas) production from nonassociated natural gas production, in order to isolate the effects of FPC price regulation. Third, we have set up a framework for simulating the effects on investment in petroleum reserves when individual policies are varied, *ceteris paribus*, and when more than one policy is varied—as might happen, for instance, if Congress cut the depletion allowance and the Texas Railroad Commission (TRRC) tightened up on the "market-demand factor" to try to protect industry after-tax profits. Let us briefly discuss the general approach we have taken to each of these three topics.

There are two alternative approaches to obtaining the displacement of equilibrium price and quantity when policies change. One approach is to estimate structural demand and supply functions, using two-stage least squares (2SLS) to account for the simultaneous system which lies behind the observed data. The 2SLS approach yields consistent estimates if the sample is sufficiently large; in our case, with only 13 or 14 time-series observations (1959–71 or 1959–72 respectively), we must resort to the pooling of cross-section and time-series data in order to justify the use of 2SLS techniques. The structural supply equation derived from our model is set forth in the appendix to this paper; see equation (46). The structural demand function is specified in an *ad hoc* fashion, following our own and others' guesses about what variables affect the demand for crude petroleum products; we have tried specifications with both nominal and real own-price, other fuel prices, and income variables, plus cyclical and stock-of-user-equipment variables.

The other approach to obtaining the displacement of equilibrium price and quantity is to solve the structural demand-supply system of equations for reduced forms in market price and quantity and then estimate the reduced-form equations with ordinary least squares (OLSQ). While simpler than the 2SLS approach, the reduced-form approach in our case presents a serious problem because of the complex manner in which market price enters the after-tax output-input relative price variables which our model implies are arguments in the structural supply function. (See equations (29), (30), (32), and (33) in the appendix for the definitions of these variables.) As a result, the explicit solutions for the reduced forms are not log-linear. Thus we have been forced to take approximations in order to achieve log-linear estimating equations. We have explored the use of a non-linear estimating technique to avoid the necessity of taking the approximations; however, this approach does not appear promising.

Turning now to the separation of oil from natural gas, jointness occurs both in exploring and drilling for new reserves and in producing from so-called "oil" wells. We have not attempted to separate oil from gas in the latter instance, preferring instead to try both oil alone (ignoring associated-and-dissolved natural gas) and an index of all "oil-well" products. To separate oil from gas in exploring and drilling for new reserves, it has been necessary to modify the treatment of finding costs in [2], which was based on a Divisia index of gross additions to crude oil and nonassociated gas reserves.[2] In the current research, finding costs are derived by applying the familiar economic

---

[2] The problem of jointness arises solely from the fact that "dry-hole" costs—outlays on drilling which do not result in commercial finds of oil or gas—cannot be allocated by any meaningful economic criterion between oil and nonassociated gas. Using an index of additions to reserves permits one to compute finding costs without having to allocate dry-hole costs; however, one must then work with an index of oil-and-nonassociated-gas as the relevant unit of reserves.

analysis of joint production; the marginal finding cost of oil depends on all marginal drilling outlays—on dry holes, successful oil wells, *and* successful gas wells—and similarly for nonassociated gas. (See equations 9, and 23–25 in the appendix.)

Turning finally to the simulations of the effects of policy changes, the simulations will be carried out using the estimated reserve-investment equations, together with the calculated displacements of equilibrium price and quantity caused by the policy changes. The logic of the simulations depends on a market-clearing condition that relates aggregate demand in the United States to the quantities supplied by (a) the five prorationing states; (b) the remaining portion of the domestic industry accounted for by the "nonprorationing" states, and (c) the import sector. The quantity of imports is viewed as an exogenous policy variable, while the quantities supplied by the prorationing and nonprorationing sectors will be those predicted by the estimated supply functions or the calculated displacements of reduced-form quantities. The reserve-investment equations and the reduced-form or structural supply and demand equations used in the simulations will, of course, be estimated from compatible data bases.

To date, we have tried three specific approaches to the task of obtaining information about the displacement of equilibrium price and quantity in petroleum markets when policies are varied; we have yet to try two other specific approaches. Those attempted so far include (1) estimation of approximated reduced-form equations using pooled cross-section and time-series data, obtained by disaggregating the prorationing sector into sub-state districts for three of the five states (Louisiana, New Mexico, and Texas), using the remaining two states (Kansas and Oklahoma) as separate districts; (2) similar estimations using the aggregated prorationing sector data employed in [2]; and (3) as a first pass, OLSQ estimation of structural demand and supply equations, using aggregated data for the two sectoral supply equations. Yet to be tried are revised reduced-form estimations similar to those in (1) above but modified to account for our past experience with the reduced-forms; and 2SLS estimations of structural supply and demand equations, using pooled data for the two sectoral supply equations.

We have encountered a number of difficulties with the three approaches tried thus far. As expected, disaggregating the prorationing sector into sub-state districts introduced considerable "noise" into the data compared to the aggregate data used in [2]. Not expected, however, was the appearance of negative "gross real investment" in reserves in several sub-state districts and Kansas for several years. The problem is worst for natural gas, for which the reserves data are notoriously suspect, but it also occurs for oil. Analysis of the detailed reserves-addition data, year by year and component by component,[3]

[3] The components of gross additions are "revisions" (due largely to information obtained during production from developed fields), "extensions" (due largely to informa-

revealed that the prime source of the negative "gross investment" was in the "revisions" category. Given the negative "gross investment" (which renders nonsensical the finding cost variable, computed as tax-adjusted drilling costs divided by gross additions to reserves), we have two alternatives: (a) drop the offending sub-state districts and Kansas from the sample, thereby doing violence to the internal consistency of the model and pretending that the reserve-additions data for the other districts and Oklahoma are free of whatever problem in measuring or reporting the data is giving rise to the negative values in a few areas; and (b) net out revisions for the years 1966–71 or 1966–72, for which data on revisions and extensions of proved reserves are given separately. To date, only the first of these alternatives has been systematically explored.

In a similar vein, when we separate the aggregated prorationing and nonprorationing sectors into oil and nonassociated gas, negative (or unusually small) "gross investment" in nonassociated gas reserves appears in a number of years. We have not yet succeeded in making suitable adjustments in the data to get rid of these anomalies. As a result, we as yet lack any reliable empirical estimates for the nonassociated gas industry as a separate unit.

In the nonprorationing sector, we have not yet succeeded in eliciting reliable, stable, and credible supply-side results from the data, even for oil. Where the estimated equations exhibit strong summary statistical properties (coefficient of determination adjusted for degrees of freedom, Durbin-Watson statistic, and F-statistic), the sign of one or more independent variables for which we have a strong hypothesis is the opposite of that predicted. Conversely, where we obtain the predicted signs and acceptable t-ratios for individual coefficients, the overall statistics are defective in some way (usually in the Durbin-Watson statistic, suggesting one or more omitted variables). As a result, we have not yet been able to carry out the simulations using an estimated nonprorationing supply function, which the logic of our simulation framework requires.

Both the pooled-data and aggregate-data reduced-form regressions for the prorationing sector attempted to date have proved disappointing. The reasons are similar to those just cited for the nonprorationing supply functions: weak summary statistics, or wrong signs on individual coefficients. As indicated earlier, however, we are in the process of revising the model and adjusting the data in an attempt to improve the results.

tion obtained from development drilling, following the discovery of new reserves), and new reserves found from successful wildcat drilling in new fields and in new pools in old fields. The inclusion of revisions in the reserves data is incompatible with our basic model, in which all gross additions to reserves are obtained by drilling, exploratory or development; however, revisions are separated from extensions only from 1966 on, thus limiting our ability to analyze investment behavior net of revision except through pooling cross-section data with the shortened time series from 1966–71 or 1966–72.

We have succeeded in obtaining estimates for aggregate prorationing sector supply and aggregate demand which are at least minimally acceptable; these estimates are reported in section III below. Two comments are in order about these estimates. First, they are biased because we have estimated structural equations from a simultaneous system with OLSQ techniques; as noted earlier, we hope to apply 2SLS techniques to the same basic model, using pooled data for which there are sufficient observations to justify 2SLS. Second, the demand-equation estimates reported below include time as an explanatory variable; this is second-order "*ad hoc*kery" (the demand equation being basically *ad hoc* to begin with) for which we can offer no greater justification than that the estimates are better with time included than without it. Removing time from the demand equation boosts the price elasticity of demand from about 0.8 in absolute value to in excess of 2.3; the former figure is at least reasonable, but the latter seems so far out of line as to call for further analysis and estimation. In the meantime, we report the demand estimates with time included as interim results.

III. Interim Empirical Results

Estimates of the supply and reserve equations (A and B below) are reported for alternative interest rates: "debt" (Moody's index of 36 industrial bond yields) and "equity" (the inverse of Standard & Poor's price-earnings ratio index). The supply equations (A below) are log-linear approximations of the supply function implied by our model; see the discussion immediately preceding equation (46) in the appendix. In interpreting the estimates for 1959–72, it should be noted that prices were controlled throughout 1972, whereas our model implicitly assumes market-clearing prices.

The numbers in parentheses beneath the regression coefficients are t-ratios. $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom. DW is the Durbin-Watson statistic; the null hypothesis of no serial correlation of the residuals is $DW = 2.0$. $F(n_1, n_2)$ is the F-statistic of the regression.

A. Prorationing sector oil supply, aggregate data

Debt interest rate:

1959–1971:

$$\ln Q_{ot} = \underset{(8.38)}{16.5887} - \underset{(.13)}{.0074} \ln h_{ot} + \underset{(1.22)}{.3008} \ln w_t + \underset{(6.49)}{.0413}\, t + \underset{(2.10)}{.0805} \ln S_t$$

$$\bar{R}^2 = .9814 \quad DW = 1.9237 \quad F(4, 8) = 159.12$$

1959–1972:

$$\ln Q_{ot} = \underset{(13.28)}{15.2379} + \underset{(.38)}{.0174} \ln h_{ot} + \underset{(.97)}{.1264} \ln w_t + \underset{(11.45)}{.0367}\, t + \underset{(2.02)}{.0750} \ln S_t$$

$$\bar{R}^2 = .9831 \quad DW = 1.7033 \quad F(4, 9) = 189.66$$

Equity interest rate:

1959–1971:

$$\ln Q_{ot} = \underset{(8.68)}{19.2129} - \underset{(1.39)}{.0892} \ln h_{ot} + \underset{(2.27)}{.6127} \ln w_t + \underset{(6.93)}{.0478}\, t + \underset{(2.52)}{.1475} \ln S_t$$

$$\bar{R}^2 = .9848 \qquad DW = 2.5065 \qquad F(4, 8) = 195.44$$

1959–1972:

$$\ln Q_{ot} = \underset{(10.89)}{16.5554} - \underset{(.60)}{.0349} \ln h_{ot} + \underset{(1.59)}{.2754} \ln w_t + \underset{(9.78)}{.0387}\, t + \underset{(1.93)}{.1125} \ln S_t$$

$$\bar{R}^2 = .9835 \qquad DW = 1.7157 \qquad F(4,9) = 194.21$$

Definition of variables:

Q_{ot}—Quantity of crude oil produced in the prorationing states (Kansas, Louisiana, New Mexico, Oklahoma, and Texas) (U.S. Bureau of Mines).

h_{ot}—Output/reserve-input, after-tax relative price variable (see equation (29) in the appendix). (Constructed from drilling costs published by the Joint Association Survey, reserves figures from American Petroleum Institute, and tax parameters.) Hypothesis: positive.

w_t—Output/non-reserve-input, after-tax relative price variable (see equation (32) in the appendix). (Constructed as an average cost per well from data published by the Joint Association Survey (section II) on operating costs, number of wells from U.S. Bureau of Mines, and tax parameters.) Hypothesis: positive.

t—Time (a measure of the rate of disembodied technological change). Hypothesis: positive.

S_t—Texas market-demand factor (MDF). Hypothesis: positive.

These estimates of the coefficients of the prorationing sector supply function for oil are only partly consistent with the hypotheses. The positive signs of the estimated coefficients of S and t are consistent with the hypotheses. However, the insignificant estimates of the coefficient of h are inconsistent with its hypothesized positive sign and most of the estimates of the coefficient of w are inconsistent with its hypothesized positive sign. If these estimates are to be believed, they imply: (a) MDP had a significant impact on prorationing sector oil supply, but (b) Federal tax policy did not have a significant impact on the quantity of oil supplied by the prorationing sector.[4]

[4] In one case—equity interest rate, 1959–71—the coefficient on $\ln w_t$ is significantly positive (using a one-tail test), as predicted. We do not have a great deal of confidence in this lone significant estimate because of the poor quality of the data used to construct w_t and the preponderance of insignificant estimates in other variants of this equation.

B. Prorationing sector oil reserves investment equations

Debt interest rate:
1959–1971:

$$\underset{}{\ln R_{ot}} = \underset{(6.36)}{12.2129} - \underset{(5.48)}{.0867} \ln S_t + \underset{(1.37)}{.0181} \ln h_{ot} + \underset{(2.36)}{.3200} \ln Q_{ot} - \underset{(2.67)}{.0131}\, t$$

$$\bar{R}^2 = .9461 \quad DW = 1.5097 \quad F(4, 8) = 53.62$$

$$\Delta \ln R_{ot} = - \underset{(3.07)}{.1012} \Delta \ln S_t + \underset{(2.10)}{.0234} \Delta \ln h_{ot} + \underset{(2.79)}{.4389} \Delta \ln Q_{ot} - \underset{(2.74)}{.0148}$$

$$\bar{R}^2 = .5842 \quad DW = 1.4473 \quad F(3, 8) = 6.15$$

$$\Delta R_{ot}/R_{ot} = - \underset{(3.25)}{.1151} \Delta S_t/S_t + \underset{(1.81)}{.0167} \Delta h_{ot}/h_{ot} + \underset{(3.16)}{.5130} \Delta Q_{ot}/Q_{ot} - \underset{(2.88)}{.0164}$$

$$\bar{R}^2 = .5865 \quad DW = 1.5703 \quad F(3, 8) = 6.20$$

Equity interest rate:
1959–1972:

$$\ln R_{ot} = \underset{(7.40)}{12.2055} - \underset{(6.29)}{.0988} \ln S_t + \underset{(1.98)}{.0232} \ln h_{ot} + \underset{(2.74)}{.3184} \ln Q_{ot} - \underset{(3.18)}{.0130}\, t$$

$$\bar{R}^2 = .9552 \quad DW = 1.6164 \quad F(4, 8) = 65.02$$

$$\Delta \ln R_{ot} = - \underset{(3.45)}{.1104} \Delta \ln S_t + \underset{(2.44)}{.0211} \Delta \ln h_{ot} + \underset{(3.40)}{.4826} \Delta \ln Q_{ot} - \underset{(3.67)}{.0168}$$

$$\bar{R}^2 = .6314 \quad DW = 1.5877 \quad F(3, 8) = 7.28$$

$$\Delta R_{ot}/R_{ot} = - \underset{(3.59)}{.1293} \Delta S_t/S_t + \underset{(2.11)}{.0168} \Delta h_{ot}/h_{ot} + \underset{(3.81)}{.5642} \Delta Q_{ot}/Q_{ot} - \underset{(3.60)}{.0177}$$

$$\bar{R}^2 = .6256 \quad DW = 1.5520 \quad F(3, 8) = 7.13$$

Definition of variables not previously defined:

R_{ot}—Stock of proved reserves of crude oil at the end of year t (American Petroleum Institute).
S_t—Hypothesis: negative.
h_{ot}—Hypothesis: positive.
Q_{ot}—Hypothesis: positive.
t—Hypothesis: negative.

These estimates are consistent with the hypotheses in all but one instance (the ln h_{ot} variable in the stock-of-reserves equation, debt interest rate, for 1959–71).[5] Thus the significantly negative coefficients on the ln S_t variable indicate that, the easier were state production

[5] Note that the respective coefficients in each group of three reserves equations (ln R_{ot}, Δln R_{ot}, and $\Delta R_{ot}/R_{ot}$) are hypothesized to be the same in all three equations. That is, reading down a column, all coefficients in the column are hypothesized to be equal: see equations (39), (41), and (47) in the appendix.

restrictions, the smaller was the stock of reserves held by oil producers in the prorationing states. The significantly positive (1-tail test) coefficients on the ln h_{ot} variable imply that (*ceteris paribus*), when oil producers held larger proved reserves then (a) the higher the market price of crude oil, (b) the more generous the Federal tax treatment of petroleum corporate income, and (c) the lower the finding cost of new reserves. The significantly positive coefficients on the ln Q_{ot} variables indicate that, the larger was prorationing sector oil output, the larger was the stock of proved reserves held by oil producers. Finally, the significantly negative coefficients on the time variable may be interpreted to mean that, the faster the rate of technological change in producing crude oil, the smaller the stock of reserves producers held.

Looking at the summary statistics, the adjusted coefficients of determination are well over 0.9 for the stock-of-reserves equations, and acceptably high for the first-difference log-linear and first-difference proportional-change equations. The Durbin-Watson statistics evidence some positive serial correlation of the residuals, although all but one are greater than 1.5 in magnitude. The F-statistics of the regression are all significant at the 0.05 level or better.

These estimates for crude oil separately may be compared with those in Table 1 (debt interest rate) and Table 2 (equity interest rate) of [2], which were obtained from data aggregating oil and nonassociated gas with Divisia indexes. The oil-only estimates are somewhat weaker statistically than the estimates in [2], but for purposes of the simulations the oil-only estimates will be more easily interpreted; this is because of the conceptual difficulty of measuring the demand for Divisia-index oil-and-nonassociated-gas output.

C. Aggregate U.S. oil demand

1959–1971:

$$\ln Q_{At} = \underset{(33.31)}{16.2338} - \underset{(2.00)}{.8155} \ln (p_o/I)_t + \underset{(3.83)}{.2585} \ln (p_g/I)_t + \underset{(3.56)}{.0238}\, t$$

$$\bar{R}^2 = .9915 \quad DW = 1.8839 \quad F(3, 9) = 466.08$$

$$\ln Q_{At} = \underset{(17.44)}{16.6189} - \underset{(1.97)}{.8615} \ln (p_o/I)_t - \underset{(.48)}{.0678} \ln (Y/I)_t + \underset{(1.76)}{.2118} \ln (p_g/I)_t + \underset{(3.39)}{.0238}\, t$$

$$\bar{R}^2 = .9907 \quad DW = 1.8677 \quad F(4, 8) = 319.66$$

1959–1972:

$$\ln Q_{At} = \underset{(31.64)}{15.9129} - \underset{(1.54)}{.6831} \ln (p_o/I)_t + \underset{(3.89)}{.1549} \ln (p_g/I)_t + \underset{(3.37)}{.0249}\, t$$

$$\bar{R}^2 = .9915 \quad DW = 1.2746 \quad F(3, 10) = 504.05$$

$$\ln Q_{At} = \underset{(19.57)}{17.0048} - \underset{(1.99)}{.8672} \ln (p_o/I)_t - \underset{(1.50)}{.1601} \ln (Y/I)_t + \underset{(2.04)}{.1037} \ln (p_g/I)_t + \underset{(3.46)}{.0242}\, t$$

$$\bar{R}^2 = .9924 \quad DW = 1.6130 \quad F(4, 9) = 425.85$$

Definition of variables not previously defined:

Q_{At}—Quantity of oil demanded, continental U.S.; in equilibrium, this equals the sum of prorationing and nonprorationing quantities supplied, plus imports (U.S. Bureau of Mines).

$(p_o/I)_t$—Weighted average price at the wellhead of crude oil, continental U.S., p_o, deflated by the implicit GNP price deflator, I (U.S. Bureau of Mines, U.S. Department of Commerce). Hypothesis: negative.

$(p_g/I)_t$—Weighted average new-contract price for interstate natural gas, continental U.S., p_g, deflated by I (see above) Foster Associates, Washington, D.C.). Hypothesis: positive.

$(Y/I)_t$—Real GNP (nominal GNP deflated by I) (President's Economic Report). Hypothesis: positive.

t—Hypothesis: none.

According to these estimates, the higher its own real price, the smaller the aggregate quantity of crude oil demanded in the continental United States; the higher the real new-contract price of interstate natural gas, the larger the aggregate quantity of crude oil demanded; demand for crude oil also grew steadily with time. Real income did not significantly affect the aggregate demand for crude oil according to our estimates for the time period 1959–71; the negative sign and the t-ratio of 1.50 for the coefficient of ln $(Y/I)_t$ for 1959–72 are not reassuring. The coefficients are relatively stable with and without real income for the period 1959–71, but less so for 1959–72. The summary statistics are generally stronger for these equations than for the prorationing-sector supply and reserves equations (A and B, respectively, above); however, the Durbin-Watson statistics suggest some positive serial correlation of the residuals for the period 1959–72. One possible reason for the relatively poorer results and the longer time series may be the price controls introduced in August 1971 and their accompanying side effects.

References

1. James C. Cox and Arthur W. Wright, "The Economics of the Oil Industry's Tax Burden," in *The Petroleum Industry's Tax Burden*, Compendium prepared for the House Committee on Ways and Means and the Senate Committee on Finance (Arlington, Va.: Taxation with Representation, 1973), pp. 5–36.
2. James C. Cox and Arthur W. Wright, "The Determinants of Investment in Petroleum Reserves and Their Implications for Public Policy," *American Economic Review* 66: 1 (March 1976), pp. 153–167.

Appendix

A Model of the Crude Oil and Natural Gas Producer

We focus on three activities of the crude petroleum producer: the acquisition of proved reserves of oil and nonassociated gas through drilling; production from oil wells; and production from gas wells.

The acquisition of proved reserves of oil and nonassociated gas is viewed as a production process in which the quantity of drilling (D) is the input in a process which yields new proved reserves of oil (I_o) and nonassociated gas (I_g) as joint products. The technological relation among D, I_o, and I_g at time t is represented by the production possibility frontier

$$F(I_o(t), I_g(t), D(t), t) = 0. \tag{1}$$

Production from gas wells is viewed as a process in which the quantity of output (Q_g) is produced from two inputs, the stock of proved reserves of non-associated gas (R_g) and an index of non-reserve inputs in gas-well production (L_g). The technological relation at time t among Q_g, R_g, and L_g is presented by the production possibility frontier

$$F(Q_g(t), R_g(t), L_g(t), t) = 0. \tag{2}$$

Production from oil wells in five major producing states (Kansas, Louisiana, New Mexico, Oklahoma, and Texas) is controlled by state commissions under the policy of "market-demand prorationing" (MDP). This policy limits the use of the productive services of proved oil reserves by setting a "market-demand factor" each month. The Texas Railroad Commission (TRRC) formerly administered MDP by imposing "shut-down days" on the operation of wells subject to its control. Thus if there were 15 shutdown days in a month of 30 days, the market-demand factor S would have been $S = 0.5 = (30\text{-}15)/30$. If MDP were actually enforced, as the name "shutdown days" suggests, by shutting wells down completely part of the time, the full-time equivalent of a stock of proved oil reserves R would be simply SR; the flow of services from the stock of reserves could then be assumed proportional to SR. In fact, MDP is enforced differently; the owner of a controlled well may operate it as many or as few days a month as he wishes, so long as total output for the month does not exceed the quantity S times the "rated allowable" capacity of the well (which the TRRC also determines). Producers may therefore be able to obtain a given flow of productive services from fewer proved reserves than if they were forced to shut down part of the time; if so, they will utilize their reserves more intensively than they would under a literal "shutdown days" prorationing scheme. To include this possibility, we specify the full-time equivalent stock of proved oil reserves, Ω, as the function

$$\Omega(t) = [S(t)]^{\theta}R(t),\ 0 \leqq S(t) \leqq 1,\ 0 < \theta\ ; \tag{3}$$

where θ is the elasticity of the full-time equivalent stock of reserves with respect to the market-demand factor.

Production from prorationing sector oil wells is viewed as a process in which the quantity of output (Q_o) is produced from two inputs, the full-time equivalent stock of proved oil reserves (Ω) and an index of non-reserve inputs in oil-well production (L_o). The technological relation at time t among Q_o, Ω, and L_o is represented by the production possibility frontier

$$F(Q_o(t), \Omega(t), L_o(t), t) = 0. \tag{4}$$

This same technological relation can be used for production in "nonprorationing" states, which do not practice MDP, by setting $S(t)$ equal to one for all t.

By definition, the time rate of change ($\dot{R}$) of a stock of proved reserves equals the rate of gross additions to the stock from drilling (I), less the rate of production from the stock (Q), plus the net rate of change of the stock from other causes such as "revisions" (B). Thus we have two accounting identities which relate stocks to flows of proved reserves:

$$\dot{R}_g(t) = I_g(t) - Q_g(t) + B_g(t)\ ; \tag{5}$$

$$\dot{R}_o(t) = I_o(t) - Q_o(t) + B_o(t). \tag{6}$$

We assume that the crude oil and gas producer maximizes the present value of after-tax cash flow from drilling wells and selling the outputs of oil and gas wells.

After-tax cash flow is conveniently represented as the algebraic sum of a positive revenue term and two negative cost terms, one for drilling (reserve acquisition) costs and one for other "non-reserve," input costs. That is, after-tax cash flow can be written as

$$N(t) = N_1(t) - N_2(t) - N_3(t), \tag{7}$$

where $N_1(t)$ is the revenue term, $N_2(t)$ is the drilling cost term, and $N_3(t)$ is the non-reserve input cost term. We discuss each of these cash-flow components in turn.

The crude oil and gas producer must make royalty payments to the landowners from whom the drilling and production rights have been leased. These payments are customarily calculated as a percentage of gross revenue. Thus, let $[1 - \pi(t)]$ be the proportion of gross revenue which must be paid in royalty at time t; then the proportion $\pi(t)$ of gross revenue accrues to the producer. Therefore, the producer's before-tax revenue from selling outputs Q_o, Q_g at market prices p_o, p_g is $\pi[p_oQ_o + p_gQ_g]$.

The after-tax revenue of the crude oil and gas producer depends on several tax provisions. State and local governments assess production and severance taxes on both quantity of production and revenue; these are represented in the after-tax cash flow equation by the average production and severance tax rate y. These taxes are deductible from net income subject to the Federal corporation income tax, which is assessed at rate u. Further deductions are allowed for a proportion z of gross revenue through the percentage depletion allowance in the Federal tax law.

Together, the royalty share and the tax provisions determine the after-tax revenue component of the cash flow equation:

$$N_1(t) = \{1\text{-}y(t)\text{-}u(t)[1\text{-}y(t)\text{-}z(t)]\}\{\pi(t)p_o(t)Q_o(t) + \pi(t)\, p_g(t)Q_g(t)\}. \tag{8}$$

Various categories of investment cost in the crude petroleum industry are treated differently under the Federal corporation income tax. Define the following terms: $D(t)$ is the quantity of drilling at time t; $k_1(t)$ is the dry-hole cost per unit of $D(t)$; $k_2(t)$ is the "intangibles cost" per unit of $D(t)$; $k_3(t)$ is the "tangibles cost" per unit of $D(t)$; and $H(t)$ is the discounted value at time t of the time stream of tax deductions from one dollar of depreciable outlays made at time t.

Dry-hole costs, k_1D, and intangibles costs, k_2D, are fully deductible from net income in the year in which they are incurred. Tangibles costs, k_3D, must be capitalized and depreciated over a number of years; therefore the discounted value at time t of the tax deductions they provide is $H(t)k_3(t)D(t)$. Thus we have the after-tax drilling cost component of the cash-flow equation at time t:

$$N_2(t) = \{[1-u(t)][k_1(t) + k_2(t)] + [1-u(t)H(t)]\, k_3(t)\}\, D(t). \tag{9}$$

Other input cost categories are also treated differently under the Federal corporation income tax. Define the following terms: L_o and L_g are indexes of the quantities of non-reserve inputs used in production from oil wells and from gas wells respectively; w_{o1} is the deductible cost per unit of L_o; w_{g1} is the deductible cost per unit of L_g; w_{o2} is the depreciable cost per unit of L_o; w_{g2} is the depreciable cost per unit of L_g. Then the "non-reserve" input cost component of the cash-flow equation at time t is

$$\begin{aligned} N_3(t) = {} & \{[1-u(t)]w_{o1}(t) + [1-u(t)H(t)]\, w_{o2}(t)\}L_o(t) \\ & + \{[1-u(t)]\, w_{g1}(t) + [1 - u(t)H(t)]w_{g2}(t)\}\, L_g(t). \end{aligned} \tag{10}$$

The assumed objective of the crude oil and natural gas producer is maximization of the present value of after-tax cash flow,

$$V = {}_o\int^{\infty} N(t)\, e^{-\rho(t)}\, dt; \tag{11}$$

where $\rho(t)$ is the force of interest at time t,

$$\rho(t) = {}_o\int^{t} r(s)\, ds, \tag{12}$$

and $r(t)$ is the after-tax instantaneous rate of interest at time t. The solution to the problem of maximizing equation (11), subject to equations (1) through (6), can be found by maximizing the Lagrangian function

$$\begin{aligned} {}_0\!\int^{\infty} \phi(t)\, dt = & \\ & {}_0\!\int^{\infty} \{N(t) + \lambda_o(t) F(Q_o(t), S(t)^{\theta} R_o(t), L_o(t), t) \\ & + \lambda_g(t)\, F(Q_g(t), R_g(t), L_g(t), t) \\ & + \mu(t)\, F(I_o(t), I_g(t); D(t), t) \\ & + \eta_o(t)\, [I_o(t) - Q_o(t) + B_o(t) - \dot{R}_o(t)] \\ & + \eta_g(t)\, [I_g(t) - Q_g(t) + B_g(t) - \dot{R}_g(t)]\}\, e^{-\rho(t)}\, dt. \end{aligned} \tag{13}$$

Substituting equations (7) through (10) and (12) into (13), we find the Euler necessary conditions to be equations (1), (2), (4), (5), (6), and the following:

$$0 = \frac{\partial \phi(t)}{\partial Q_o(t)} = \{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t) p_o(t) + \lambda_o(t) \frac{\partial F}{\partial Q_o(t)} - \eta_o(t); \tag{14}$$

$$0 = \frac{\partial \phi(t)}{\partial Q_g(t)} = \{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t) p_g(t) + \lambda_g(t) \frac{\partial F}{\partial Q_g(t)} - \eta_g(t); \tag{15}$$

$$0 = \frac{\partial \phi(t)}{\partial L_o(t)} = -\{[1 - u(t)]w_{o1}(t) + [1 - u(t)H(t)]w_{o2}(t)\} + \lambda_o(t) \frac{\partial F}{\partial L_o(t)}; \tag{16}$$

$$0 = \frac{\partial \phi(t)}{\partial L_g(t)} = -\{[1 - u(t)]w_{g1}(t) + [1 - u(t)H(t)]w_{g2}(t)\} + \lambda_g(t) \frac{\partial F}{\partial L_g(t)}; \tag{17}$$

$$0 = \frac{\partial \phi(t)}{\partial D(t)} = -\{[1 - u(t)][k_1(t) + k_2(t)] + [1 - u(t)H(t)]k_3(t)\} + \mu(t) \frac{\partial F}{\partial D(t)}; \tag{18}$$

$$0 = \frac{\partial \phi(t)}{\partial I_o(t)} = \mu(t) \frac{\partial F}{\partial I_o(t)} + \eta_o(t); \tag{19}$$

$$0 = \frac{\partial \phi(t)}{\partial I_g(t)} = \mu(t) \frac{\partial F}{\partial I_g(t)} + \eta_g(t); \tag{20}$$

$$0 = \frac{\partial \phi(t)}{\partial R_o(t)} - \frac{d}{dt}\left(\frac{\partial \phi(t)}{\partial \dot{R}_o(t)}\right) = \dot{\eta}_o(t) - r(t)\eta_o(t) + \lambda_o(t) S(t)^{\theta} \frac{\partial F}{\partial \Omega(t)}; \tag{21}$$

$$0 = \frac{\partial \phi(t)}{\partial R_g(t)} - \frac{d}{dt}\left(\frac{\partial \phi(t)}{\partial \dot{R}_g(t)}\right) = \dot{\eta}_g(t) - r(t)\eta_g(t) + \lambda_g(t) \frac{\partial F}{\partial R_g(t)}. \tag{22}$$

The preceding necessary conditions can be used to derive the investment functions implicit in the model. We first derive and interpret the output-reserve input relative price variables, which will be shown to be arguments of the investment functions. Then we assume a specific form of the production function to derive the investment functions; their discrete analogues comprise the estimating equations used in the empirical work.

Using equations (18) and (19), and the implicit function theorem, we find

$$\eta_o(t) = -\frac{\mu(t)\partial F/\partial I_o(t)}{\mu(t)\partial F/\partial D(t)}\, k^*(t) = \left[\frac{\partial I_o(t)}{\partial D(t)}\right]^{-1} k^*(t); \tag{23}$$

where

$$k^*(t) = \{[1 - u(t)][k_1(t) + k_2(t)] + [1 - u(t)H(t)]k_3(t)\} \tag{24}$$

is the after-tax cost per unit of drilling at time t. Since $\partial I_o(t)/\partial D(t)$ is the marginal change in oil reserves due to drilling at time t and $k^*(t)$ is the after-tax cost per unit of drilling at

time t, $\eta_o(t)$ is the marginal after-tax cost of acquiring oil reserves from drilling at time t. By an analogous argument we find that

$$\eta_g(t) = -\frac{\mu(t)\partial F/\partial I_g(t)}{\mu(t)\partial F/\partial D(t)} k^*(t) = \left[\frac{\partial I_g(t)}{\partial D(t)}\right]^{-1} k^*(t) \tag{25}$$

is the marginal after-tax cost of acquiring nonassociated gas reserves at time t.

η_o and η_g are elements, respectively, of the oil and gas output-reserve input relative price variables. To derive these variables, we use equations (3) and (4) and the implicit function theorem; thus we find

$$\frac{\partial Q_o(t)}{\partial R_o(t)} = -S(t)^\theta \frac{\lambda_o(t)\partial F/\partial\Omega(t)}{\lambda_o(t)\partial F/\partial Q_o(t)}. \tag{26}$$

Equations (14) and (21) imply

$$-S(t)^\theta \frac{\lambda_o(t)\partial F/\partial\Omega(t)}{\lambda_o(t)\partial F/\partial Q_o(t)} = \frac{r(t)\eta_o(t) - \dot{\eta}_o(t)}{\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t)p_o(t) - \eta_o(t)}. \tag{27}$$

Equations (26) and (27) imply

$$\frac{\partial Q_o(t)}{\partial R_o(t)} = \frac{\eta_o(t)\{r(t) - \dot{\eta}_o(t)/\eta_o(t)\}}{\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t)p_o(t) - \eta_o(t)}. \tag{28}$$

The left-hand side of (28) is, of course, the marginal product of the stock of oil reserves. We now show that the right-hand side of (28) is the ratio of the marginal after-tax net cost of holding oil reserves to the marginal after-tax net return from producing them.

Consider first the numerator of (28). From (23) we see that $\eta_o(t)$ is the marginal after-tax cost of oil reserves at time t; therefore $\dot{\eta}_o(t)/\eta_o(t)$ is the own-rate of interest on oil reserves at time t. Since $r(t)$ is the after-tax monetary rate of interest, the numerator of the right-hand side of (28) is the marginal after-tax net cost of holding a unit of oil reserves. Next consider the denominator of (28). The first term, $\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\ \pi(t)p_o(t)$, is the marginal after-tax revenue from selling a unit of oil-well output. The second term, $\eta_o(t)$, is the marginal after-tax cost of a unit of oil reserves to replace that which is produced. The difference between the two terms is the marginal after-tax net return from producing a unit of reserves.

The right-hand side of (28) is the inverse of the oil output-reserve input, after-tax relative price variable $h_o(t)$, which is shown below to be an argument of the oil investment and supply functions:

$$h_o(t) = \frac{\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t)p_o(t) - \eta_o(t)}{\eta_o(t)\{r(t) - \dot{\eta}_o(t)/\eta_o(t)\}}. \tag{29}$$

A similar argument to that above leads to the gas output-reserve input, after-tax relative price variable $h_g(t)$, which is shown below to be an argument of the gas investment and supply functions:

$$h_g(t) = \frac{\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t)p_g(t) - \eta_g(t)}{\eta_g(t)\{r(t) - \dot{\eta}_g(t)/\eta_g(t)\}}. \tag{30}$$

We now derive the oil and gas output/non-reserve input after-tax relative price variables. Using equations (14) and (16) and the implicit function theorem, we find

$$\frac{\partial Q_o(t)}{\partial L_o(t)} = -\frac{\lambda_o(t)\partial F/\partial L_o(t)}{\lambda_o(t)\partial F/\partial Q_o(t)} = \frac{[1 - u(t)]w_{o1}(t) + [1 - u(t)H(t)]w_{o2}(t)}{\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t)p_o(t) - \eta_o(t)}. \tag{31}$$

The left hand side of (31) is the marginal product of the "non-reserve" input in production from oil wells. The right-hand side of (31) is the ratio of the marginal after-tax cost of

the non-reserve input to the marginal after-tax net return from producing a unit of output. The right-hand side of (31) is the inverse of the oil output/non-reserve input, after-tax relative price variable

$$w_o(t) = \frac{\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t)p_o(t) - \eta_o(t)}{[1 - u(t)]w_{o1}(t) + [1 - u(t)H(t)]w_{o2}(t)}, \tag{32}$$

which is shown below to be an argument of the oil supply function.

A similar argument to the preceding one leads to the gas output/non-reserve input, after-tax relative price variable $w_g(t)$, which is shown below to be an argument of the gas supply function:

$$w_g(t) = \frac{\{1 - y(t) - u(t)[1 - y(t) - z(t)]\}\pi(t)p_g(t) - \eta_g(t)}{[1 - u(t)]w_{g1}(t) + [1 - u(t)H(t)]w_{g2}(t)}. \tag{33}$$

In order to derive estimable investment functions, we now assume that the gas and oil production frontiers, (2) and (4), can be represented with CES production functions. The oil production function is assumed to be

$$Q_o(t) = Ae^{\gamma t}\,\{a\Omega(t)^{-v} + (1 - a)\,L_o(t)^{-v}\}^{-b/v} \tag{34}$$

where: $A > 0$ is the scale parameter; $\gamma \geqq 0$ is the rate of (disembodied) technological change; $a\epsilon(0,1)$ is the input-intensity parameter; $(1 + v)^{-1}\,\epsilon(0,1)$ is the elasticity of factor substitution, restricted so that both inputs are necessary for positive production; and $b\,\epsilon(0,1)$ is the degree of homogeneity, restricted so that the production function is concave.

Equations (3) and (34) imply that the marginal product of oil reserves is

$$\frac{\partial Q_o(t)}{\partial R_o(t)} = \frac{\partial Q_o(t)}{\partial \Omega(t)}\,\frac{\partial \Omega(t)}{\partial R_o(t)} = abA^{-v/b}[e^{\gamma t}]^{-v/b}[Q_o(t)]^{(1+v/b)}[R_o(t)]^{-(1+v)}[S(t)]^{-\theta v}. \tag{35}$$

From equations (28), (29), and (35) we obtain the following expression for the optimal stock of proved oil reserves:

$$R_o(t) = [abA^{-v/b}]^{1/(1+v)}[h_o(t)]^{1/(1+v)}[Q_o(t)]^{(b+v)/b(1+v)}[S(t)]^{-\theta v/(1+v)}[e^{\gamma t}]^{-v/b(1+v)}. \tag{36}$$

Taking a logarithmic transformation of (36) yields the optimal reserves stock equation

$$\ln R_o(t) = \alpha_0 + \alpha_1 \ln h_o(t) + \alpha_2 \ln Q_o(t) + \alpha_3 \ln S(t) + \alpha_4 t, \tag{37}$$

where:

$$\begin{aligned}
\alpha_0 &= \frac{1}{1+v}[\ln a + \ln b] - \frac{v}{b(1+v)}\ln A;\\
\alpha_1 &= \frac{1}{1+v};\\
\alpha_2 &= \frac{b+v}{b(1+v)};\\
\alpha_3 &= -\frac{\theta v}{1+v};\\
\alpha_4 &= -\frac{\gamma v}{b(1+v)}.
\end{aligned} \tag{38}$$

Replacing continuous time by discrete time in (37), and adding the error term ϵ_t, we get the reserves stock estimating equation

$$\ln R_{ot} = \alpha_0 + \alpha_1 \ln h_{ot} + \alpha_2 \ln Q_{ot} + \alpha_3 \ln S_t + \alpha_4 t + \epsilon_t. \tag{39}$$

Differentiation of (37) yields the proportional net investment equation

$$\frac{dR_o(t)/dt}{R_o(t)} = \alpha_4 + \alpha_1 \frac{dh_o(t)/dt}{h_o(t)} + \alpha_2 \frac{dQ_o(t)/dt}{Q_o(t)} + \alpha_3 \frac{dS(t)/dt}{S(t)} . \tag{40}$$

Replacing continuous time by discrete time in (40), and adding the error term ϵ'_t, we get the proportional net investment estimating equation

$$\frac{\Delta R_{ot}}{R_{ot}} = \alpha_4 + \alpha_1 \frac{\Delta h_{ot}}{h_{ot}} + \alpha_2 \frac{\Delta Q_{ot}}{Q_{ot}} + \alpha_3 \frac{\Delta S_t}{S_t} + \epsilon'_t . \tag{41}$$

The net investment estimating equation, with error term ϵ''_t, is

$$\Delta \ln R_{ot} = \alpha_4 + \alpha_1 \Delta \ln h_{ot} + \alpha_2 \Delta \ln Q_{ot} + \alpha_3 \Delta \ln S_t + \epsilon''_t . \tag{42}$$

Equation (42) can be derived either by taking a first-order Taylor series approximation of (37), or by taking first differences in (39).

A similar argument to the preceding one leads to the gas investment estimating equations (which omit the variable S_t because the policy of MDP did not apply to nonassociated gas production):

$$\ln R_{gt} = \beta_0 + \beta_1 \ln h_{gt} + \beta_2 \ln Q_{gt} + \beta_3 t + \xi_t ; \tag{43}$$

$$\Delta R_{gt}/R_{gt} = \beta_3 + \beta_1 \frac{\Delta h_{gt}}{h_{gt}} + \beta_2 \frac{\Delta Q_{gt}}{Q_{gt}} + \xi'_t ; \tag{44}$$

$$\Delta \ln R_{gt} = \beta_3 + \beta_1 \Delta \ln h_{gt} + \beta_2 \Delta \ln Q_{gt} + \xi''_t . \tag{45}$$

The CES functional form is self-dual; in the present context, self-duality implies that the oil supply function has the following form:

$$Q_o(t) = \delta_0 e^{\delta_1 t} \, [\delta_2 S(t)^{\delta_3} \, h_o(t)^{\delta_4} + \delta_5 \, w_o(t)^{\delta_4}]^{\delta_6} , \tag{46}$$

where:

$$\begin{aligned} \delta_0 &= [b^b A^{2(1+v)}]^{1/(1-b)} ; \\ \delta_1 &= \frac{\gamma}{1-b} ; \\ \delta_2 &= a^{1/(1+v)} ; \\ \delta_3 &= -\frac{\theta v}{1+v} ; \\ \delta_4 &= -\frac{v}{1+v} ; \\ \delta_5 &= (1-a)^{1/(1+v)} ; \\ \delta_6 &= -\frac{b(1+v)}{v(1-b)} . \end{aligned} \tag{47}$$

A similar argument leads to a gas supply function with the same functional form except that the market-demand factor, S, does not appear.

Empirical Evidence on Economic Depreciation of Structures

Charles R. Hulten
Johns Hopkins University

Frank C. Wykoff
Pomona College and Claremont Graduate School

Empirical Evidence on Economic Depreciation of Structures[1]

Charles R. Hulten[2]

Johns Hopkins University

Frank C. Wykoff[2]

Pomona College and Claremont Graduate School

This paper reports preliminary results from our study of the economic depreciation of nonresidential structures. Our analysis is based on a sample of vintage asset prices collected by the Office of Industrial Economics; this sample contains 6305 observations on twenty-one types of industrial and commercial buildings. In 1973, industrial and commercial buildings accounted for approximately 30 percent of gross investment in producers' durable equipment and nonresidential structures.

The results are presented in two parts. Part I describes the polynomial approach to the estimation of economic depreciation, and presents detailed numerical results for warehouses. The chief advantage of the polynominal approach is that the regressions of vintage price (per square foot) on age and time are linear in the parameters, so that estimation is straightforward. Furthermore, variables can be added easily. The main drawback is that geometric depreciation cannot be derived as a special case of the polynomial model. Thus, statistical discrimination between these alternative patterns of depreciation is not straightforward. This problem is taken up in part II, along with the "survivors" problem. In order to discriminate between alternative functional forms, a Box-Cox estimation procedure is used. In the Box-Cox model, parameters determining the functional form are estimated jointly with the coefficients of age and time. Discrimination between linear, geometric, and s-shaped depreciation patterns is therefore possible within the framework of classical parametric statistical hypothesis testing. This is the main strength of this approach. Its chief drawback is that the model is non-linear in the parameters, and estimation is rather difficult and expensive. Furthermore, it is not easy to include additional information

[1] This report is of work in progress on Phase II of Contract #TOS–74–27. The empirical results are not to be construed as final conclusions. The analysis and evidence do not necessarily reflect views held by any agency of the Treasury Department.

[2] We would like to express our appreciation to our research assistants Ron Boyer and Flint Brayton, who have contributed a great deal to this project.

available from the OIE survey (as is done in the polynominal approach). Numerical results are present for two classes: medical buildings and office buildings. The general conclusion for these classes is that depreciation is neither geometric nor straight-line.

Part I

Summary Statistics by Asset Class

In this Part, a linear statistical model is used to determine the patterns of depreciation and the values of the parameters of the appropriate depreciation scheme for assets by class. As indicated in an earlier report, over 90% of sampled structures fall into classes with enough usable data to warrant analysis. Sixteen classes of buildings were analyzed. Table 1 contains the classes evaluated and some

TABLE 1.—*Summary statistics by asset class*

Class	Number of observations	Number of used	Price/Square foot Mean	Price/Square foot S.D.	Age Mean	Age S.D.
Retail Trade	1666	1013	7.31	7.89	20.50	24.22
Offices	1654	889	12.20	9.80	16.68	23.84
Warehouse	580	275	5.20	5.10	11.77	19.66
Factory	526	282	5.60	5.92	15.84	22.14
Repair garage	230	101	4.72	3.62	10.59	16.25
Service station	225	28	18.34	13.8	3.94	8.40
Shopping center	216	64	10.14	5.27	3.18	7.95
Bank	207	70	20.26	14.88	9.88	18.09
Apartment	203	148	7.96	6.79	28.36	26.50
Restaurant/Bar	142	81	11.60	10.51	15.48	22.93
Medical	94	49	10.68	7.23	16.90	29.34
Machine shop	81	38	4.46	3.97	11.51	19.09
Terminal	69	17	9.86	10.94	6.09	13.67
Motel	65	33	10.92	6.42	10.14	16.32
Recreational	58	26	11.46	17.69	14.02	23.11
Hotel	42	37	7.51	12.86	46.31	35.94
Others	243					

indicators of the relative richness of the raw data for each class. Over 100 observations are available for each of 10 asset classes and over 40 are available for six more classes. Four very small classes are not analyzed, theatres, parking garages, supermarkets, and stadiums, and one class is too heterogeneous to be meaningful.

Since we are analyzing the relation between price and age, some simple indicators are presented in Table 1 to illustrate the number of used structures and the range of values on price and age across observations. For every large class, except service stations, a comparatively large number of buildings were acquired when used. Only 28 service stations are priced when used, so little information about service station depreciation will be provided. Considerable variations in price are revealed in the table. For example, the mean factory price per square foot is $5.60 with a standard deviation of 5.92. The average factory was acquired at 15.84 years of age and the standard deviation on age is

22.14, so that age at acquisition varies considerably across the observations. Of the smaller classes, only terminals have very few used price values, 17. Considerable variations appear in prices and age.

Detailed empirical work was undertaken for two pilot classes, warehouses and medical buildings, and then extended to the remaining classes. Warehouses and medical buildings are representative classes. Neither is the largest nor smallest but warehouses represented the larger classes and medical buildings the smaller classes. Both are rich in used asset prices, 275 and 49 observations respectively. Warehouses are rather simple structures selling on average for around $5 a square foot whereas medical buildings are more complex and sell for about $10 to $11 a square foot. Average warehouse age is 12 years with a standard deviation of about 20, and the average medical building in the sample was acquired at age 17 with a standard deviation of 30. Both are rich in age variations.

With the warehouse class, the relatively conventional polynominal functional forms were estimated for the price equations. Also the attempts to fix land values were undertaken in this class. Finally, variable rates of return were combined with the estimated prices to obtain service prices and efficiency functions. Consequently, relative efficiencies vary across age and date, and one can explore the assumption that relative efficiencies are constant over date with these results. The medical buildings provided the data for analysis of the Winfrey adjustment for nonsurvivors and the Box-Cox nonlinear estimation procedure. These experiments were then extended to the other classes. We have not yet been able to combine the two distinct sets of experiments. Consequently several different sets of estimates will be reported and we will remain somewhat eclectic about choice of the correct depreciation form and parameter values at this time. This evidence is all preliminary.

It should be noted that in addition to providing estimates of depreciation and productive efficiencies, we shall be testing hypotheses about well known functional forms and about commonly employed parameter values. Our ultimate objective here is, of course, to obtain usable estimates of relative efficiencies and of economic depreciation by asset classes.

Part II

Warehouse Experiments

Warehouse acquisition prices q are assumed to depend upon age at acquisition s, year of acquisition t and on income y as a proxy for land value:[3]

$$q = f(s, t, y) \tag{1}$$

where q is the price per square foot paid at acquisition; s is age

[3] Some very crude estimates of population size were also introduced to measure land values but were not successful.

measured in yearly increments; t is the date acquired minus 1961, and y is total adjusted gross income of individual income taxpayers in the 3-digit ZIP code area in which the structure is located.

Various degrees of polynominals over s and t were estimated for equation (1). The general form is:

$$q = a_0 + a_1 s + a_2 s^2 + \cdots + b_1 t + b_2 t^2 + \cdots + cy \qquad (2)$$

The choice of polynominal will determine the form of the price equation from which a rectangular array of prices will be derived. The resultant price array forms the basis for estimation of economic depreciation.

The following decision rule was employed to select the appropriate form of the polynominal: if the addition of a particular variable increased the coefficient of determination adjusted for degrees of freedom, $\overline{R}^2$, then it was included in the equation. This rule corresponds to selecting the form which minimizes the sum of squared residuals about the regression adjusted for degrees of freedom. It is shown by Theil[4] that if the correct specification is among those estimated, then on average it will have the smallest adjusted sum of squared residuals. Consequently, on average this procedure will lead to the correct form.

It must be noted that the choice of a particular form does not correspond to rejection of the null hypothesis that some other form is correct. For example, if we select a polynominal which is cubic in age and quadratic in date, then on average this form will be the correct specification. There is still some probability, however, that the linear form is in fact the correct specification. To test the null hypothesis that the polynomial is linear, one must state an alternative form and construct an F statistic under the null hypothesis of linearity. These tests were undertaken.

In addition to estimating polynomial forms, we assumed equation (1) to be semi-log:

$$\ln q = a_o + a_1 s + bt + cy \qquad (3)$$

computing antilogs of (3):

$$q = Ae^{as}\, e^{bt}\, e^{cy} \qquad (4)$$

where e is the exponential e and A is $\ln a_0$. Under (4), age alters q at a constant geometric rate. In other words, the semi-log form corresponds to the assumption of geometric depreciation and the coefficient a of s is the constant rate of geometric decay. Thus, we will estimate geometric depreciation, linear depreciation (when $q = a_o + a_1 s + b_1 t + \cdots + cy$) and the other polymonial forms such as quadratic and cubic.

Comparison of the polynomial equations (2) with the semi-log results of (3) is difficult. The polynomials are all minimizing sums of squares calculated on price q whereas the semi-log equation calculates and minimizes the sum of squares about the log of q. Therefore the usual equation statistics are not directly comparable. Theil suggests a means of converting the sum of squared residuals from (3) to comparable units

[4] Henri Theil. *Principles of Econometrics*. New York: John Wiley & Sons, 1971.

of the statistics of (2) by taking anti-logs of the predicted values of (3) then squaring the difference from q and summing them. The resultant statistic may then be compared against the sum of squared residuals of the polynomial forms, each statistic adjusted for degrees of freedom. Note that we are comparing equation (3) to some polynomial version of (2), and (3) differs from the polynomials in more ways than just geometric depreciation. Nevertheless, this test may indicate the relative success of geometric decay.

Table 2 contains a summary of the results obtained for the warehouse class price equation. A few select equations are presented to illustrate the estimation and test procedure. Equation 1, row 1, is the semi-log form, and the coefficient of s is the geometric rate of depreciation. The point estimate of the rate of depreciation is 1.75% with a 95% confidence region ranging from 1.40% to 2.09%. 1.75% corresponds to a double declining balance depreciation scheme on an asset lifetime of 114 years. The *Bulletin F* life for warehouses was 75 years, and the 60 year life published in *Revenue Procedure 62-21* results in a double declining balance rate of 3.33%, well beyond the confidence region.

The choice of functional form may be illustrated with the Table 2 figures. Equations (2) and (3) illustrate the contribution of income to explaining price variations. Equation (3) is equation (2) with the income variable added. The adjusted coefficient of determination, $\overline{R}^2$, rises slightly from .089 to .093. While not particularly impressive, income does contribute to explaining price variations of warehouses and subsequent experiments are presented with income in the equation. That income performs at all well is rather surprising since only very large 3-digit Zip code areas are distinguished. We are optimistic that when refined to the 5-digit Zip code area level, income will explain even more price variations. We conclude that additional experimentation with location variables, while costly, may well prove rewarding. Note that the coefficient of the age variable remained unchanged when income entered the equation.

Equations (3), (4), (5) and (7) depict the decision rule for choosing the polynomial forms of the age variable. Equation (3) is linear, (4) is quadratic, (5) is cubic, and (7) contains s, s^2 and s^4. The $\overline{R^2}$ rises from .093 for linear to .1044 for quadratic to .1070 for cubic. Each additional variable has reduced the adjusted sum of squares. Equation (7) replaces s^3 with s^4 from equation (5) and $\overline{R^2}$ becomes .1069. We choose the cubic form.

Higher powers in date did not contribute to price determination as illustrated by equations (5) and (6). The addition of t^2 reduces $\overline{R^2}$ from .1070 to .1058. Thus the final form selected was cubic in age, linear in date with income included as an explanatory variable.

The explanatory power of higher degree polynomials, s^2, s^3, s^4 is comparatively small and differences in $\overline{R^2}$ across these forms are really pretty negligible. Cubic and quadratic have $\overline{R^2}$ respectively of .1070 and .1044. Given the least squares parameter values, the actual curves are

TABLE 2.—*Warehouse polynomial price equations coefficient values (t-Statistics)*

No.	Age	Date	a_0	s	s^2	s^3	s^4	t	t^2	Y	$\bar{R}^2$	s.e.	d.f.
1	Semilog	Semilog	1.45	−.017				.023		.643	.2085	.808	576
			(28.9)	(10.1)				(7.6)		(3.15)			
2	Linear	Linear	6.15	−.069				.085			.0893	4.87	577
			(25.6)	(6.7)				(4.7)					
3	Linear	Linear	5.81	−.069				.083		2.27	.0931	4.86	576
			(19.3)	(6.6)				(4.7)		(1.85)			
4	Quadratic	Linear	6.05	−.115	.60 (−03)*			.086		2.24	.1044	4.83	575
			(19.5)	(6.0)	(2.9)			(4.8)		(1.84)			
5	Cubic	Linear	6.17	−.164	.20 (−2)	−.69 (−5)		.088		2.21	.1070	4.82	574
			(19.4)	(4.6)	(2.3)	(1.6)		(4.9)		(1.81)			
6	Cubic	Quadratic	6.13	−.164	.20 (−2)	−.69 (−5)		.098	.45 (−3)	2.21	.1085	4.83	573
			(18.6)	(4.6)	(2.3)	(1.6)		(3.4)	(.46)	(1.18)			
7	"U"**	Linear	6.16	−.152	.14 (−2)		−.24 (−7)	.087		2.21	.1069	4.82	574
			(19.4)	(5.0)	(2.6)		(1.6)	(4.9)		(1.82)			

* (−03) is notation for 10^{-3}: thus, .60 (−03) is .00060.
** s, s^2, s^4 only.

very close over the sample ages. The cubic equation is actually very nearly a parabola over the first 50–60 years of asset life. We computed the roots of the cubic at the mean income level for the year 1961 to depict the general shape of the price equation by age. Figure 1 depicts the equation. From new to 62 years of age, prices fall in convex fashion from \$6.48 per square foot to \$2.19 at age 62. Age 62 is the point at which the cubic reaches its local minimum. Actually, the price then remains quite flat as the local maximum occurs at age 126 and price \$3.09 per square foot. In other words, the cubic form merely allows the price equation to flatten out considerably over the later years of age. The actual values of local extreme are relatively unimportant. It is the flatness of the curve after the first 50 years or so that seems interesting because a geometric line would not permit a leveling off of the depreciation rate. Thus the polynomial experiments suggests that the true form may be neither linear nor geometric, but convex and very flat in later years.

The null hypothesis that aging influences price in a linear fashion may be tested against the cubic form, equation (5). If the null hypothesis is correct, the adjusted sums of squares for equations (3) and (5) would differ only because of random sampling differences. If the hypothesis of linearity is incorrect, (5) would have a significantly smaller adjusted sum of squares. Where Q_n is the sum of squares for equation n, the F-statistic is:

$$F(2{,}574) = \frac{(Q_3 - Q_5)/2}{Q_5/574}$$

F = 5.38. The critical value of F is 3. We can reject the null hypothesis that the warehouse price equation is linear.

The acquisition price array is now constructed from equation (5), the cubic form. At the average income level, \$1.4 million, acquisition prices are predicted using the equation for each age from new to 50 and for each date from 1942 to 1972. This price array forms the basis for estimation of service prices, relative efficiencies and economic depreciation over the post World War II period. The array is illustrated for selected years in Table 3. Tracing acquisition prices down any column, we observe how price declines as only the age index changes. As buildings age the rate of decline falls. The decline in 1972 from new warehouse values to one year old was \$.16. In the same year, 40-year-old buildings fell by \$.02. In percentage terms, the rate of decline falls from 2.15 percent to .61 percent.

From the acquisition price array, Table 3, one can construct an array of service prices with the additional information of rates of return. The prime commercial paper rate, 4–6 months to maturity, formed the basis for our rates of return per year. This rate is charged prime borrowers for short-term loans. While it seemed reasonable that movements over time in rates of return on structures might approximate borrowers rate

TABLE 3.—*Estimated acquisition prices warehouse class*

p (S,T) Age	1972	1971	1970	1969	1968	1967	1966	1965	1964	1963	1962	1961	1960
New		7.36	7.27	7.19	7.10	7.01	6.92	6.84	6.75	6.66	6.57	6.48	6.40
1	7.29	7.20	7.11	7.02	6.94	6.85	6.76	6.67	6.59	6.50	6.41	6.32	6.23
2	7.13	7.04	6.95	6.87	6.78	6.69	6.60	6.52	6.43	6.34	6.25	6.16	6.08
3	6.98	6.89	6.80	6.71	6.62	6.54	6.45	6.36	6.27	6.19	6.10	6.01	5.92
4	6.82	6.74	6.65	6.56	6.47	6.39	6.30	6.21	6.12	6.03	5.95	5.86	5.77
5	6.68	6.59	6.50	6.41	6.33	6.24	6.15	6.06	5.98	5.89	5.80	5.71	5.62
6	6.53	6.45	6.36	6.27	6.18	6.10	6.01	5.92	5.83	5.74	5.66	5.57	5.48
7	6.39	6.31	6.22	6.13	6.04	5.96	5.87	5.78	5.69	5.60	5.52	5.43	5.34
8	6.26	6.17	6.08	6.00	5.91	5.82	5.73	5.64	5.56	5.47	5.38	5.29	5.21
9	6.13	6.04	5.95	5.86	5.78	5.69	5.60	5.51	5.42	5.34	5.25	5.16	5.07
10	6.00	5.91	5.82	5.73	5.65	5.56	5.47	5.38	5.30	5.21	5.12	5.03	4.94
11	5.87	5.78	5.70	5.61	5.52	5.43	5.35	5.26	5.17	5.08	4.99	4.91	4.82
12	5.75	5.66	5.57	5.49	5.40	5.31	5.22	5.14	5.05	4.96	4.87	4.78	4.70
13	5.63	5.54	5.46	5.37	5.28	5.19	5.11	5.02	4.93	4.84	4.75	4.67	4.58
14	5.52	5.43	5.34	5.25	5.17	5.08	4.99	4.90	4.81	4.73	4.64	4.55	4.46
15	5.41	5.32	5.23	5.14	5.05	4.97	4.88	4.79	4.70	4.62	4.53	4.44	4.35
16	5.30	5.21	5.12	5.03	4.95	4.86	4.77	4.68	4.59	4.51	4.42	4.33	4.24
17	5.19	5.10	5.02	4.93	4.84	4.75	4.66	4.58	4.49	4.40	4.31	4.23	4.14
18	5.09	5.00	4.91	4.83	4.74	4.65	4.56	4.48	4.39	4.30	4.21	4.12	4.04
19	4.99	4.90	4.82	4.73	4.64	4.55	4.46	4.38	4.29	4.20	4.11	4.03	3.94
20	4.90	4.81	4.72	4.63	4.54	4.46	4.37	4.28	4.19	4.11	4.02	3.93	3.84
21	4.80	4.71	4.63	4.54	4.45	4.36	4.28	4.19	4.10	4.01	3.92	3.84	3.75
22	4.71	4.63	4.54	4.45	4.36	4.27	4.19	4.10	4.01	3.92	3.84	3.75	3.66
23	4.63	4.54	4.45	4.36	4.28	4.19	4.10	4.01	3.92	3.84	3.75	3.66	3.57
24	4.54	4.46	4.37	4.28	4.19	4.10	4.02	3.93	3.84	3.75	3.67	3.58	3.49
25	4.46	4.37	4.29	4.20	4.11	4.02	3.94	3.85	3.76	3.67	3.58	3.50	3.41
26	4.38	4.30	4.21	4.12	4.03	3.95	3.86	3.77	3.68	3.59	3.51	3.42	3.33
27	4.31	4.22	4.13	4.05	3.96	3.87	3.78	3.70	3.61	3.52	3.43	3.34	3.26
28	4.24	4.15	4.06	3.97	3.89	3.80	3.71	3.62	3.54	3.45	3.36	3.27	3.18
29	4.17	4.08	3.99	3.90	3.82	3.73	3.64	3.55	3.47	3.38	3.29	3.20	3.11
30	4.10	4.01	3.93	3.84	3.75	3.66	3.58	3.49	3.40	3.31	3.22	3.14	3.05
31	4.04	3.95	3.86	3.77	3.69	3.60	3.51	3.42	3.34	3.25	3.16	3.07	2.98
32	3.98	3.89	3.80	3.71	3.63	3.54	3.45	3.36	3.27	3.19	3.10	3.01	2.92
33	3.92	3.83	3.74	3.65	3.57	3.48	3.39	3.30	3.22	3.13	3.04	2.95	2.86
34	3.86	3.77	3.69	3.60	3.51	3.42	3.33	3.25	3.16	3.07	2.98	2.90	2.81

35	3.81	3.72	3.63	3.54	3.46	3.37	3.28	3.19	3.11	3.02	2.93	2.84	2.75
36	3.76	3.67	3.58	3.49	3.41	3.32	3.23	3.14	3.05	2.97	2.88	2.79	2.70
37	3.71	3.62	3.53	3.44	3.36	3.27	3.18	3.09	3.01	2.92	2.83	2.74	2.65
38	3.66	3.57	3.49	3.40	3.31	3.22	3.13	3.05	2.96	2.87	2.78	2.70	2.61
39	3.62	3.53	3.44	3.35	3.27	3.18	3.09	3.00	2.92	2.83	2.74	2.65	2.56
40	3.58	3.49	3.40	3.31	3.22	3.14	3.05	2.96	2.87	2.79	2.70	2.61	2.52
41	3.54	3.45	3.36	3.27	3.19	3.10	3.01	2.92	2.83	2.75	2.66	2.57	2.48
42	3.50	3.41	3.32	3.24	3.15	3.06	2.97	2.88	2.80	2.71	2.62	2.53	2.45
43	3.46	3.38	3.29	3.20	3.11	3.02	2.94	2.85	2.76	2.67	2.59	2.50	2.41
44	3.43	3.34	3.26	3.17	3.08	2.99	2.90	2.82	2.73	2.64	2.55	2.47	2.38
45	3.40	3.31	3.22	3.14	3.05	2.96	2.87	2.79	2.70	2.61	2.52	2.43	2.35
46	3.37	3.28	3.20	3.11	3.02	2.93	2.84	2.76	2.67	2.58	2.49	2.41	2.32
47	3.34	3.26	3.17	3.08	2.99	2.91	2.82	2.73	2.64	2.55	2.47	2.38	2.29
48	3.32	3.23	3.14	3.06	2.97	2.88	2.79	2.71	2.62	2.53	2.44	2.35	2.27
49	3.30	3.21	3.12	3.03	2.95	2.86	2.77	2.68	2.59	2.51	2.42	2.33	2.24
50	3.28	3.19	3.10	3.01	2.92	2.84	2.75	2.66	2.57	2.49	2.40	2.31	2.22

Figure 1


Warehouse Price Equation Cubic in Age
(Date = 1961, Income = $1.4 million)
Price
8
6
4
2
0
0
20
40
60
80
100
120
140
Age

movements, the prime rate levels seemed to be too small. To compensate for such low levels, we raised the level in 1950 of the prime rate to 10 percent then maintained the absolute level of the changes per year in the prime. Table 4 lists these rates. 1950 was chosen largely arbitrarily. We felt 10 percent was a reasonably low return and 1950 was the low nonwar year rate of interest. The resultant rates of return range from 9.21 percent to 16.38 percent.

TABLE 4.—*Rates of return used to estimate service prices*

Date	Prime	Adjusted	Date	Prime	Adjusted
1971	5.11	13.66	1956	3.31	11.86
1970	7.72	16.27	1955	2.18	10.73
1969	7.83	16.38	1954	1.58	10.13
1968	5.90	14.45	1953	2.52	11.07
1967	5.10	13.65	1952	2.33	10.88
1966	5.55	14.10	1951	2.16	10.71
1965	4.38	12.93	1950	1.45	10.00
1964	3.97	12.52	1949	1.49	10.04
1963	3.55	12.10	1948	1.44	9.99
1962	3.26	11.81	1947	1.03	9.58
1961	2.97	11.52	1946	.81	9.36
1960	3.85	12.40	1945	.75	9.30
1959	3.97	12.52	1944	.73	9.28
1958	2.46	11.01	1943	.69	9.24
1957	3.81	12.36	1942	.66	9.21

The equation for calculating the service prices is

$$c\ (s,\ t) = r\ \ (t)\ q\ (s,\ t) + q\ (s,\ t) - q\ (s + 1,\ t + 1) \tag{5}$$

where r is the rate of return. The service prices for select years (in Table 5) were computed for the postwar period. However, 1972 is lost because acquisition prices were computed only up to 1972 and one year is lost in using equation (5) to obtain service prices. Service prices move from year to year more than the acquisition prices in Table 3, because variable rates of return have been combined with the acquisition prices. For example, acquisition prices were smaller in 1961 than 1960 and smaller in 1971 and 1970. Lower rates of return in these years resulted in reduced opportunity costs on using capital. Table 5 tells us the rental cost of warehouse square footage to users. Old warehouses cost less to use than newer ones because of physical deterioration and obsolescence. In 1971, for example, it cost about twice as much to rent a new warehouse as it did to use a 30-year-old warehouse. These rents are assumed to apply to owner-used warehouses as shadow rents as well as to market rental warehouse space.

The marginal rates of substitution between new and used warehouses may be seen more directly by computation of the relative efficiency function. The relative efficiency function is:

$$\Phi\ (s,\ t) = c\ (s,\ t)/c\ (o,\ t) \tag{6}$$

From $c\ (s,\ t)$ values in Table 5, we compute $\Phi\ (s,\ t)$ over the period 1942 to 1972 for warehouses aged new to 50 years old. Table 6 contains the

TABLE 5.—*Service prices warehouse class*

C (S,T) Age	1971	1970	1969	1968	1967	1966	1965	1964	1963	1962	1961	1960
New	1.08	1.25	1.25	1.10	1.03	1.05	0.95	0.91	0.88	0.85	0.82	0.86
1	1.05	1.22	1.22	1.07	1.00	1.02	.93	.89	.85	.82	.79	.84
2	1.02	1.19	1.19	1.04	.98	.99	.90	.87	.83	.80	.77	.82
3	1.00	1.17	1.16	1.02	.95	.97	.88	.84	.81	.78	.75	.79
4	.98	1.14	1.13	.99	.93	.94	.86	.82	.78	.76	.73	.77
5	.95	1.11	1.10	.97	.90	.92	.84	.80	.76	.74	.71	.75
6	.93	1.08	1.07	.94	.88	.89	.81	.78	.74	.72	.69	.73
7	.91	1.06	1.05	.92	.86	.87	.79	.76	.72	.70	.67	.71
8	.88	1.03	1.02	.89	.83	.85	.77	.74	.70	.68	.65	.69
9	.85	1.01	1.00	.87	.81	.83	.75	.72	.68	.66	.63	.67
10	.84	.98	.97	.85	.79	.80	.73	.70	.66	.64	.61	.65
11	.82	.96	.95	.83	.77	.78	.71	.68	.64	.62	.59	.63
12	.80	.93	.93	.81	.75	.76	.69	.66	.63	.60	.58	.61
13	.78	.91	.90	.79	.73	.74	.67	.64	.61	.58	.56	.59
14	.76	.89	.88	.77	.71	.72	.65	.62	.59	.57	.54	.57
15	.74	.87	.86	.75	.69	.70	.64	.61	.57	.55	.53	.56
16	.72	.85	.84	.73	.68	.69	.62	.59	.56	.53	.51	.54
17	.71	.83	.82	.71	.66	.67	.60	.57	.54	.52	.50	.52
18	.69	.81	.80	.69	.64	.65	.59	.56	.53	.50	.48	.51
19	.67	.79	.78	.67	.62	.63	.57	.54	.51	.49	.47	.49
20	.66	.77	.76	.66	.61	.62	.55	.53	.50	.47	.45	.48
21	.64	.75	.74	.64	.59	.60	.54	.51	.48	.46	.44	.46
22	.63	.73	.72	.62	.58	.58	.52	.50	.47	.45	.43	.45
23	.61	.72	.71	.61	.56	.57	.51	.48	.46	.43	.41	.43
24	.60	.70	.69	.59	.55	.55	.50	.47	.44	.42	.40	.42
25	.58	.58	.67	.58	.53	.54	.48	.46	.43	.41	.39	.41
26	.57	.67	.66	.57	.52	.53	.47	.44	.42	.40	.38	.40
27	.56	.65	.64	.55	.51	.51	.46	.43	.41	.39	.37	.38
28	.54	.64	.63	.54	.50	.50	.45	.42	.39	.37	.35	.37
29	.53	.62	.61	.53	.48	.49	.43	.41	.38	.36	.34	.36
30	.52	.61	.60	.51	.47	.48	.42	.40	.37	.35	.33	.35
31	.51	.60	.59	.50	.46	.46	.41	.39	.36	.34	.32	.34
32	.50	.59	.57	.49	.45	.45	.40	.38	.35	.33	.31	.33
33	.49	.57	.56	.48	.44	.44	.39	.37	.34	.32	.30	.32
34	.48	.56	.55	.47	.43	.43	.38	.36	.33	.31	.30	.31

35	.47	.55	.54	.46	.42	.42	.37	.35	.32	.31	.29	.30
36	.46	.54	.53	.45	.41	.41	.36	.34	.32	.30	.28	.29
37	.45	.53	.52	.44	.40	.40	.35	.33	.31	.29	.27	.28
38	.44	.52	.51	.43	.39	.39	.35	.32	.30	.28	.26	.28
39	.43	.51	.50	.42	.38	.39	.34	.31	.29	.27	.26	.27
40	.42	.50	.49	.41	.38	.38	.33	.31	.28	.27	.25	.26
41	.42	.49	.48	.41	.37	.37	.32	.30	.28	.26	.24	.25
42	.41	.48	.47	.40	.36	.36	.32	.29	.27	.25	.23	.25
43	.40	.48	.47	.39	.35	.36	.31	.29	.26	.25	.23	.24
44	.40	.47	.46	.38	.35	.35	.30	.28	.26	.24	.22	.23
45	.39	.46	.45	.38	.34	.34	.30	.27	.25	.23	.22	.23
46	.38	.45	.44	.37	.33	.34	.29	.27	.25	.23	.21	.22
47	.38	.45	.44	.37	.33	.33	.29	.26	.24	.22	.21	.22
48	.37	.44	.43	.36	.32	.32	.28	.26	.24	.22	.20	.21
49	.37	.44	.43	.35	.32	.32	.28	.25	.23	.21	.20	.21
50	.36	.43	.42	.35	.31	.31	.27	.25	.23	.21	.19	.20

TABLE 6.—*Relative efficiencies warehouse class*

Phi (S,T) Age	1971	1970	1969	1968	1967	1966	1965	1964	1963	1962	1961	1960
New	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1	.97	.97	.97	.97	.97	.97	.97	.97	.97	.97	.97	.97
2	.95	.95	.95	.95	.95	.94	.94	.94	.94	.94	.94	.94
3	.92	.92	.92	.92	.92	.92	.92	.92	.92	.92	.91	.91
4	.90	.90	.90	.90	.90	.90	.89	.89	.89	.89	.89	.89
5	.88	.88	.88	.88	.87	.87	.87	.87	.87	.87	.86	.86
6	.86	.86	.86	.85	.85	.85	.85	.85	.84	.84	.84	.84
7	.84	.84	.84	.83	.83	.83	.83	.82	.82	.82	.82	.81
8	.82	.82	.82	.81	.81	.81	.80	.80	.80	.79	.79	.79
9	.80	.80	.80	.79	.79	.79	.78	.78	.78	.77	.77	.77
10	.78	.78	.78	.77	.77	.77	.76	.76	.75	.75	.75	.75
11	.76	.76	.76	.75	.75	.74	.74	.74	.73	.73	.72	.72
12	.74	.74	.74	.73	.73	.73	.72	.72	.71	.71	.70	.70
13	.72	.72	.72	.71	.71	.71	.70	.70	.69	.69	.68	.68
14	.70	.71	.70	.70	.69	.69	.68	.68	.67	.67	.66	.66
15	.69	.69	.68	.68	.67	.67	.66	.66	.65	.65	.64	.64
16	.67	.67	.67	.66	.65	.65	.65	.64	.63	.63	.62	.62
17	.65	.66	.65	.64	.64	.63	.63	.62	.62	.61	.61	.60
18	.64	.64	.64	.63	.62	.62	.61	.60	.60	.59	.59	.58
19	.62	.62	.62	.61	.61	.60	.59	.59	.58	.58	.57	.57
20	.61	.61	.61	.60	.59	.59	.58	.57	.56	.56	.55	.55
21	.59	.60	.59	.58	.57	.57	.56	.56	.55	.54	.54	.53
22	.58	.58	.58	.57	.56	.56	.55	.54	.53	.53	.52	.52
23	.57	.57	.56	.55	.55	.54	.53	.53	.52	.51	.50	.50
24	.55	.55	.55	.54	.53	.53	.52	.51	.50	.50	.49	.49
25	.54	.54	.54	.53	.52	.51	.50	.50	.49	.48	.47	.47
26	.53	.53	.52	.51	.51	.50	.49	.48	.47	.47	.46	.46
27	.51	.52	.51	.50	.49	.49	.48	.47	.46	.45	.45	.44
28	.50	.51	.50	.49	.48	.48	.46	.46	.45	.44	.43	.43
29	.49	.49	.49	.48	.47	.46	.45	.44	.44	.43	.42	.42
30	.48	.48	.48	.47	.46	.45	.44	.43	.42	.41	.41	.40
31	.47	.47	.47	.46	.45	.44	.43	.42	.41	.40	.39	.39
32	.46	.46	.46	.45	.44	.43	.42	.41	.40	.39	.38	.38
33	.45	.45	.45	.44	.43	.42	.41	.40	.39	.38	.37	.37
34	.44	.44	.44	.43	.42	.41	.40	.39	.38	.37	.36	.36

35	.43	.44	.43	.42	.41	.40	.39	.38	.37	.36	.35	.35
36	.42	.43	.42	.41	.40	.39	.38	.37	.36	.35	.34	.34
37	.41	.42	.41	.40	.39	.38	.37	.36	.35	.34	.33	.33
38	.41	.41	.41	.39	.38	.37	.36	.35	.34	.33	.32	.32
39	.40	.40	.40	.38	.37	.37	.35	.34	.33	.32	.31	.31
40	.39	.40	.39	.38	.36	.36	.34	.33	.32	.31	.30	.30
41	.38	.39	.38	.37	.36	.35	.34	.33	.32	.31	.29	.29
42	.38	.38	.38	.36	.35	.34	.33	.32	.31	.30	.29	.28
43	.37	.38	.37	.35	.34	.34	.32	.31	.30	.29	.28	.28
44	.37	.37	.36	.35	.34	.33	.32	.31	.29	.28	.27	.27
45	.36	.37	.36	.34	.33	.32	.31	.30	.29	.28	.27	.26
46	.35	.36	.35	.34	.32	.32	.30	.29	.28	.27	.26	.26
47	.35	.36	.35	.33	.32	.31	.30	.29	.28	.26	.25	.25
48	.34	.35	.34	.33	.31	.31	.29	.28	.27	.26	.25	.24
49	.34	.35	.34	.32	.31	.30	.29	.28	.26	.25	.24	.24
50	.34	.34	.33	.32	.30	.30	.28	.27	.26	.25	.24	.23

results. The relative efficiency function declines rather sharply at first and then tends to level out and become quite flat. From 45 to 50 years old, relative efficiencies fell only .02 of new structure capacity from .36 to .34. On the other hand, over the first 5 years of age, relative efficiencies declined from 1.00 to .88 for a decline of .12 of new asset efficiency. In other words, most of the loss in productive efficiency relative to new assets occurs for warehouses early. Once old, warehouses display only very gradual decline in relative efficiency.

The relative efficiency function, derived from a cubic price equation and discussed above, may be compared to other relative efficiency patterns. For example, consider the geometric Φ-function derived from the semi-log price equation. If the relation between new and used asset prices is exponential, then the Φ-function can easily be shown to be geometric at the same rate. The relative efficiency function under geometric decay for warehouses, is

$$\Phi\,(s) = e^{-1.75s} \tag{7}$$

Figure 2 depicts equation (7) along with the Φ-function plotted from Table 6 for the year 1961. The Φ-functions are clearly similar in shape over the first 50 years but the cubic is quite a bit flatter for the older buildings. The geometric rate of 1.75 is already quite flat compared to the usual treatment of structures. Consequently, the cubic price equation yields relative efficiency declines which are considerably slower than usually thought to be reasonable for structures.

Preliminary results for all classes

We have not yet extended all of our results to all asset classes. Nor have we yet combined our two sets of experiments, on warehouses and on medical buildings, to derive final estimates on economic depreciation. Nevertheless we are prepared to report some preliminary results for every asset class. We must emphasize that these results are quite tentative and further experimentation, currently in progress, may lead to very different estimates. The results are presented in two distinct sets: (1) extensions of warehouse studies and (2) extensions of medical building studies.

Table 7 contains estimates of geometric depreciation based upon the semilog equation:

$$\ln q_i = \alpha + \beta_1 s_i + \beta_2 t_i + \beta_3 Y_i \tag{8}$$

With the point estimate for each of the 16 classes of the rate of geometric depreciation, 95 percent confidence regions are reported. Apartments for example are estimated to depreciate at a rate of 1.52 percent. If equation (1) is the correct specification of the price equation, one can state with a 95 percent level of confidence that the true rate of depreciation on apartments is somewhere between 1.12 and 1.92 percent. This range of values is extraordinarily narrow. Extreme

TABLE 7.—*Geometric depreciation results by class*

Class	Rate of depreciation	95 percent confidence region	Tax rates[1]	$\bar{R}^2$	Degree of freedom
Apartment	1.52	(1.92, 1.12)	5.0	.2670	199
Bank	2.23	(2.85, 1.61)	4.0	.2814	203
Factory	1.61	(1.98, 1.24)	4.4	.1906	522
Hotel	.79	(1.73, −.15)	5.0	.1759	38
Machine shop	1.56	(2.75, .37)	4.4	.1777	77
Medical building	1.51	(1.99, .03)		.3716	90
Motel	1.40	(2.74, .06)		.1239	61
Office building	1.52	(1.69, 1.35)	4.4	.2125	1654
Recreational building	2.50	(3.60, 1.40)		.2513	54
Repair garage	2.04	(2.71, 1.37)		.1678	226
Restaurant	1.40	(2.04, .76)		.1960	138
Retail trade	1.16	(1.33, .99)	4.0	.1989	1666
Service station	3.11	(4.31, 1.91)		.3272	221
Shopping center	.93	(1.99, −.13)		.0141	214
Terminal	1.31	(1.91, −.29)		.2179	65
Warehouse	1.75	(2.09, 1.40)	3.3	.2085	576

[1] Taken from U.S. Treasury, *Revenue Procedure 62–21*, Revised, Aug. 1974.

estimates are obtained for two small comparatively unreliable classes: hotels .79 percent and service stations 3.11 percent. Not much data is available on used prices for these classes, hotels have 37 observations and service stations 28. Shopping centers also appear to depreciate quite slowly .93 percent. However, little aging has occurred on shopping centers as indicated in Table 1. Shopping centers are a relatively new phenomenon. The average age of shopping centers in the sample is 4 years. Zero is included in the confidence region of three classes, hotels, shopping centers, and terminals, all very small classes. For the remaining classes, depreciation rates fall in the range from 1.2 percent to 2.5 percent. These estimates typically have very narrow confidence regions. Restaurants and bars, for example, have a confidence region of 1.4 percent to 3.6 percent about an estimate of 2.5 percent.

The geometric rates estimated from the sample are generally slow compared to those implied by employing double declining balance to published lives in Treasury *Revenue Procedure 62–21*. The latter rates are also reported in the table. Let us stress that one cannot conclude that *Revenue Procedure 62–21* depreciation is, in any sense, inappropriate from these results. Very complex chains of reasoning about tax incidence, resource allocation, and so on must be considered before any such normative judgment can be entertained. We compare our estimates to the tax rates merely for illustrative purposes. Generally, the estimated rates are less than half the tax rates, and the tax rate is well outside the respective confidence region in every case.

Let us now turn to estimates of straight line price equations. Table 8 contains the computed lives from the linear price equation:

$$q_i = \alpha + \beta_1 s_i + \beta_2 t_i + \beta_2 Y_i \tag{9}$$

The lives for each class were computed at the median income level for 1961. Again confidence regions about the age coefficients were usually

Figure 2

Relative Efficiency Functions
for Cubic and Geometric Price Equations
Warehouse Class

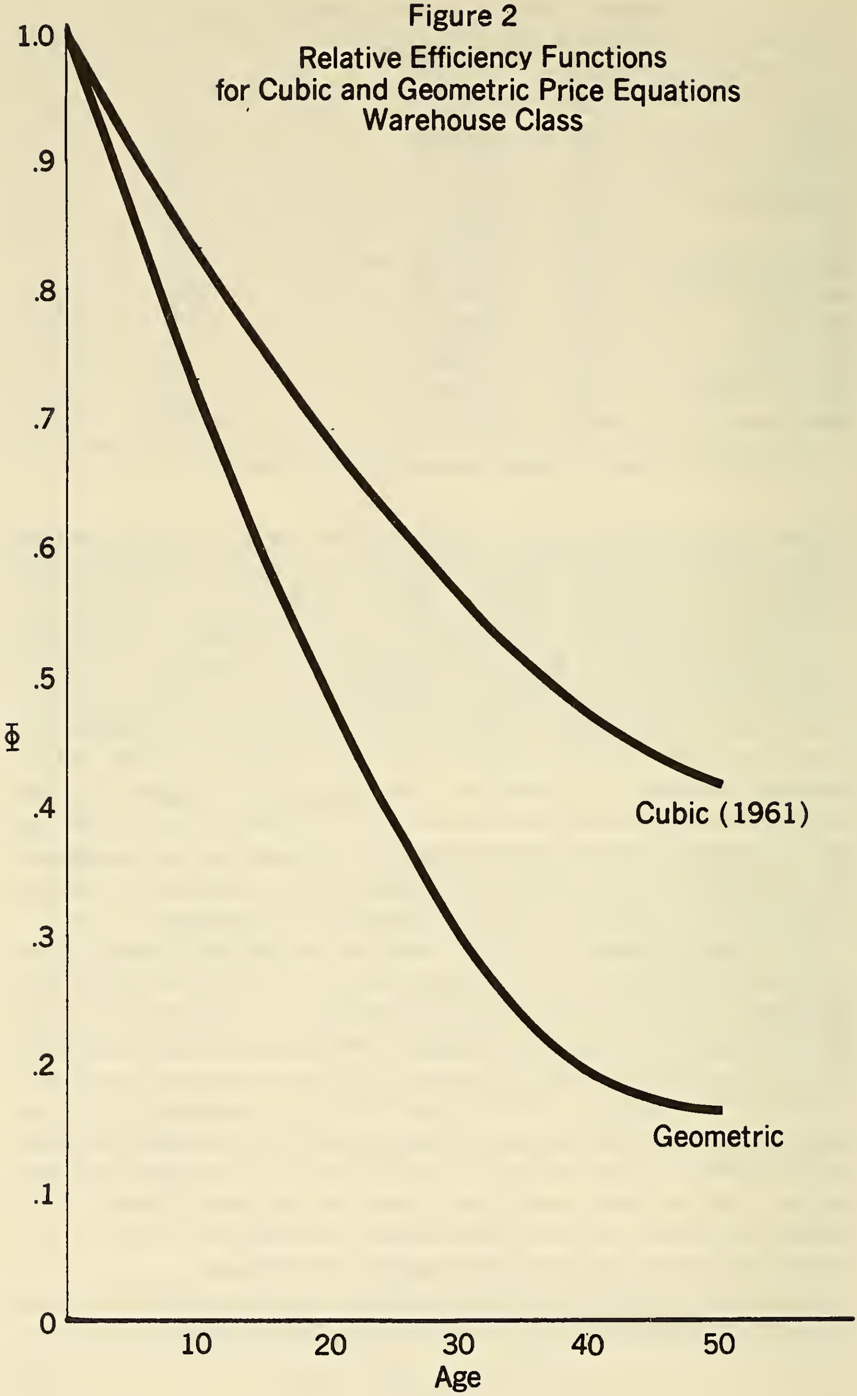


TABLE 8.—*Linear depreciation results by class**

Class	Life	Coefficient values: Age	Date	Income	$\bar{R}^2$	Degree of freedom
Apartment	114	−.096	.191	1.40	.1966	199
		(5.8)	(5.4)	(2.43)		
Bank	87	−.309	.318	2.54	.2128	203
		(6.0)	(5.4)	(.49)		
Factory	105	−.608	.086	.36	.0788	522
		(5.9)	(4.6)	(.33)		
Hotel	79	−.100	.085	.40	.3526	38
		(1.9)	(.74)	(4.53)		
Machine shop	133	−.051	.120	3.89	.1089	38
		(2.1)	(3.0)	(2.00)		
Medical building	169	−.089	.189	5.39	.2273	90
		(3.9)	(3.5)	(1.36)		
Motel	112	−.117	.302	2.02	.1114	61
		(2.34)	(2.88)	(.22)		
Office building	107	−.131	.781	.422	.1317	1654
		(13.70)	(9.27)	(4.11)		
Recreational building	97	−.219	.260	11.06	.0377	54
		(.209)	(1.45)	(.79)		
Repair garage	93	−.074	.094	2.40	.1219	226
		(5.06)	(4.14)	(1.90)		
Restaurant	131	−.134	.318	11.99	.1850	138
		(3.80)	(4.50)	(2.04)		
Retail trade	111	−.079	.180	.56	.1195	1666
		(10.47)	(10.91)	(4.94)		
Service station	55	−.476	.620	9.44	.2675	221
		(5.02)	(8.00)	(1.87)		
Shopping center	328	−.013	.134	.41	.0043	214
		(.65)	(1.97)	(.18)		
Terminal	65	−.144	.182	−7.27	.0804	65
		(1.53)	(2.21)	(.93)		
Warehouse	89	−.069	.083	2.27	.0931	576
		(6.6)	(4.7)	(1.85)		

* t-statistics in parentheses beneath the coefficients.

narrow. The main exception is again the shopping center class. Consequently its estimate is extremely unreliable, and of course a life of 328 years is quite unlikely. The remaining lives range from 55 for service stations to 133 for machine shops.

These estimated lives may also be compared to those published in *Revenue Procedure 62–21*. Table 9 makes this comparison. It seems

TABLE 9.—*Lives straight line estimates and published lives from Bulletin F*

Class	Estimated	Revenue Procedure 62-21
Apartment	114	40
Bank	87	50
Factory	105	45
Garage	93	45
Hotels	79	40
Machine shop	133	45
Office building	107	45
Retail trade	111	50
Warehouse	89	60

worth noting that if classes are ranked by age, the order of buildings would be changed by the estimated lives from that of the published tax

lives. Warehouses would move towards lower lives and apartments and machine shops towards longer lives. Of course, all estimated lives are longer than their corresponding tax lives. Again we must emphasize that the linear equation may not properly specify the form of the price function and thus we must wait for the correct specification before we can accurately compare our estimates to the other more traditional assumptions about depreciation. In fact, we have seen that the null hypothesis that the warehouse price equation is linear rather than cubic has been rejected at a 95 percent level of confidence.

The above two types of price equations, geometric and linear, lead to relatively simple mathematical formulations of depreciation. Probably for this reason, they dominate the accounting treatment of assets. Because they are well known and commonly used, we have presented parameter estimates for these forms. However, we are by no means confident that geometric or linear price functions are correct specifications. Next we report our estimates of the functional form chosen

TABLE 10.—*Polynomial price equation by asset class*

Class	Form: Age	Form: Date	$\bar{R}^2$	Degree of freedom	S.E.
Apartment	"U"	Linear	.2239	197	5.995
Bank	Cubic	Linear	.2192	201	13.176
Factory	Quadratic	Linear	.0819	522	5.674
Hotel	Quadratic	"U"	.4601	34	9.558
Machine shop	Quadratic	Linear	.1494	76	3.684
Medical building	Quadratic	Quadratic	.2715	88	6.205
Motel	Quadratic	Linear	.1313	61	6.027
Office building	Cubic	Quadratic	.1623	1647	8.990
Recreational building	Linear	Linear	.0443	55	17.447
Repair garage	Linear	Linear	.1219	226	3.404
Restaurant	Quadratic	Quadratic	.2374	136	9.209
Retail trade	Cubic	Cubic	.1446	1658	7.306
Service station	Linear	Cubic	.2929	.219	11.593
Shopping center	Quadratic	Linear	.1343	212	4.911
Terminal	Quadratic	Quadratic	.1083	64	10.410
Warehouse	Cubic	Linear	.1070	574	4.823

among the polynomial class of forms for each type of building. Tables 10 and 11 contain these results. Table 10 contains the specification, among the class of equations

$$q = a_0 + a_1 s + a_2 s^2 + \cdots + b_1 t + b_2 t^2 + \cdots + cY, \tag{10}$$

which reduced the adjusted sum of squared residuals. Three of the sixteen classes had linear price equations in age: recreational buildings, repair garages, and service stations. Eight classes had quadratic price equations in age: factory, hotel, machine shop, medical building, motel, restaurant and bar, shopping center, and terminal. The price equations of the three largest classes were cubic in age: office building, retail trade, and warehouses, as were banks. The apartment price equation was U-shaped. With a great many degrees of freedom, 34 to 1658, the adjusted coefficients of determination ranged from .08 to .46.

TABLE 11.—*Coefficient values and t-statistics of polynomial price equations*

Class	s	s^2	s^3	s^4	t	t^2	t^3	t^4	inc.
Apartment	−.3352	.0048		−.25 (−6)	.1918				3.6415
	(−3.865)	(2.505)		(−1.777)	(4.2928)				(2.6233)
Bank	−1.0401	.0287	−.2613 (−3)		0.3075				3.8550
	(−2.6776)	(1.7898)	(−1.6344)		(5.2054)				(.6712)
Factory	−.1003	.0005			.0884				
	(−3.7761)	(1.3688)			(4.6598)				
Hotel	−.6476	.0071	.2263 (−4)		.5589	.0210		−3.043 (−5)	36.9745
	(−2.6452)	(1.7597)	(−1.4084)		(2.1355)	(1.7807)		(−1.6871)	(4.3635)
Machine shop	.0874	−.0024			.1145				4.4053
	(1.2831)	(−2.1591)			(2.8955)				(2.2824)
Medical building	−.1593	.0007			.3628	.0049			4.0191
	(−2.8957)	(1.4512)			(4.1524)	(2.4700)			(1.0321)
Motel	−.2578	.0022			.3329				
	(−2.0667)	(1.2026)			(3.1411)				
Office building	−.3140	.0034	.00001		.2917	.0037			4.6718
	(7.6886)	(3.4707)	(2.0403)		(9.2317)	(4.0580)			(4.6023)
Recreational building	−.2031				.2316				
	(−1.9823)				(1.3189)				
Repair garage	−.0738				.0940				2.4001
	(−5.0612)				(4.1401)				(1.9030)
Restaurant	−.2631	.0015			.5632	.0100			14.0279
	(−3.5462)	(1.8923)			(5.3101)	(2.9902)			(2.4492)
Retail trade	−.2092	.0025	−.0001		.2885	.0102	.0001		5.4792
	(−6.5869)	(3.1998)	(1.9718)		(10.9991)	(4.1037)	(2.5987)		(4.9456)
Service station	−.5350				.9541	.02879	.0033		9.2361
	(−5.6044)				(7.2705)	(2.2971)	(−1.4016)		(1.8618)
Shopping center	−.4672	.0087			.2446				
	(−5.2229)	(5.6458)			(3.7060)				
Terminal	−.6467	.0012			.3676	.0030			
	(−2.2972)	(1.8801)			(2.2671)	(1.0375)			
Warehouse	−2.2972	.0020	−.6901 (−5)		.0878				2.2078
	(−4.5747)	(2.2792)	(−1.6286)		(4.9189)				(1.8113)

Note: Standard error in parentheses.

The explanatory power of the equations is not too bad for cross-section data. Considerable price variation remains to be explained, but our objective is not really to explain all price variations of structures in the United States. That task would be quite different than ours. Rather we wish to estimate the form of economic depreciation and the pertinent parameters of that form for each asset class. The t-statistics on the age coefficients are very encouraging. Rarely are they below 2 and often well above this critical value. Thus we are able to be relatively confident about the actual depreciation schedules we will derive from these price equations.

Part III

Medical Buildings Results

There are three sets of results to report: (1) regressions of acquisition cost per square foot (q_i) on age (s_i) and time (t_i) using ordinary least squares (OLS), (2) OLS regressions on q_i transformed to allow for asset retirement, and (3) Box-Cox nonlinear regressions which test hypotheses about the functional form of the regression equations.

The class studied is Medical Buildings. There are 94 observations, half of which represent new assets. Ten observations are for assets older than 50 years, and 15 observations are for acquisitions before 1950. The data are summarized in Table A1 of Appendix A. It should be noted that the earliest point, an 11-year-old asset sold in 1893, has a q_i value of \$12.10, while a new asset in 1920 has a q_i of \$3.43 and a new asset in 1940 a q_i value of \$1.67. It thus appears that the 1893 observation is an outlier. The analysis has therefore been repeated with this observation omitted and has been summarized in Appendix B.

Untransformed OLS regressions

The following results were obtained using OLS (t-statistics are given in parentheses):

$$\underset{}{q_i} = \underset{(-.17)}{-0.61} - \underset{(-4.00)}{0.09s_i} + \underset{(3.54)}{0.19t_i} \qquad R^2 = 0.237 \tag{11}$$

$$\ln q_i = \underset{(1.85)}{0.733} - \underset{(-6.28)}{.015s_i} + \underset{(4.12)}{.024t_i} \qquad R^2 = 0.318 \tag{12}$$

$$\ln q_i = \underset{(3.89)}{2.589} - \underset{(-3.16)}{.013s_i} - \underset{(-0.50)}{.037\ln s_i} + \underset{(5.19)}{.053t_i} - \underset{(-3.39)}{0.909\ln t_i} \qquad R^2 = 0.452 \tag{13}$$

It is somewhat surprising that the variables s_i and t_i are able to explain so much of the variation in each regression. (Because the dependent variable differs as between (11) and (12)–(13), the R^2's are not comparable). The point estimates are of plausible magnitude and have

the expected sign. All explanatory variables are significant except the constant in (11) and $\ln s_i$ in (13).

Transformed OLS regressions

Our sample is based on a study of surviving assets, and does not consider the retirement of assets. It thus gives a picture of the assets which are, in some sense, the most successful and *not* a picture of the average asset. In other words, we have information about assets which continue to be productive after many other similar assets have been retired; our estimates thus reflect the experiences of the best (in some sense) assets and not the typical asset.

To correct for this, we have weighed the q by the estimated survival probability. This is conceptually equivalent to adding zeros back into the sample corresponding to the retired assets, i.e., of blowing the sample of survivors up to the level of all assets in the class. The survival probabilities are derived from Winfrey, who has calculated the percentage of assets surviving to various vintages. Two of Winfrey's distributions are used: the L_0 and the S_3. They are tabled below:

Winfrey actually used 5 percent intervals instead of the 20 percent

TABLE 12.—*Two Winfrey distributions*

Age, percent of average life	Percent surviving	
	L_0	S_3
0	100	100
20	92.4	100
40	80.9	99.6
60	68.5	94.9
80	56.3	78.5
100	44.8	50.0
120	34.4	21.5
140	22.5	5.1
160	18.0	0
180	12.1	0
200	7.7	0
220	4.6	0

intervals shown in column 1 of Table 12. Note that "age" is expressed as a percentage of average life. The life used in the calculations is 67 years, which was derived from the 1942 Bulletin F.

The L_0 and S_3 distributions were chosen because they represent platykurtotic and leptokurtotic distributions respectively. Since we don't know anything about the actual retirement distribution, some assumptions must be made. The L_0 distribution is relatively flat and allows assets to survive to very old ages. The S_3 distribution has little retirement in the early vintages, and falls off rapidly after the mean life is reached. It is highly kurtotic. Together, L_0 and S_3 seem to represent reasonable alternatives in the absence of hard information. Other distributions can be tried. The results for the L_0 transformation are:

$$q_1^* = \underset{(-.05)}{.183} - \underset{(-5.65)}{.122s_i} + \underset{(3.55)}{.182t_i} \qquad R^2 = .3268 \tag{14}$$

$$\ln q_i^* = \underset{(2.01)}{.796} - \underset{(-12.12)}{.029s_i} + \underset{(4.11)}{.024t_i} \qquad R^2 = .6415 \tag{15}$$

$$\ln q_i^* = \underset{(3.83)}{2.528} - \underset{(-7.59)}{.032s_i} + \underset{(.74)}{.054 \ln s_i} + \underset{(5.13)}{.052t_i} + \underset{(-3.30)}{.878 \ln t_i} \qquad R^2 = .6861 \tag{16}$$

The results for the S_3 transformation are

$$q^{**} = \underset{(-.08)}{-2.72} - \underset{(-5.47)}{1.20s_i} + \underset{(3.60)}{.188t_i} \qquad R^2 = .3183 \tag{17}$$

$$\ln q_i^{**} = \underset{(1.78)}{0.972} - \underset{(-15.20)}{.051s_i} + \underset{(3.10)}{.025t_i} \qquad R^2 = .7249 \tag{18}$$

$$\ln q_i^{**} = \underset{(2.41)}{1.898} - \underset{(-15.16)}{0.075s_i} + \underset{(6.02)}{.523 \ln s_i} + \underset{(3.86)}{.047t_i} - \underset{(-2.06)}{.656 \ln t_i} \qquad R^2 = .8191 \tag{19}$$

Again, a large part of the variation in the dependent variables is explained. Because of the transformations, the R^2's are not uniformly comparable. All explanatory variables are significant with the exception of the constants in (14) and (17), and ln s_i in (16); the coefficient of ln s_i is significant under the S_3 transformation. Coefficients continue to have plausible magnitudes and expected signs.

As expected, the inclusion of the retirement distribution causes the measured rate of depreciation to increase: from 1.5 percent in (12) to 2.9 percent in (15) and 5.1 percent in (18). Note, however, that the rate of inflation remains approximately constant across transformations.

Rental prices and efficiency functions

Rental prices and efficiency functions were estimated in a rather crude way in order to break the path for future work. The following methodology was used:

(1) The acquisition prices q_i were predicted using the semi-log regressions (12), (15), and (18). The predictions are for $t = 1970$; varying t_i will change the magnitude of economic depreciation (in 15 and 18), but not the estimates of the efficiency function since inflation enters in a multiplicative way.

(2) Rental prices, c_s, were predicted under the assumption that the rate of return r was 10 percent. The following simple formula was used ($q_s = q_i | t = 1970$)

$$c_s = (1 + r)q_s - q_{s+1} \tag{20}$$

A more sophisticated formulation would take into account the tax structure; e.g. for structures owned by corporations

$$(1 - u)c_s = (1 + r)q_s - uD_s q_0 + (1 - u)\tau q_s - q_{s+1} \qquad (21)$$

where u = corporation income tax rate, D_s = depreciation deduction allowed at vintage s, and τ = corporate property tax rate. Other adjustments are possible: for the deductibility of interest and for the treatment of capital gain. An important problem arises from the fact that we cannot assume all medical buildings are owned by corporations.

(3) The efficiency functions, Φ_s, calculated assuming

$$\Phi s = c_s/c_0 \qquad (22)$$

where the c_s defined in (20) are used. If $-uD_s q_0 + (1 - u)\ \tau q_s$ is assumed to be of a second order of smallness (which is essentially true), then (22) gives an approximation to the more sophisticated model of rental prices in each year. The actual values of the projections are given in tables 13A, 13B, and 13C. Note that, as expected, the efficiency function declines much more rapidly for the transformed data, implying correspondingly smaller estimates for the capital stock.

TABLE 13A.—*Projection of asset prices, rentals, and efficiencies using regression (12) (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	φ_{s-1}	s
13.3431	-0.0000	-0.0800	-0.0000	-0.0000	1.
13.1445	0.1987	0.0149	1.5330	1.0000	2.
12.9488	0.1957	0.0149	1.5101	0.9851	3.
12.7560	0.1928	0.0149	1.4877	0.9704	4.
12.5661	0.1899	0.0149	1.4655	0.9560	5.
12.3790	0.1871	0.0149	1.4437	0.9418	6.
12.1947	0.1843	0.0149	1.4222	0.9277	7.
12.0131	0.1816	0.0149	1.4010	0.9139	8.
11.8343	0.1789	0.0149	1.3802	0.9003	9.
11.6581	0.1762	0.0149	1.3596	0.8869	10.
11.4845	0.1736	0.0149	1.3394	0.8737	11.
11.3135	0.1710	0.0149	1.3194	0.8607	12.
11.1451	0.1684	0.0149	1.2998	0.8479	13.
10.9792	0.1659	0.0149	1.2804	0.8353	14.
10.8157	0.1635	0.0149	1.2614	0.8228	15.
10.6547	0.1610	0.0149	1.2426	0.8106	16.
10.4961	0.1586	0.0149	1.2241	0.7985	17.
10.3398	0.1563	0.0149	1.2059	0.7866	18.
10.1859	0.1539	0.0149	1.1879	0.7749	19.
10.0342	0.1516	0.0149	1.1702	0.7634	20.
9.8848	0.1494	0.0149	1.1528	0.7520	21.
9.7377	0.1472	0.0149	1.1356	0.7408	22.
9.5927	0.1450	0.0149	1.1187	0.7298	23.
9.4499	0.1428	0.0149	1.1021	0.7189	24.
9.3092	0.1407	0.0149	1.0857	0.7082	25.
9.1706	0.1386	0.0149	1.0695	0.6977	26.
9.0340	0.1365	0.0149	1.0536	0.6873	27.
8.8995	0.1345	0.0149	1.0379	0.6771	28.
8.7670	0.1325	0.0149	1.0225	0.6670	29.
8.6365	0.1305	0.0149	1.0072	0.6570	30.
8.5079	0.1286	0.0149	0.9922	0.6473	31.
8.3813	0.1267	0.0149	0.9775	0.6376	32.
8.2565	0.1248	0.0149	0.9629	0.6281	33.
8.1336	0.1229	0.0149	0.9486	0.6188	34.
8.0125	0.1211	0.0149	0.9345	0.6096	35.
7.8932	0.1193	0.0149	0.9205	0.6005	36.
7.7757	0.1775	0.0149	0.9068	0.5916	37.

TABLE 13A.—*Projection of asset prices, rentals, and efficiencies using regression 12 (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	φ_{s-1}	s
7.6599	-0.1158	-0.0149	-0.8933	-0.5827	38.
7.5459	0.1140	0.0149	0.8800	0.5741	39.
7.4335	0.1123	0.0149	0.8669	0.5655	40.
7.3229	0.1107	0.0149	0.8540	0.5571	41.
7.2138	0.1090	0.0149	0.8413	0.5488	42.
7.1064	0.1074	0.0149	0.8288	0.5406	43.
7.0006	0.1058	0.0149	0.8164	0.5326	44.
6.8964	0.1042	0.0149	0.8043	0.5247	45.
6.7937	0.1027	0.0149	0.7923	0.5169	46.
6.6926	0.1011	0.0149	0.7805	0.5092	47.
6.5929	0.0996	0.0149	0.7689	0.5016	48.
6.4948	0.0982	0.0149	0.7575	0.4941	49.
6.3981	0.0967	0.0149	0.7462	0.4868	50.
6.3028	0.0953	0.0149	0.7351	0.4795	51.
6.2090	0.0938	0.0149	0.7241	0.4724	52.
6.1166	0.0924	0.0149	0.7133	0.4653	53.
6.0255	0.0911	0.0149	0.7027	0.4584	54.
5.9358	0.0897	0.0149	0.6923	0.4516	55.
5.8474	0.0884	0.0149	0.6820	0.4449	56.
5.7604	0.0871	0.0149	0.6718	0.4382	57.
5.6746	0.0858	0.0149	0.6618	0.4317	58.
5.5901	0.0845	0.0149	0.6519	0.4253	59.
5.5069	0.0832	0.0149	0.6422	0.4190	60.
5.4249	0.0820	0.0149	0.6327	0.4127	61.
5.3441	0.0808	0.0149	0.6233	0.4066	62.
5.2646	0.0796	0.0149	0.6140	0.4005	63.
5.1862	0.0784	0.0149	0.6048	0.3946	64.
5.1090	0.0772	0.0149	0.5958	0.3887	65.
5.0329	0.0761	0.0149	0.5870	0.3829	66.
4.9580	0.0749	0.0149	0.5782	0.3772	67.
4.8842	0.0738	0.0149	0.5696	0.3716	68.
4.8115	0.0727	0.0149	0.5611	0.3668	69.
4.7398	0.0716	0.0149	0.5528	0.3606	70.
4.6693	0.0706	0.0149	0.5445	0.3552	71.
4.5997	0.0695	0.0149	0.5364	0.3499	72.
4.5313	0.0685	0.0149	0.5285	0.3447	73.
4.4638	0.0675	0.0149	0.5206	0.3396	74.
4.3973	0.0665	0.0149	0.5128	0.3345	75.
4.3319	0.0655	0.0149	0.5052	0.3296	76.
4.2674	0.0645	0.0149	0.4977	0.3247	77.
4.2038	0.0635	0.0149	0.4903	0.3198	78.
4.1413	0.0626	0.0149	0.4830	0.3151	79.
4.0796	0.0617	0.0149	0.4758	0.3104	80.
4.0189	0.0607	0.0149	0.4687	0.3057	81.
3.9590	0.0598	0.0149	0.4617	0.3012	82.
3.9001	0.0589	0.0149	0.4548	0.2967	83.
3.8420	0.0581	0.0149	0.4481	0.2923	84.
3.7848	0.0572	0.0149	0.4414	0.2879	85.
3.7285	0.0563	0.0149	0.4348	0.2837	86.
3.6730	0.0555	0.0149	0.4284	0.2794	87.
3.6183	0.0547	0.0149	0.4220	0.2753	88.
3.5644	0.0539	0.0149	0.4157	0.2712	89.
3.5113	0.0531	0.0149	0.4095	0.2671	90.
3.4591	0.0523	0.0149	0.4034	0.2632	91.
3.4076	0.0515	0.0149	0.3974	0.2592	92.
3.3568	0.0507	0.0149	0.3915	0.2554	93.
3.3069	0.0500	0.0149	0.3857	0.2516	94.
3.2576	0.0492	0.0149	0.3799	0.2478	95.
3.2091	0.0485	0.0149	0.3743	0.2441	96.
3.1614	0.0478	0.0149	0.3687	0.2405	97.
3.1143	0.0471	0.0149	0.3632	0.2369	98.
3.0679	0.0464	0.0149	0.3578	0.2334	99.
3.0222	0.0457	0.0149	0.3525	0.2299	100.

TABLE 13B.—*Projection of asset prices, rentals, and efficiencies using regression 15 (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	φ_{s-1}	s
13.9992	−0.0000	−0.0000	−0.0133	−0.0000	1.
13.5991	0.4001	0.0286	1.8001	1.0000	2.
13.2103	0.3887	0.0286	1.7486	0.9714	3.
12.8327	0.3776	0.0286	1.6986	0.9436	4.
12.4659	0.3668	0.0286	1.6501	0.9167	5.
12.1096	0.3563	0.0286	1.6029	0.8905	6.
11.7635	0.3461	0.0286	1.5571	0.8650	7.
11.4272	0.3362	0.0286	1.5126	0.8403	8.
11.1006	0.3266	0.0286	1.4694	0.8163	9.
10.7833	0.3173	0.0286	1.4274	0.7929	10.
10.4751	0.3082	0.0286	1.3866	0.7703	11.
10.1757	0.2994	0.0286	1.3469	0.7483	12.
9.8848	0.2909	0.0286	1.3084	0.7269	13.
9.6023	0.2825	0.0286	1.2710	0.7061	14.
9.3278	0.2745	0.0286	1.2347	0.6859	15.
9.0612	0.2666	0.0286	1.1994	0.6663	16.
8.8022	0.2590	0.0286	1.1651	0.6473	17.
8.5506	0.2516	0.0286	1.1318	0.6288	18.
8.3062	0.2444	0.0286	1.0995	0.6108	19.
8.0688	0.2374	0.0286	1.0680	0.5933	20.
7.8381	0.2306	0.0286	1.0375	0.5764	21.
7.6141	0.2240	0.0286	1.0079	0.5599	22.
7.3964	0.2176	0.0286	0.9790	0.5439	23.
7.1850	0.2114	0.0286	0.9511	0.5283	24.
6.9797	0.2054	0.0286	0.9239	0.5132	25.
6.7802	0.1995	0.0286	0.8975	0.4986	26.
6.5864	0.1938	0.0286	0.8718	0.4843	27.
6.3981	0.1883	0.0286	0.8469	0.4705	28.
6.2152	0.1829	0.0286	0.8227	0.4570	29.
6.0376	0.1777	0.0286	0.7992	0.4440	30.
5.8650	0.1726	0.0286	0.7763	0.4313	31.
5.6973	0.1676	0.0286	0.7541	0.4190	32.
5.5345	0.1629	0.0286	0.7326	0.4070	33.
5.3763	0.1582	0.0286	0.7116	0.3953	34.
5.2226	0.1537	0.0286	0.6913	0.3840	35.
5.0733	0.1493	0.0286	0.6715	0.3731	36.
4.9283	0.1450	0.0286	0.6523	0.3624	37.
4.7875	0.1409	0.0286	0.6337	0.3520	38.
4.6506	0.1368	0.0286	0.6156	0.3420	39.
4.5177	0.1329	0.0286	0.5980	0.3322	40.
4.3886	0.1291	0.0286	0.5809	0.3227	41.
4.2631	0.1254	0.0286	0.5643	0.3135	42.
4.1413	0.1219	0.0286	0.5482	0.3045	43.
4.0229	0.1184	0.0286	0.5325	0.2958	44.
3.9079	0.1150	0.0286	0.5173	0.2874	45.
3.7962	0.1117	0.0286	0.5025	0.2792	46.
3.6877	0.1085	0.0286	0.4881	0.2712	47.
3.5823	0.1054	0.0286	0.4742	0.2634	48.
3.4799	0.1024	0.0286	0.4606	0.2559	49.
3.3804	0.0995	0.0286	0.4475	0.2486	50.
3.2838	0.0966	0.0286	0.4347	0.2415	51.
3.1899	0.0939	0.0286	0.4222	0.2346	52.
3.0988	0.0912	0.0286	0.4102	0.2279	53.
3.0102	0.0886	0.0286	0.3984	0.2214	54.
2.9241	0.0860	0.0286	0.3871	0.2150	55.
2.8406	0.0836	0.0286	0.3760	0.2089	56.
2.7594	0.0812	0.0286	0.3652	0.2029	57.
2.6805	0.0789	0.0286	0.3548	0.1971	58.
2.6039	0.0766	0.0286	0.3447	0.1915	59.
2.5294	0.0744	0.0286	0.3348	0.1860	60.
2.4571	0.0723	0.0286	0.3252	0.1807	61.
2.3869	0.0702	0.0286	0.3159	0.1755	62.
2.3187	0.0682	0.0286	0.3069	0.1705	63.

TABLE 13B.—*Projection of asset prices, rentals, and efficiencies using regression 15 (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	φ_{s-1}	s
2.2524	−0.0663	−0.0286	−0.2981	−0.1656	64.
2.1880	0.0644	0.0286	0.2896	0.1609	65.
2.1255	0.0625	0.0286	0.2813	0.1563	66.
2.0647	0.0608	0.0286	0.2733	0.1518	67.
2.0057	0.0590	0.0286	0.2655	0.1475	68.
1.9484	0.0573	0.0286	0.2579	0.1433	69.
1.8927	0.0557	0.0286	0.2505	0.1392	70.
1.8386	0.0541	0.0286	0.2434	0.1352	71.
1.7860	0.0526	0.0286	0.2364	0.1313	72.
1.7350	0.0511	0.0286	0.2297	0.1276	73.
1.6854	0.0496	0.0286	0.2231	0.1239	74.
1.6372	0.0482	0.0286	0.2167	0.1204	75.
1.5904	0.0468	0.0286	0.2105	0.1170	76.
1.5450	0.0455	0.0286	0.2045	0.1136	77.
1.5008	0.0442	0.0286	0.1987	0.1104	78.
1.4579	0.0429	0.0286	0.1930	0.1072	79.
1.4162	0.0417	0.0286	0.1875	0.1041	80.
1.3758	0.0405	0.0286	0.1821	0.1012	81.
1.3364	0.0393	0.0286	0.1769	0.0983	82.
1.2982	0.0382	0.0286	0.1718	0.0955	83.
1.2611	0.0371	0.0286	0.1669	0.0927	84.
1.2551	0.0360	0.0286	0.1622	0.0901	85.
1.1901	0.0350	0.0286	0.1575	0.0875	86.
1.1560	0.0340	0.0286	0.1530	0.0850	87.
1.1230	0.0330	0.0286	0.1486	0.0826	88.
1.0909	0.0321	0.0286	0.1444	0.0802	89.
1.0597	0.0312	0.0286	0.1403	0.0779	90.
1.0294	0.0303	0.0286	0.1363	0.0757	91.
1.0000	0.0294	0.0286	0.1324	0.0735	92.
0.9714	0.0286	0.0286	0.1286	0.0714	93.
0.9436	0.0278	0.0286	0.1249	0.0694	94.
0.9167	0.0270	0.0286	0.1213	0.0674	95.
0.8905	0.0262	0.0286	0.1179	0.0655	96.
0.8650	0.0255	0.0286	0.1145	0.0636	97.
0.8403	0.0247	0.0286	0.1112	0.0618	98.
0.8163	0.0240	0.0286	0.1080	0.0600	99.
0.7929	0.0233	0.0286	0.1050	0.0583	100.

TABLE 13C.—*Projection of asset prices, rentals, and efficiencies using regression 18 ($t = 1970$)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	φ_{s-1}	s
17.6547	−0.0000	−0.0000	−0.0310	−0.0000	1.
16.7768	0.8778	0.0497	2.6433	1.0000	2.
15.9427	0.8342	0.0497	2.5119	0.9583	3.
15.1500	0.7927	0.0497	2.3870	0.9030	4.
14.3967	0.7533	0.0497	2.2683	0.8581	5.
13.6809	0.7158	0.0497	2.1555	0.8155	6.
13.0007	0.6802	0.0497	2.0483	0.7749	7.
12.3542	0.6464	0.0497	1.9465	0.7364	8.
11.7400	0.6143	0.0497	1.8497	0.6998	9.
11.1563	0.5837	0.0497	1.7577	0.6650	10.
10.6015	0.5547	0.0497	1.6703	0.6319	11.
10.0744	0.5271	0.0497	1.5873	0.6005	12.
9.5735	0.5009	0.0497	1.5084	0.5706	13.
9.0975	0.4760	0.0497	1.4334	0.5423	14.
8.6452	0.4523	0.0497	1.3621	0.5153	15.
8.2153	0.4298	0.0497	1.2944	0.4897	16.
7.8068	0.4085	0.0497	1.2300	0.4653	17.
7.4187	0.3882	0.0497	1.1688	0.4422	18.
7.0498	0.3689	0.0497	1.1107	0.4202	19.
6.6993	0.3505	0.0497	1.0555	0.3993	20.
6.3662	0.3331	0.0497	1.0030	0.3795	21.
6.0496	0.3165	0.0497	0.9532	0.3606	22.
5.7489	0.3008	0.0497	0.9058	0.3427	23.
5.4630	0.2858	0.0497	0.8607	0.3256	24.
5.1914	0.2716	0.0497	0.8179	0.3094	25.
4.9333	0.2581	0.0497	0.7773	0.2941	26.
4.6880	0.2453	0.0497	0.7386	0.2794	27.
4.4549	0.2331	0.0497	0.7019	0.2655	28.
4.2334	0.2215	0.0497	0.6670	0.2523	29.
4.0229	0.2105	0.0497	0.6338	0.2398	30.
3.8229	0.2000	0.0497	0.6023	0.2279	31.
3.6328	0.1901	0.0497	0.5724	0.2165	32.
3.4522	0.1806	0.0497	0.5439	0.2058	33.
3.2805	0.1716	0.0497	0.5169	0.1955	34.
3.1174	0.1631	0.0497	0.4912	0.1858	35.
2.9624	0.1550	0.0497	0.4667	0.1766	36.
2.8151	0.1473	0.0497	0.4435	0.1678	37.
2.6751	0.1400	0.0497	0.4215	0.1595	38.
2.5421	0.1330	0.0497	0.4005	0.1515	39.
2.4157	0.1264	0.0497	0.3806	0.1440	40.
2.2956	0.1201	0.0497	0.3617	0.1368	41.
2.1815	0.1141	0.0497	0.3437	0.1300	42.
2.0730	0.1085	0.0497	0.3266	0.1236	43.
1.9699	0.1031	0.0497	0.3104	0.1174	44.
1.8720	0.0979	0.0497	0.2949	0.1116	45.
1.7789	0.0931	0.0497	0.2802	0.1060	46.
1.6905	0.0884	0.0497	0.2663	0.1008	47.
1.6064	0.0841	0.0497	0.2531	0.0958	48.
1.5265	0.0759	0.0497	0.2405	0.0910	49.
1.4506	0.0759	0.0497	0.2286	0.0865	50.
1.3785	0.0721	0.0497	0.2172	0.0822	51.
1.3100	0.0685	0.0497	0.2064	0.0781	52.
1.2448	0.0651	0.0497	0.1961	0.0742	53.
1.1829	0.0619	0.0497	0.1864	0.0705	54.
1.1241	0.0588	0.0497	0.1771	0.0670	55.
1.0682	0.0559	0.0497	0.1683	0.0637	56.
1.0151	0.0531	0.0497	0.1599	0.0605	57.
0.9646	0.0505	0.0497	0.1520	0.0575	58.
0.9167	0.0480	0.0497	0.1444	0.0546	59.
0.8711	0.0456	0.0497	0.1372	0.0519	60.
0.8278	0.0433	0.0497	0.1304	0.0493	61.
0.7866	0.0412	0.0497	0.1239	0.0469	62.
0.7475	0.0391	0.0497	0.1178	0.0446	63.

TABLE 13C.—*Projection of asset prices, rentals, and efficiencies using regression 18 (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	φ_{s-1}	s
0.7103	−0.0372	−0.0497	−0.1119	−0.0423	64.
0.6750	0.0353	0.0497	0.1064	0.0402	65.
0.6415	0.0336	0.0497	0.1011	0.0382	66.
0.6096	0.0319	0.0497	0.0960	0.0363	67.
0.5793	0.0303	0.0497	0.0913	0.0345	68.
0.5505	0.0288	0.0497	0.0867	0.0328	69.
0.5231	0.0274	0.0497	0.0824	0.0312	70.
0.4971	0.0260	0.0497	0.0783	0.0296	71.
0.4724	0.0247	0.0497	0.0744	0.0282	72.
0.4489	0.0235	0.0497	0.0707	0.0268	73.
0.4266	0.0223	0.0497	0.0672	0.0254	74.
0.4054	0.0212	0.0497	0.0639	0.0242	75.
0.3852	0.0202	0.0497	0.0607	0.0230	76.
0.3660	0.0192	0.0497	0.0577	0.0218	77.
0.3478	0.0182	0.0497	0.0548	0.0207	78.
0.3305	0.0173	0.0497	0.0521	0.0197	79.
0.3141	0.0164	0.0497	0.0495	0.0187	80.
0.2985	0.0156	0.0497	0.0470	0.0178	81.
0.2837	0.0148	0.0497	0.0447	0.0169	82.
0.2696	0.0141	0.0497	0.0425	0.0161	83.
0.2561	0.0134	0.0497	0.0404	0.0153	84.
0.2434	0.0127	0.0497	0.0384	0.0145	85.
0.2313	0.0121	0.0497	0.0364	0.0138	86.
0.2198	0.0115	0.0497	0.0346	0.0131	87.
0.2089	0.0109	0.0497	0.0329	0.0125	88.
0.1985	0.0104	0.0497	0.0313	0.0118	89.
0.1886	0.0099	0.0497	0.0297	0.0112	90.
0.1792	0.0094	0.0497	0.0282	0.0107	91.
0.1703	0.0089	0.0497	0.0268	0.0102	92.
0.1619	0.0085	0.0497	0.0255	0.0096	93.
0.1538	0.0080	0.0497	0.0242	0.0092	94.
0.1462	0.0076	0.0497	0.0230	0.0087	95.
0.1389	0.0073	0.0497	0.0219	0.0083	96.
0.1320	0.0069	0.0497	0.0208	0.0079	97.
0.1254	0.0066	0.0497	0.0198	0.0075	98.
0.1192	0.0062	0.0497	0.0188	0.0071	99.
0.1133	0.0059	0.0497	0.0178	0.0068	100.

Projections using regressions (13), (16), and (19) were also made. They lead to increasing efficiency functions in the early vintages in (16), and strange results in (19). They are given in A2.A, A2.B, A2.C of Appendix A.

Box-Cox regressions

Since regression estimates are used to predict vintage acquisition prices, the form of the regression (i.e., linear, log-linear, semi-log, or other) is obviously of great importance. Yet, we have so far said little about the correct functional form. Only (15–16 or 18–19) are comparable on the basis of the R^2 criterion, and they represent only a limited range of possibilities. Fortunately, the Box-Cox procedure permits the simultaneous estimation of parameters determining the form of the regression and the economic parameters. This procedure is based on a regression of the form

$$\frac{q_i^{\xi_1} - 1}{\xi_1} = \alpha + \beta \frac{s_i^{\xi_2} - 1}{\xi_2} + \gamma \frac{t_i^{\xi_3} - 1}{\xi_3} + u_i \tag{23}$$

The parameters ξ_1, ξ_2, ξ_3 are the Box-Cox parameters and α, β, γ the economic parameters. When

$$\xi_1 = \xi_2 = \xi_3 = 1, \text{ the regression is linear} \tag{24}$$

$$\xi_1 \to 0, \xi_2 = 1, \xi_3 = 1, \text{ the regression is semi-log} \tag{25}$$

$$\xi_1 \to 0, \xi_2 \to 0, \xi_3 \to 0, \text{ the regression is log-log} \tag{26}$$

These restrictions are, of course, not the only possibilities. The strength of this technique for our purposes is that, in principle, the point estimates of the ξ_1 (and α, β, and γ) can be used to predict the acquisition prices without imposing the exact functional form *a priori* (i.e., a more general class of forms can be considered). It should, of course, be pointed out that the Box-Cox form does not include all possible functional forms.

The Box-Cox regression is non-linear in the parameters. OLS is therefore not a feasible estimation technique. One possible strategy is to use maximum likelihood estimation and likelihood ratio testing. These procedures have desirable asymptotic properties, and are feasible in our study given the relatively large number of data points. In particular, it is useful to exploit the fact that the statistic $-2 \ln \lambda$, where λ is the ratio of the constrained likelihood function to the unconstrained likelihood function, is approximately distributed as chi-square when the sample is large.

A steepest ascent gradient method was used to derive estimates of the Box-Cox and economic parameters. Because of the computer costs involved in using this technique, only the untransformed and L_0 transformations were analyzed. The resulting log-likelihood ratios are given in table 14 for the cases where (i) there are no restrictions placed on the ξ_1, (ii) $\xi_2 = \xi_3$, and (iii) $\xi_1 = \xi_2 = \xi_3$ (there were convergence problems with (i); this is a problem which unfortunately limits this technique). Table 14 also gives the log-likelihood ratios for the OLS regressions of the preceding sections, since OLS parameter estimates are also maximum likelihood estimates (MLE). Regressions 11–13 (14–16, 17–19) are special cases of the Box-Cox formulation (23), and the corresponding likelihood ratios can therefore be used to test the restrictions implied by the regressions (i.e., in (11): $\xi_1 = \xi_2 = \xi_3 = 1$, in (12) $\xi_1 \to 0$, $\xi_2 = \xi_3 = 1$, etc.). Regressions (13), (16), and (19) are not derivable from (23), but are included anyway.

The statistic $-2 \ln \lambda$ can be computed from table 14; $\ln \lambda$ is the difference between the least constrained log-likelihood value and the log-likelihoods corresponding to additional constraints (i.e. corresponding to nested constraints; only (13), (16), and (19) are not nested). Because of "nesting," λ varies between 0 and 1, implying that $-2 \ln \lambda$

TABLE 14.—*Summary of log-likelihood ratios*

	Outlier in	Outlier out
Untransformed		
Box-Cox:		
Unconstrained	NC	NC
$\xi_2 = \xi_3$	−288.17	−279.25
$\xi_1 = \xi_2 = \xi_3$	−293.60	−281.5
A.1	−306.64	−300.43
A.2	−292.02	−282.88
* A.3	−286.31	−282.47
L_0		
Box-Cox:		
Unconstrained	−267.77	−261.0
$\xi_2 = \xi_3$	−270.30	−261.02
$\xi_1 = \xi_2 = \xi_3$	−280.8	−268.2
B.1	−302.32	−296.20
B.2	−274.50	−265.11
* B.3	−268.25	−264.35
S_3		
Box-Cox:		
Unconstrained		
$\xi_2 = \xi_3$		
$\xi_1 = \xi_2 = \xi_3$		
C.1	−303.91	−297.51
C.2	−293.16	−286.15
* C.3	−273.45	−268.87

NC = Non-convergent
* = Non-nested hypothesis

varies between 0 and ∞. When $\lambda = 1$, the constraint is valid and $-2 \ln \lambda = 0$. This is the null hypothesis; failure to reject the null hypothesis implies the failure to reject the validity of the constraint. Recalling that $-2 \ln \lambda$ is approximately chi-squared, Figure 3 portrays the region of rejection at the 95 percent level of significance (regions II and III), and at the 99 percent level of significance (region III).

The results of the test may be summarized as follows:

(a) with the *untransformed data* and the *outlier in*, all the constraints are rejected at the 95 percent level of significance.[1]

(b) with *untransformed data* and the *outlier out*, all constraints are rejected at the 95 percent level of significance.[2]

(c) with the L_0 *transformation* and the *outlier in*, all constraints are rejected at the 95 percent level of significance.[3]

(d) with the L_0 *transformation* and the *outlier out*, only the constraint $\xi_2 = \xi_3$ cannot be rejected, at the 95 percent level of significance.[4]

The following tentative conclusions can be drawn from the analysis: (1) there is no support for the hypothesis that depreciation is of the

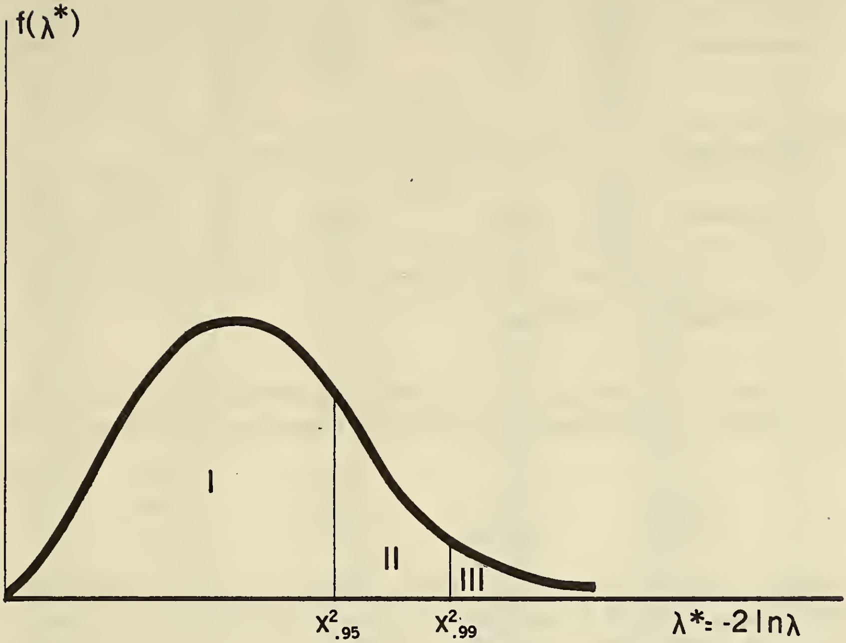
[1] The constraint (12) cannot be rejected at the 99% level of significance. Note, also, that the unconstrained case did not converge, so that the constraint $\xi_2 = \xi_3$ cannot be tested.

[2] The constraints (12) and $\xi_1 = \xi_2 = \xi_3$ cannot be rejected at the 99 percent level of significance. Note, again, that the unconstrained case did not converge.

[3] The constraint $\xi_2 = \xi_3$ cannot be rejected at the 99 percent level of significance.

[4] The constraint (12) cannot be rejected at the 99 percent level of significance.

Figure 3



straight-line form, (2) there is only marginal support of the hypothesis that depreciation is of the declining balance form, and (3) the only strong evidence forthcoming is that, in case (d), depreciation and inflation enter the analysis in the same form. Of the cases considered, (d) is perhaps intuitively the most appealing.

These conclusions are generally supported by the asymptotic Normal test on the individual parameter estimates. These estimates are given in Table 15 for the Box-Cox unconstrained and $\xi_2 = \xi_3$ cases. Standard errors are given in parentheses and the statistics $\hat{\xi}_i/\hat{\sigma}\xi_i$, which are asymptotically normal, are easily computed. When this is done, it is found that ξ_1 is significantly different (at the 95 percent level) from both 0 and 1, ξ_2 is not significantly different from 1 (both separately and under the restriction $\xi_2 = \xi_3$), and that the individual ξ_3 are not significantly different from either 0 or 1. This constitutes further evidence that depreciation is not of the declining balance form.

The MLE of the α, β, and γ are rather disappointing—no estimate is significant at the 95 percent level. Several of the point estimates are, however, significant at the 90 percent level.

In order to analyze the problem further, a grid search of values of ξ_1 and ξ_2 (= ξ_3) was done using OLS on the untransformed data. It was

TABLE 15.—*Summary of Box-Cox point estimates*

	ξ_1	ξ_2		ξ_3	α	β	γ
Outlier in:							
L_0 Untransformed:							
$\xi_2 = \xi_3$	0.288		1.109		0.764	−0.016	0.028
	(0.048)		(0.302)		(0.713)	(0.018)	(0.033)
Unconstrained	0.223	0.909		2.569	1.688	−0.050	0.00007
	(0.050)	(0.181)		(1.150)	(0.676)	(−.033)	(0.0005)
$\xi_2 = \xi_3$	0.201		1.070		0.836	−0.027	0.026
	(0.049)		(0.175)		(0.535)	(0.018)	(0.019)
Outlier out:							
L_0 Untransformed:							
$\xi_2 = \xi_3$	0.250		0.770		−1.454	−0.050	0.148
	(0.048)		(0.253)		(1.377)	(0.043)	(0.154)
Unconstrained	0.198	0.928		0.695	−1.483	−0.045	0.179
	(0.049)	(0.170)		(0.903)	(4.096)	(0.028)	(0.646)
$\xi_2 = \xi_3$	0.202		0.915		−0.679	−0.048	0.074
	(0.049)		(0.159)		(−.812)	(0.028)	(0.051)

Note.—Standard error in parentheses.

found that $\xi_1 = 0.288$, $\xi_2 = \xi_3 = 1.109$ did indeed give a unique maximum. Also, the estimates of the economic parameters were found to be

$$\begin{array}{cccc} \alpha & \beta & \gamma & R^2 \\ 0.764 & -0.0158 & 0.0276 & 0.346 \\ (1.18) & (-5.58) & (4.17) & \end{array} \tag{27}$$

t-statistics are in parentheses. The standard errors are 0.646, 0.003, 0.007 respectively. It is interesting to note that these are far below the standard errors given by MLE, while the point estimates remain unchanged. This comes about because the estimates of economic parameters are very sensitive to the functional form, even in a close neighborhood of the maximum. In other words, varying ξ_1 and $\xi_2 = \xi_3$ in the vicinity of $\xi_1 = .288$ $\xi_2 = \xi_3 = 1.109$ causes the estimates of α, β, and γ to jump around a great deal. Fixing ξ_1 and $\xi_2 = \xi_3$ in this region results in significant estimates of α, β, γ.

It should be noted, finally, that the point estimates in (27) are close to those in (12).

Projections using Box-Cox estimates

Projections of asset prices, rentals, the rate of depreciation, and the relative asset efficiency were made using Box-Cox point estimates under the L_0 transformation and the restriction $\xi_2 = \xi_3$ (this restriction was not rejected by the likelihood-ratio test with the outlier out). The projections are for $t = 1970$ and are presented in table 16.A for the *outlier in* and in table 16.B for the *outlier out*. In both cases, the rate of depreciation is not constant. With the outlier in, the rate is uniformly increasing with vintage (from 1.64 percent to 3.7 percent). With the outlier out, the rate of depreciation is U-shaped (it declines from 2.66 percent to 2.34 percent, and then rises uniformly to 3.12 percent.

Comparison of Table 16A and Table 13B, the projection assuming the L_o transformation (outlier in) and geometric depreciation, reveals that there exists a divergence between the efficiency function Φ_s. This divergence implies that an error would be made if capital stocks were estimated by a perpetual inventory method using the constant rate of depreciation from regression (15).

TABLE 16A.—*Projections of asset prices, rentals, and efficiencies using Box-Cox estimates with the L_0 transformation, outlier in (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
13.2095	−0.0000	−0.0000	−0.0000	−0.0000	1.
12.9928	0.2167	0.0164	1.5377	1.0000	2.
12.7710	0.2218	0.0171	1.5210	0.9892	3.
12.5471	0.2240	0.0175	1.5011	0.9762	4.
12.3224	0.2247	0.0179	1.4794	0.9621	5.
12.0977	0.2246	0.0182	1.4568	0.9474	6.
11.8738	0.2239	0.0185	1.4337	0.9324	7.
11.6510	0.2228	0.0188	1.4102	0.9171	8.
11.4297	0.2214	0.0190	1.3865	0.9017	9.
11.2100	0.2197	0.0192	1.3627	0.8862	10.
10.9922	0.2178	0.0194	1.3388	0.8707	11.
10.7764	0.2158	0.0196	1.3150	0.8552	12.
10.5628	0.2136	0.0198	1.2912	0.8398	13.
10.3515	0.2113	0.0200	1.2676	0.8244	14.
10.1425	0.2090	0.0202	1.2441	0.8091	15.
9.9359	0.2066	0.0204	1.2208	0.7939	16.
9.7319	0.2041	0.0205	1.1977	0.7789	17.
9.5303	0.2015	0.0207	1.1747	0.7640	18.
9.3314	0.1989	0.0209	1.1520	0.7492	19.
9.1351	0.1963	0.0210	1.1295	0.7345	20.
8.9414	0.1937	0.0212	1.1072	0.7200	21.
8.7504	0.1910	0.0214	1.0852	0.7057	22.
8.5621	0.1883	0.0215	1.0634	0.6916	23.
8.3764	0.1856	0.0217	1.0418	0.6776	24.
8.1935	0.1829	0.0218	1.0206	0.6637	25.
8.0132	0.1802	0.0220	0.9996	0.6501	26.
7.8357	0.1775	0.0222	0.9788	0.6366	27.
7.6609	0.1748	0.0223	0.9584	0.6233	28.
7.4888	0.1721	0.0225	0.9382	0.6102	29.
7.3194	0.1694	0.0226	0.9183	0.5972	30.
7.1527	0.1667	0.0228	0.8987	0.5844	31.
6.9886	0.1640	0.0229	0.8793	0.5718	32.
6.8273	0.1614	0.0231	0.8602	0.5594	33.
6.6685	0.1587	0.0232	0.8414	0.5472	34.
6.5125	0.1561	0.0234	0.8229	0.5352	35.
6.3590	0.1534	0.0236	0.8047	0.5233	36.
6.2082	0.1508	0.0237	0.7867	0.5116	37.
6.0600	0.1482	0.0239	0.7691	0.5001	38.
5.9143	0.1457	0.0240	0.7517	0.4888	39.
5.7712	0.1431	0.0242	0.7345	0.4777	40.
5.6307	0.1406	0.0244	0.7177	0.4667	41.
5.4926	0.1380	0.0245	0.7011	0.4560	42.
5.3571	0.1356	0.0247	0.6848	0.4454	43.
5.2240	0.1331	0.0248	0.6688	0.4349	44.
5.0933	0.1306	0.0250	0.6530	0.4247	45.
4.9651	0.1282	0.0252	0.6375	0.4146	46.
4.8393	0.1258	0.0253	0.6223	0.4047	47.
4.7159	0.1234	0.0255	0.6074	0.3950	48.
4.5948	0.1211	0.0257	0.5927	0.3854	49.
4.4761	0.1187	0.0258	0.5782	0.3760	50.

TABLE 16A.—*Projections of asset prices, rentals, and efficiencies using Box-Cox estimates with the L_0 transformation, outlier in (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
4.3597	−0.1164	−0.0260	−0.5640	−0.3668	51.
4.2455	0.1142	0.0262	0.5501	0.3578	52.
4.1336	0.1119	0.0264	0.5365	0.3489	53.
4.0239	0.1097	0.0265	0.5230	0.3402	54.
3.9164	0.1075	0.0267	0.5099	0.3316	55.
3.8111	0.1053	0.0269	0.4970	0.3232	56.
3.7079	0.1032	0.0271	0.4843	0.3149	57.
3.6069	0.1010	0.0272	0.4718	0.3069	58.
3.5080	0.0989	0.0274	0.4596	0.2989	59.
3.4111	0.0969	0.0276	0.4477	0.2911	60.
3.3162	0.0948	0.0278	0.4359	0.2835	61.
3.2234	0.0928	0.0280	0.4245	0.2760	62.
3.1326	0.0908	0.0282	0.4132	0.2687	63.
3.0437	0.0889	0.0284	0.4021	0.2615	64.
2.9567	0.0869	0.0286	0.3913	0.2545	65.
2.8717	0.0850	0.0288	0.3807	0.2476	66.
2.7885	0.0832	0.0290	0.3703	0.2408	67.
2.7072	0.0813	0.0292	0.3602	0.2342	68.
2.6277	0.0795	0.0294	0.3502	0.2278	69.
2.5501	0.0777	0.0296	0.3405	0.2214	70.
2.4741	0.0759	0.0298	0.3309	0.2152	71.
2.4000	0.0742	0.0300	0.3216	0.2091	72.
2.3275	0.0725	0.0302	0.3125	0.2032	73.
2.2567	0.0708	0.0304	0.3035	0.1974	74.
2.1876	0.0691	0.0306	0.2948	0.1917	75.
2.1202	0.0675	0.0308	0.2862	0.1861	76.
2.0543	0.0658	0.0311	0.2779	0.1807	77.
1.9901	0.0643	0.0313	0.2697	0.1754	78.
1.9274	0.0627	0.0315	0.2617	0.1702	79.
1.8662	0.0612	0.0317	0.2539	0.1651	80.
1.8066	0.0597	0.0320	0.2463	0.1602	81.
1.7484	0.0582	0.0322	0.2388	0.1553	82.
1.6917	0.0567	0.0324	0.2316	0.1506	83.
1.6364	0.0553	0.0327	0.2244	0.1460	84.
1.5825	0.0539	0.0329	0.2175	0.1415	85.
1.5300	0.0525	0.0332	0.2107	0.1371	86.
1.4789	0.0511	0.0334	0.2041	0.1328	87.
1.4291	0.0498	0.0337	0.1977	0.1286	88.
1.3806	0.0485	0.0339	0.1914	0.1245	89.
1.3334	0.0472	0.0342	0.1853	0.1205	90.
1.2875	0.0459	0.0344	0.1793	0.1166	91.
1.2428	0.0447	0.0347	0.1734	0.1128	92.
1.1993	0.0435	0.0350	0.1678	0.1091	93.
1.1571	0.0423	0.0353	0.1622	0.1055	94.
1.1159	0.0411	0.0355	0.1568	0.1020	95.
1.0760	0.0400	0.0358	0.1516	0.0986	96.
1.0371	0.0388	0.0361	0.1464	0.0952	97.
0.9994	0.0377	0.0364	0.1415	0.0920	98.
0.9627	0.0367	0.0367	0.1366	0.0888	99.
0.9271	0.0356	0.0370	0.1319	0.0858	100.

TABLE 16B.—*Projections of asset prices, rentals, and efficiencies using Box-Cox estimates with the L_0 transformation, outlier out (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
14.9266	−0.0000	−0.0000	−0.0370	−0.0000	1.
14.5293	0.3974	0.0266	1.8900	1.0000	2.
14.1572	0.3721	0.0256	1.8250	0.9656	3.
13.8030	0.3542	0.0250	1.7699	0.9364	4.
13.4632	0.3398	0.0246	1.7201	0.9101	5.
13.1357	0.3275	0.0243	1.6738	0.8856	6.
12.8191	0.3166	0.0241	1.6302	0.8625	7.
12.5124	0.3067	0.0239	1.5887	0.8405	8.
12.2147	0.2977	0.0238	1.5489	0.8195	9.
11.9254	0.2893	0.0237	1.5108	0.7993	10.
11.6440	0.2814	0.0236	1.4740	0.7799	11.
11.3700	0.2740	0.0235	1.4384	0.7610	12.
11.1031	0.2669	0.0235	1.4039	0.7428	13.
10.8428	0.2602	0.0234	1.3705	0.7251	14.
10.5890	0.2538	0.0234	1.3381	0.7080	15.
10.3414	0.2476	0.0234	1.3065	0.6913	16.
10.0997	0.2417	0.0234	1.2759	0.6750	17.
9.8637	0.2360	0.0234	1.2460	0.6592	18.
9.6332	0.2305	0.0234	1.2169	0.6438	19.
9.4080	0.2252	0.0234	1.1885	0.6288	20.
9.1879	0.2200	0.0234	1.1608	0.6142	21.
8.9729	0.2151	0.0234	1.1338	0.5999	22.
8.7627	0.2102	0.0234	1.1075	0.5860	23.
8.5572	0.2055	0.0235	1.0818	0.5724	24.
8.3562	0.2009	0.0235	1.0567	0.5591	25.
8.1597	0.1965	0.0235	1.0321	0.5461	26.
7.9676	0.1922	0.0235	1.0081	0.5334	27.
7.7796	0.1879	0.0236	0.9847	0.5210	28.
7.5958	0.1838	0.0236	0.9618	0.5089	29.
7.4160	0.1798	0.0237	0.9394	0.4970	30.
7.2400	0.1759	0.0237	0.9175	0.4855	31.
7.0679	0.1721	0.0238	0.8961	0.4741	32.
6.8996	0.1684	0.0238	0.8752	0.4630	33.
6.7348	0.1647	0.0239	0.8547	0.4522	34.
6.5736	0.1612	0.0239	0.8347	0.4416	35.
6.4159	0.1577	0.0240	0.8151	0.4313	36.
6.2616	0.1543	0.0241	0.7959	0.4211	37.
6.1106	0.1510	0.0241	0.7772	0.4112	38.
5.9628	0.1478	0.0242	0.7588	0.4015	39.
5.8182	0.1446	0.0242	0.7409	0.3920	40.
5.6768	0.1415	0.0243	0.7233	0.3827	41.
5.5384	0.1384	0.0244	0.7061	0.3736	42.
5.4029	0.1354	0.0245	0.6893	0.3647	43.
5.2704	0.1325	0.0245	0.6728	0.3560	44.
5.1407	0.1297	0.0246	0.6567	0.3475	45.
5.0138	0.1269	0.0247	0.6409	0.3391	46.
4.8897	0.1241	0.0248	0.6255	0.3310	47.
4.7682	0.1215	0.0248	0.6104	0.3230	48.
4.6494	0.1188	0.0249	0.5957	0.3152	49.
4.5331	0.1163	0.0250	0.5812	0.3075	50.
4.4194	0.1137	0.0251	0.5670	0.3000	51.
4.3082	0.1113	0.0252	0.5532	0.2927	52.
4.1993	0.1088	0.0253	0.5397	0.2855	53.
4.0929	0.1065	0.0254	0.5264	0.2785	54.
3.9887	0.1041	0.0254	0.5134	0.2716	55.
3.8869	0.1019	0.0255	0.5007	0.2649	56.
3.7873	0.0996	0.0256	0.4883	0.2584	57.
3.6898	0.0974	0.0257	0.4762	0.2519	58.
3.5946	0.0953	0.0258	0.4643	0.2456	59.
3.5014	0.0932	0.0259	0.4526	0.2395	60.
3.4103	0.0911	0.0260	0.4412	0.2335	61.
3.3212	0.0891	0.0261	0.4301	0.2276	62.
3.2341	0.0871	0.0262	0.4192	0.2218	63.

TABLE 16B.—*Projections of asset prices, rentals, and efficiencies using Box-Cox estimates with the L_0 transformation, outlier out (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
3.1490	−0.0851	−0.0263	−0.4086	−0.2162	64.
3.0657	0.0832	0.0264	0.3981	0.2106	65.
2.9843	0.0814	0.0265	0.3879	0.2053	66.
2.9048	0.0795	0.0267	0.3780	0.2000	67.
2.8271	0.0777	0.0268	0.3682	0.1948	68.
2.7511	0.0760	0.0269	0.3587	0.1898	69.
2.6769	0.0742	0.0270	0.3494	0.1848	70.
2.6043	0.0726	0.0271	0.3402	0.1800	71.
2.5334	0.0709	0.0272	0.3313	0.1753	72.
2.4642	0.0693	0.0273	0.3226	0.1707	73.
2.3965	0.0677	0.0275	0.3141	0.1662	74.
2.3304	0.0661	0.0276	0.3057	0.1618	75.
2.2658	0.0646	0.0277	0.2976	0.1575	76.
2.2028	0.0631	0.0278	0.2896	0.1532	77.
2.1412	0.0616	0.0280	0.2819	0.1491	78.
2.0811	0.0601	0.0281	0.2743	0.1451	79.
2.0223	0.0587	0.0282	0.2668	0.1412	80.
1.9650	0.0573	0.0283	0.2596	0.1373	81.
1.9091	0.0560	0.0285	0.2525	0.1336	82.
1.8544	0.0546	0.0286	0.2455	0.1299	83.
1.8011	0.0533	0.0288	0.2388	0.1263	84.
1.7491	0.0520	0.0289	0.2321	0.1228	85.
1.6983	0.0508	0.0290	0.2257	0.1194	86.
1.6487	0.0495	0.0292	0.2194	0.1161	87.
1.6004	0.0483	0.0293	0.2132	0.1128	88.
1.5532	0.0472	0.0295	0.2072	0.1096	89.
1.5072	0.0460	0.0296	0.2013	0.1065	90.
1.4624	0.0449	0.0298	0.1956	0.1035	91.
1.4186	0.0438	0.0299	0.1900	0.1005	92.
1.3760	0.0427	0.0301	0.1845	0.0976	93.
1.3344	0.0416	0.0302	0.1792	0.0948	94.
1.2938	0.0405	0.0304	0.1740	0.0921	95.
1.2543	0.0395	0.0305	0.1689	0.0894	96.
1.2158	0.0385	0.0307	0.1640	0.0867	97.
1.1782	0.0375	0.0309	0.1591	0.0842	98.
1.1417	0.0366	0.0310	0.1544	0.0817	99.
1.1060	0.0356	0.0312	0.1498	0.0793	100.

Appendix A

TABLE A1.—*Data summary for medical building class*

	Vintage					
Date	0	1–10	11–30	31–50	Older	Row Total
1968–73	11.7767[1]	16.6396	16.6700	7.3113	2.0044	13.2055
	5[2]	8	4	2	2	21
1962–67	14.1586	13.1558	11.4603	10.9601	3.1995	12.8677
	19	5	2	2	2	80
1956–61	11.4657	0	13.3937	8.7202	4.0193	9.5230
	9	0	2	4	4	19
1950–55	10.9527	11.6637	0	0	2.0660	10.2023
	5	3	0	0	1	9
Earlier	5.2983	3.3333	5.8897	4.6071	1.0714	4.9510
	7	1	4	2	1	15
Column total	11.6208	13.9542	11.6623	8.2245	2.9712	10.7488
	45	17	12	10	10	94

[1] Cost per square foot
[2] Number of observations

TABLE A2–A.—*Projections of asset prices, rentals and efficiencies using regression (13) (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
15.6544	−0.0000	−0.0000	−0.0149	−0.0000	1.
15.6609	0.5934	0.0379	2.1589	1.0000	2.
14.6451	0.4159	0.0276	1.9220	0.8903	3.
14.3028	0.3422	0.0234	1.8067	0.8369	4.
14.0020	0.3008	0.0210	1.7311	0.8019	5.
13.7283	0.2738	0.0196	1.6740	0.7754	6.
13.4739	0.2544	0.0185	1.6272	0.7537	7.
13.2343	0.2396	0.0178	1.5870	0.7351	8.
13.0066	0.2277	0.0172	1.5512	0.7185	9.
12.7886	0.2179	0.0168	1.5186	0.7034	10.
12.5790	0.2096	0.0164	1.4885	0.6895	11.
12.3766	0.2024	0.0161	1.4603	0.6764	12.
12.1806	0.1960	0.0158	1.4336	0.6641	13.
11.9904	0.1902	0.0156	1.4083	0.6523	14.
11.8054	0.1850	0.0154	1.3841	0.6411	15.
11.6251	0.1803	0.0153	1.3608	0.6303	16.
11.4492	0.1759	0.0151	1.3384	0.6199	17.
11.2775	0.1718	0.0150	1.3167	0.6099	18.
11.1096	0.1679	0.0149	1.2957	0.6001	19.
10.9453	0.1643	0.0148	1.2752	0.5907	20.
10.7844	0.1609	0.0147	1.2554	0.5815	21.
10.6268	0.1576	0.0146	1.2360	0.5725	22.
10.4723	0.1545	0.0145	1.2172	0.5638	23.
10.3208	0.1515	0.0145	1.1988	0.5553	24.
10.1721	0.1487	0.0144	1.1808	0.5469	25.
10.0262	0.1459	0.0143	1.1632	0.5388	26.
9.8829	0.1433	0.0143	1.1459	0.5308	27.
9.7421	0.1408	0.0142	1.1291	0.5230	28.
9.6038	0.1383	0.0142	1.1125	0.5153	29.
9.4679	0.1359	0.0142	1.0963	0.5078	30.
9.3343	0.1336	0.0141	1.0804	0.5004	31.
9.2029	0.1314	0.0141	1.0648	0.4932	32.
9.0737	0.1292	0.0140	1.0495	0.4861	33.
8.9466	0.1271	0.0140	1.0345	0.4792	34.
8.8216	0.1250	0.0140	1.0197	0.4723	35.
8.6986	0.1230	0.0139	1.0052	0.4656	36.
8.5775	0.1211	0.0139	0.9909	0.4590	37.
8.4584	0.1191	0.0139	0.9769	0.4525	38.
8.3411	0.1173	0.0139	0.9681	0.4461	39.
8.2257	0.1154	0.0138	0.9496	0.4398	40.
8.1120	0.1137	0.0138	0.9362	0.4337	41.
8.0001	0.1119	0.0138	0.9231	0.4276	42.
7.8899	0.1102	0.0138	0.9102	0.4216	43.
7.7814	0.1085	0.0138	0.8975	0.4157	44.
7.6745	0.1069	0.0137	0.8850	0.4099	45.
7.5692	0.1053	0.0137	0.8727	0.4043	46.
7.4655	0.1037	0.0137	0.8606	0.3986	47.
7.3634	0.1022	0.0137	0.8487	0.3931	48.
7.2627	0.1006	0.0137	0.8370	0.3877	49.
7.1636	0.0992	0.0137	0.8254	0.3823	50.

TABLE A2-A.—*Projections of asset prices, rentals and efficiencies using regression (13) (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
7.0659	-0.0977	-0.0136	-0.8141	-0.3771	51.
6.9696	0.0963	0.0136	0.8029	0.3719	52.
6.8747	0.0949	0.0136	0.7918	0.3668	53.
6.7812	0.0935	0.0136	0.7810	0.3617	54.
6.6891	0.0921	0.0136	0.7703	0.3568	55.
6.5983	0.0908	0.0136	0.7597	0.3519	56.
6.5088	0.0895	0.0136	0.7493	0.3471	57.
6.4206	0.0882	0.0136	0.7391	0.3423	58.
6.3337	0.0869	0.0135	0.7290	0.3377	59.
6.2480	0.0857	0.0135	0.7191	0.3331	60.
6.1635	0.0845	0.0135	0.7093	0.3285	61.
6.0803	0.0833	0.0135	0.6996	0.3241	62.
5.9982	0.0821	0.0135	0.6901	0.3197	63.
5.9173	0.0809	0.0135	0.6807	0.3153	64.
5.8375	0.0798	0.0135	0.6715	0.3110	65.
5.7588	0.0787	0.0135	0.6624	0.3068	66.
5.6813	0.0775	0.0135	0.6534	0.3027	67.
5.6048	0.0765	0.0135	0.6446	0.2986	68.
5.5295	0.0754	0.0134	0.6359	0.2945	69.
5.4551	0.0743	0.0134	0.6273	0.2906	70.
5.3819	0.0733	0.0134	0.6188	0.2866	71.
5.3096	0.0723	0.0134	0.6104	0.2828	72.
5.2383	0.0713	0.0134	0.6022	0.2789	73.
5.1681	0.0703	0.0134	0.5941	0.2752	74.
5.0988	0.0693	0.0134	0.5861	0.2715	75.
5.0305	0.0683	0.0134	0.5782	0.2678	76.
4.9631	0.0674	0.0134	0.5704	0.2642	77.
4.8967	0.0664	0.0134	0.5628	0.2607	78.
4.8311	0.0655	0.0134	0.5552	0.2572	79.
4.7665	0.0646	0.0134	0.5477	0.2537	80.
4.7028	0.0637	0.0134	0.5404	0.2503	81.
4.6399	0.0628	0.0134	0.5331	0.2469	82.
4.5780	0.0620	0.0134	0.5260	0.2436	83.
4.5168	0.0611	0.0134	0.5189	0.2404	84.
4.4565	0.0603	0.0133	0.5120	0.2371	85.
4.3971	0.0595	0.0133	0.5051	0.2340	86.
4.3384	0.0586	0.0133	0.4984	0.2308	87.
4.2806	0.0578	0.0133	0.4917	0.2278	88.
4.2235	0.0571	0.0133	0.4851	0.2247	89.
4.1673	0.0563	0.0133	0.4786	0.2217	90.
4.1118	0.0555	0.0138	0.4722	0.2187	91.
4.0570	0.0547	0.0133	0.4659	0.2158	92.
4.0030	0.0540	0.0133	0.4597	0.2129	93.
3.9497	0.0533	0.0133	0.4536	0.2101	94.
3.8972	0.0525	0.0133	0.4475	0.2073	95.
3.8454	0.0518	0.0133	0.4415	0.2045	96.
3.7943	0.0511	0.0133	0.4357	0.2018	97.
3.7438	0.0504	0.0133	0.4299	0.1991	98.
3.6041	0.0497	0.0133	0.4241	0.1965	99.
3.6450	0.0491	0.0133	0.4185	0.1938	100.

TABLE A2-B.—*Projections of asset prices, rentals, and efficiencies using regression (16) (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
15.3146	−0.0000	−0.0000	−0.0286	−0.0000	1.
15.3980	0.0834	0.0054	1.4481	1.0000	2.
15.2432	0.1548	0.0101	1.6946	1.1702	3.
14.9943	0.2489	0.0163	1.7732	1.2246	4.
14.6981	0.2962	0.0198	1.7956	1.2406	5.
14.3761	0.3221	0.0219	1.7919	1.2374	6.
14.0397	0.3364	0.0234	1.7740	1.2251	7.
13.6959	0.3438	0.0245	1.7477	1.2069	8.
13.3492	0.3467	0.0253	1.7163	1.1852	9.
13.0026	0.3466	0.0260	1.6816	1.1612	10.
12.6581	0.3445	0.0265	1.6448	1.1358	11.
12.3172	0.3409	0.0269	1.6067	1.1095	12.
11.9809	0.3362	0.0273	1.5680	1.0828	13.
11.6501	0.3308	0.0276	1.5289	1.0558	14.
11.3254	0.3248	0.0279	1.4898	1.0288	15.
11.0070	0.3184	0.0281	1.4509	1.0020	16.
10.6953	0.3117	0.0283	1.4124	0.9754	17.
10.3905	0.3048	0.0285	1.3743	0.9491	18.
10.0927	0.2978	0.0287	1.3369	0.9232	19.
9.8019	0.2907	0.0288	1.3000	0.8977	20.
9.5183	0.2837	0.0289	1.2638	0.8728	21.
9.2417	0.2766	0.0291	1.2284	0.8483	22.
8.9722	0.2695	0.0292	1.1937	0.8243	23.
8.7096	0.2626	0.0293	1.1598	0.8009	24.
8.4539	0.2557	0.0294	1.1266	0.7780	25.
8.2050	0.2489	0.0294	1.0943	0.7557	26.
7.9628	0.2422	0.0295	1.0627	0.7339	27.
7.7272	0.2356	0.0296	1.0319	0.7126	28.
7.4981	0.2292	0.0297	1.0019	0.6919	29.
7.2752	0.2228	0.0297	0.9726	0.6717	30.
7.0586	0.2166	0.0298	0.9442	0.6520	31.
6.8480	0.2106	0.0298	0.9164	0.6329	32.
6.6434	0.2046	0.0299	0.8894	0.6142	33.
6.4445	0.1988	0.0299	0.8632	0.5961	34.
6.2514	0.1932	0.0300	0.8376	0.5784	35.
6.0637	0.1877	0.0300	0.8128	0.5613	36.
5.8814	0.1823	0.0301	0.7886	0.5446	37.
5.7044	0.1770	0.0301	0.7652	0.5284	38.
5.5325	0.1719	0.0301	0.7423	0.5126	39.
5.3656	0.1669	0.0302	0.7202	0.4973	40.
5.2036	0.1620	0.0302	0.6986	0.4824	41.
5.0462	0.1573	0.0302	0.6777	0.4680	42.
4.8935	0.1527	0.0303	0.6573	0.4539	43.
4.7453	0.1482	0.0303	0.6376	0.4403	44.
4.6014	0.1439	0.0303	0.6184	0.4270	45.
4.4618	0.1396	0.0303	0.5998	0.4142	46.
4.3263	0.1355	0.0304	0.5817	0.4017	47.
4.1948	0.1315	0.0304	0.5641	0.3896	48.
4.0673	0.1276	0.0304	0.5471	0.3778	49.
3.9435	0.1238	0.0304	0.5305	0.3664	50.

TABLE A2–B.—*Projections of asset prices, rentals, and efficiencies using regression (16) (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
3.8234	−0.1201	−0.0305	−0.5145	−0.3553	51.
3.7068	0.1165	0.0305	0.4989	0.3445	52.
3.5938	0.1130	0.0305	0.4837	0.3340	53.
3.4841	0.1097	0.0305	0.4690	0.3239	54.
3.3777	0.1064	0.0305	0.4548	0.3141	55.
3.2745	0.1032	0.0306	0.4410	0.3045	56.
3.1745	0.1001	0.0306	0.4275	0.2953	57.
3.0774	0.0971	0.0306	0.4145	0.2863	58.
2.9832	0.0942	0.0306	0.4019	0.2775	59.
2.8919	0.0913	0.0306	0.3896	0.2691	60.
2.8033	0.0886	0.0306	0.3778	0.2609	61.
2.7174	0.0859	0.0306	0.3662	0.2529	62.
2.6341	0.0833	0.0307	0.3550	0.2452	63.
2.5533	0.0808	0.0307	0.3442	0.2377	64.
2.4750	0.0783	0.0307	0.3337	0.2304	65.
2.3990	0.0760	0.0307	0.3235	0.2234	66.
2.3253	0.0737	0.0307	0.3136	0.2165	67.
2.2539	0.0714	0.0307	0.3040	0.2099	68.
2.1846	0.0693	0.0307	0.2947	0.2035	69.
2.1175	0.0672	0.0307	0.2856	0.1972	70.
2.0524	0.0651	0.0308	0.2769	0.1912	71.
1.9892	0.0631	0.0308	0.2684	0.1853	72.
1.9280	0.0612	0.0308	0.2601	0.1796	73.
1.8687	0.0593	0.0308	0.2521	0.1741	74.
1.8111	0.0575	0.0308	0.2444	0.1688	75.
1.7554	0.0558	0.0308	0.2369	0.1636	76.
1.7013	0.0541	0.0308	0.2296	0.1586	77.
1.6488	0.0524	0.0308	0.2226	0.1537	78.
1.5980	0.0508	0.0308	0.2157	0.1490	79.
1.5487	0.0493	0.0308	0.2091	0.1444	80.
1.5010	0.0478	0.0308	0.2026	0.1399	81.
1.4547	0.0463	0.0309	0.1964	0.1356	82.
1.4098	0.0449	0.0309	0.1904	0.1315	83.
1.3663	0.0435	0.0309	0.1845	0.1274	84.
1.3241	0.0422	0.0309	0.1788	0.1235	85.
1.2832	0.0409	0.0309	0.1733	0.1197	86.
1.2435	0.0396	0.0309	0.1680	0.1160	87.
1.2051	0.0384	0.0309	0.1628	0.1124	88.
1.1679	0.0372	0.0309	0.1578	0.1089	89.
1.1318	0.0361	0.0309	0.1529	0.1056	90.
1.0968	0.0350	0.0309	0.1482	0.1023	91.
1.0629	0.0339	0.0309	0.1436	0.0992	92.
1.0300	0.0329	0.0309	0.1392	0.0961	93.
0.9981	0.0319	0.0309	0.1349	0.0931	94.
0.9673	0.0309	0.0309	0.1307	0.0903	95.
0.9373	0.0299	0.0309	0.1267	0.0875	96.
0.9083	0.0290	0.0310	0.1227	0.0848	97.
0.8802	0.0281	0.0310	0.1190	0.0821	98.
0.8530	0.0273	0.0310	0.1153	0.0796	99.
0.8265	0.0264	0.0310	0.1117	0.0771	100.

TABLE A2–C.—*Projections of asset prices, rentals and efficiencies using regression (19) (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
13.9154	−0.0000	−0.0000	0.0497	−0.0000	1.
18.5508	−4.6354	−0.3331	−3.2439	1.0000	2.
21.2759	−2.7250	−0.1469	−0.8699	0.2682	3.
22.9434	−1.6676	−0.0784	0.4600	−0.1418	4.
23.9205	−0.9771	−0.0426	1.3173	−0.4061	5.
24.4124	−0.4919	−0.0206	1.9002	−0.5858	6.
24.5500	−0.1376	−0.0056	2.3036	−0.7102	7.
24.4235	0.1264	0.0051	2.5814	−0.7958	8.
24.0984	0.3251	0.0133	2.7674	−0.8531	9.
23.6237	0.4747	0.0197	2.8846	−0.8892	10.
23.0369	0.5868	0.0248	2.9492	−0.9002	11.
22.3674	0.6695	0.0291	2.9732	−0.9166	12.
21.6384	0.7291	0.0326	2.9658	−0.9143	13.
20.8682	0.7702	0.0356	2.9340	−0.9045	14.
20.0717	0.7965	0.0382	2.8833	−0.8889	15.
19.2606	0.8110	0.0404	2.8182	−0.8688	16.
18.4446	0.8161	0.0424	2.7421	−0.8453	17.
17.6311	0.8135	0.0441	2.6579	−0.8194	18.
16.8263	0.8048	0.0456	2.5679	−0.7916	19.
16.0349	0.7914	0.0470	2.4740	−0.7627	20.
15.2608	0.7741	0.0483	2.3776	−0.7330	21.
14.5068	0.7540	0.0494	2.2801	−0.7029	22.
13.7751	0.7317	0.0504	2.1823	−0.6728	23.
13.0674	0.7077	0.0514	2.0852	−0.6428	24.
12.3848	0.6826	0.0522	1.9893	−0.6133	25.
11.7281	0.6568	0.0530	1.8952	−0.5843	26.
11.0975	0.6805	0.0538	1.8033	−0.5559	27.
10.4934	0.6042	0.0544	1.7139	−0.5284	28.
9.9155	0.5779	0.0551	1.6272	−0.5016	29.
9.3636	0.5619	0.0557	1.5434	−0.4758	30.
8.8372	0.5263	0.0562	1.4627	−0.4509	31.
8.3360	0.5013	0.0567	1.3850	−0.4270	32.
7.8591	0.4769	0.0572	1.3105	−0.4040	33.
7.4060	0.4531	0.0577	1.2390	−0.3820	34.
6.9758	0.4302	0.0581	1.1708	−0.3609	35.
6.5678	0.4080	0.0585	1.1056	−0.3408	36.
6.1812	0.3866	0.0589	1.0434	−0.3217	37.
5.8151	0.3661	0.0592	0.9842	−0.3034	38.
5.4687	0.3464	0.0596	0.9279	−0.2860	39.
5.1412	0.3275	0.0599	0.8744	−0.2696	40.
4.8317	0.3095	0.0602	0.8236	−0.2539	41.
4.5394	0.2923	0.0605	0.7754	−0.2390	42.
4.2636	0.2759	0.0608	0.7298	−0.2250	43.
4.0033	0.2602	0.0610	0.6866	−0.2117	44.
3.7580	0.2454	0.0613	0.6457	−0.1991	45.
3.5268	0.2312	0.0615	0.6070	−0.1871	46.
3.3089	0.2178	0.0618	0.5705	−0.1759	47.
3.1038	0.2051	0.0620	0.5360	−0.1652	48.
2.9108	0.1931	0.0622	0.5034	−0.1552	49.
2.7291	0.1816	0.0624	0.4727	−0.1457	50.

TABLE A2-C.—*Projections of asset prices, rentals, and efficiencies using regression (19) (t = 1970)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
2.5583	0.1708	0.0626	0.4438	−0.1368	51.
2.3977	0.1606	0.0628	0.4165	−0.1284	52.
2.2467	0.1510	0.0630	0.3907	−0.1205	53.
2.1048	0.1419	0.0631	0.3665	−0.1130	54.
1.9716	0.1333	0.0633	0.3437	−0.1060	55.
1.8464	0.1251	0.0635	0.3223	−0.0994	56.
1.7290	0.1175	0.0636	0.3021	−0.0931	57.
1.6187	0.1103	0.0638	0.2832	−0.0873	58.
1.5152	0.1035	0.0639	0.2653	−0.0818	59.
1.4181	0.0971	0.0641	0.2486	−0.0766	60.
1.3271	0.0910	0.0642	0.2329	−0.0718	61.
1.2417	0.0854	0.0643	0.2181	−0.0672	62.
1.1617	0.0800	0.0645	0.2042	−0.0630	63.
1.0866	0.0760	0.0646	0.1912	−0.0589	64.
1.0163	0.0703	0.0647	0.1790	−0.0552	65.
0.9505	0.0659	0.0648	0.1675	−0.0516	66.
0.8087	0.0617	0.0649	0.1568	−0.0483	67.
0.8309	0.0578	0.0650	0.1467	−0.0452	68.
0.7768	0.0541	0.0651	0.1372	−0.0423	69.
0.7261	0.0507	0.0652	0.1284	−0.0396	70.
0.6787	0.0475	0.0653	0.1201	−0.0370	71.
0.6343	0.0444	0.0654	0.1123	−0.0346	72.
0.5927	0.0416	0.0655	0.1050	−0.0324	73.
0.5538	0.0389	0.0656	0.0982	−0.0303	74.
0.5174	0.0364	0.0657	0.0918	−0.0283	75.
0.4833	0.0340	0.0658	0.0858	−0.0264	76.
0.4515	0.0318	0.0659	0.0802	−0.0247	77.
0.4217	0.0298	0.0660	0.0749	−0.0231	78.
0.3939	0.0279	0.0661	0.0700	−0.0216	79.
0.3678	0.0260	0.0661	0.0654	−0.0202	80.
0.3435	0.0244	0.0662	0.0611	−0.0188	81.
0.3207	0.0228	0.0663	0.0571	−0.0176	82.
0.2994	0.0213	0.0664	0.0533	−0.0164	83.
0.2795	0.0199	0.0664	0.0498	−0.0154	84.
0.2609	0.0186	0.0665	0.0465	−0.0143	85.
0.2436	0.0174	0.0666	0.0435	−0.0134	86.
0.2273	0.0162	0.0666	0.0406	−0.0125	87.
0.2122	0.0152	0.0667	0.0379	−0.0117	88.
0.1980	0.0142	0.0668	0.0354	−0.0109	89.
0.1848	0.0132	0.0668	0.0330	−0.0102	90.
0.1724	0.0124	0.0669	0.0308	−0.0095	91.
0.1609	0.0115	0.0669	0.0288	−0.0089	92.
0.1501	0.0108	0.0670	0.0269	−0.0083	93.
0.1400	0.0101	0.0671	0.0251	−0.0077	94.
0.1306	0.0094	0.0671	0.0234	−0.0072	95.
0.1219	0.0088	0.0672	0.0218	−0.0067	96.
0.1137	0.0082	0.0672	0.0204	−0.0063	97.
0.1060	0.0076	0.0673	0.0190	−0.0059	98.
0.0989	0.0071	0.0673	0.0177	−0.0055	99.
0.0922	0.0067	0.0674	0.0166	−0.0051	100.

Appendix B

TABLE B-1.—*OLS regression analysis with one observation removed*

Regression	Dependent variable	Independent variables					R^2
		Constant	s_t	$\ln s_t$	t_t	$\ln t_t$	
11′	q_t	−7.04	−0.089		0.282		0.291
		(−1.62)	(−4.07)		(4.50)		
12′	$\ln q_t$	−0.172	−0.015		0.037		0.456
		(−0.38)	(−6.60)		(5.62)		
13′	$\ln q_t$	−4.34	−0.014	−0.018	0.011	1.42	0.461
		(−0.80)	(3.36)	(0.24)	(0.32)	(0.78)	
14′	q_t^*	−6.29	−0.121		0.269		0.374
		(−1.52)	(−5.78)		(4.50)		
15′	$\ln q_t^*$	−0.129	−0.029		0.037		0.687
		(−0.29)	(−12.84)		(5.67)		
16′	$\ln q_t^*$	−4.82	−0.033	0.073	0.007	1.59	0.692
		(−0.90)	(−7.75)	(0.99)	(0.22)	(0.88)	
17′	q_t^{**}	−6.70	−0.119		0.280		0.376
		(−1.59)	(−5.60)		(4.17)		
18′	$\ln q_t^{**}$	−0.005	−0.050		0.039		0.745
		(−0.01)	(−15.67)		(4.17)		
19′	$\ln q_t^{**}$	−8.66	−0.077	0.551	−0.017	2.89	0.824
		(−1.36)	(−15.34)	(6.29)	(0.43)	(1.34)	

Note: Standard error in parentheses.

Addendum

Preliminary Results: Office Buildings

The Office Building class consists of 1656 usable observations. As indicated by Table B.2, there is a fairly good distribution of observations by year and vintage. The statistical analysis was performed on the raw data and on the data transformed using the Winfrey L_0 distribution. The Winfrey S_3 transformation was not used because of the costs involved.

The OLS point estimates from regressions (11), (12), (13), and (14), (15), (16) are given on pages 128 and 130, with the corresponding standard errors. The Box-Cox maximum likelihood estimates are given in Table B.4, with the corresponding standard errors. In Table B.5, the log-likelihoods for the Box-Cox and OLS regressions are tabled. The following results were obtained:

- The Box-Cox methodology results in extremely precise point estimates. Only the constants and the coefficient of t_i in the L_0-unconstrained case are not highly significant. Point estimates are of plausible magnitude and have the expected sign.
- The likelihood-ratio test rejects any constraint (in particular, straight line and geometric are not accepted.) technically.

It is worth noting that the good fit caused convergence problems in the maximum likelihood estimation. This technique turned out to be very expensive to use.

Projections of asset prices, rentals, the rate of depreciation, and the relative asset efficiency were made using (1) the Box-Cox L_0-unconstrained point estimates, and (2) the geometric (15) regression (for t = 1970). The Box-Cox projections (Tables B.6A and B.6B) indicate a U-shaped rate of depreciation.

TABLE B-2.—*Mean price per square foot by age and date*
Class: Office Buildings

Date	Age: New	1–6	7–12	13–18	19–24	25–30	31–36	37–42	43–48	Older	Row Total
1971–1973	18.19 4[1]	19.03 17	18.93 26	13.88 12	11.03 8	7.74 5	9.51 6	8.77 6	6.22 11	11.07 30	13.21 125 9.04[2]
1968–1970	19.61 63	19.72 33	14.27 30	15.44 21	8.93 12	12.44 9	6.42 6	11.17 13	8.75 12	6.28 50	14.01 249 9.50
1965–1967	16.16 132	15.36 31	10.35 19	12.51 13	8.87 11	10.57 5	14.87 12	8.45 13	7.88 13	8.78 33	13.46 282 10.33
1962–1964	16.81 136	14.16 21	10.31 9	10.88 15	14.20 9	10.36 2	11.77 9	12.22 10	9.43 8	6.62 21	13.44 240 9.75
1959–1961	14.43 113	14.82 17	9.01 11	6.01 2	9.73 4	17.08 4	11.08 8	12.27 9	5.17 4	7.71 18	12.95 190 9.51
1956–1958	14.89 95	10.50 4	13.00 5	6.94 6	5.61 2	10.03 4	6.23 12	9.14 7	6.75 5	5.32 12	12.17 152 9.95
1953–1955	13.15 73	11.76 14	1.56 1	9.83 3	9.89 5	8.14 8	5.06 3	3.67 1	6.10 4	6.15 12	11.19 124 11.12
1950–1952	11.31 39	5.15 3	0 0	0 0	6.66 3	3.95 3	3.50 1	3.62 1	3.62 3	3.28 10	8.56 63 8.66

1947–1949	9.82 31	9.66 1	0 0	5.95 1	7.37 4	10.51 4	2.31 1	6.40 4	3.54 4	4.15 5	8.26 55 5.85
Vintage	10.82 81	4.62 8	6.10 10	7.41 16	5.46 18	5.81 12	2.85 8	0.75 3	1.99 6	3.04 14	7.61 176 8.46
Column total	14.54 767 10.89	15.32 149 10.13	12.47 111 7.61	11.48 89 7.76	8.77 76 8.24	9.31 56 8.04	8.90 66 8.01	9.54 67 7.43	6.67 70 5.24	7.20 205 8.94	12.19 1656 9.80

[1] Second number indicates number of observations.
[2] Third number indicates standard deviation of price/sq. ft.

NOTE.—8 Outlier observations removed.

TABLE B-3.—*OLS point estimates*
Class: Office Buildings

Dependent variable		Constant	s_i	$\ln s_i$	t_i	$\ln t_i$	R^2
Untransformed							
(11)	q_i	3.63	−.127		.177		.123
			(.010)		(.020)		
(12)	$\ln q_i$	1.104	−.015		.022		.203
			(.001)		(.002)		
(13)	$\ln q_i$	2.228	−.006	−.135	.034	−.445	.223
			(.002)	(.024)	(.004)	(.132)	
L_0 transformation:							
(14)	q_i^*	4.517	−.182		.162		.220
			(.009)		(.018)		
(15)	$\ln q_i^*$	1.110	−.027		.022		.403
			(.001)		(.002)		
(16)	$\ln q_i^*$	2.210	−.023	−.056	.034	−.439	.409
			(.002)	(.024)	(.004)	(.132)	

Standard errors in parentheses.

TABLE B-4.—*Box-cox point estimates*
Class: Office Buildings

	ξ_1	ξ_2		ξ_3	α	β	γ
Untransformed data:							
$\xi_2 = \xi_3$	.248		.610		.603	−.0.83	.153
	(.011)		(.092)		(.339)	(.022)	(.056)
$\xi_1 = \xi_2 = \xi_3$		.277			.105	−.207	.480
		(.001)			(.321)	(.012)	(.043)
L_0 transformed data:							
Unconstrained	.212	.714		3.154	2.34	−.094	.000006
	(.011)	(.060)		(0.634)	(0.165)	(.017)	(.000016)
$\xi_2 = \xi_3$	.209		.801		1.010	−.071	.069
	(.011)		(.059)		(.208)	(.013)	(.017)
$\xi_1 = \xi_2 = \xi_3$		.294			.231	−.317	.467
		(.011)			(.319)	(.011	(.041)

Standard errors in parentheses.

TABLE B-5.—*Log-likelihood values*
Class: Office Buildings

	Log-likelihood
Untransformed data:	
Box-Cox	
Unconstrained	------
$\xi_2 = \xi_3$	−55.03.5
$\xi_1 = \xi_2 = \xi_3$	−5515.2
OLS:	
(11)	−6021.1
(12)	−5583.9
(13)	−5564.0
L_0 transformed data:	
Box-Cox	
Unconstrained	−5220.4
$\xi_2 = \xi_3$	−5236.2
$\xi_1 = \xi_2 = \xi_3$	−5282.7
OLS:	
(14)	−5893.3
(15)	−5302.6
(16)	−5295.4

Box-Cox formulation:

$$\frac{q_i^{\xi_1} - 1}{\xi_1} = \alpha + \beta\, \frac{s_i^{\xi_2} - 1}{\xi_2} + \gamma\, \frac{t_i^{\xi_3} - 1}{\xi_3}$$

TABLE B-6A.—*Projections of asset prices, rentals, and efficiencies using Box-Cox L_o-unconstrained point estimates* (t = *1970*)

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
16.9388	−0.0000	−0.0000	−0.0000	−0.0000	1.
16.1697	0.7691	0.0454	2.4630	1.0000	2.
15.5303	0.6394	0.0395	2.2564	0.9161	3.
14.9675	0.5627	0.0362	2.1158	0.8590	4.
14.4566	0.5090	0.0340	2.0057	0.8143	5.
13.9907	0.4679	0.0324	1.9138	0.7770	6.
13.5558	0.4349	0.0311	1.8339	0.7446	7.
13.1485	0.4073	0.0300	1.7629	0.7158	8.
12.7647	0.3838	0.0292	1.6987	0.6897	9.
12.4014	0.3633	0.0285	1.6398	0.6658	10.
12.0561	0.3452	0.0278	1.5854	0.6437	11.
11.7271	0.3290	0.0273	1.5346	0.6231	12.
11.4127	0.3144	0.0268	1.4871	0.6038	13.
11.1115	0.3011	0.0264	1.4424	0.5856	14.
10.8226	0.2889	0.0260	1.4001	0.5685	15.
10.5449	0.2777	0.0257	1.3600	0.5522	16.
10.2776	0.2673	0.0253	1.3218	0.5367	17.
10.0200	0.2576	0.0251	1.2854	0.5219	18.
9.7715	0.2486	0.0248	1.2506	0.5077	19.
9.5314	0.2401	0.0246	1.2172	0.4942	20.
9.2993	0.2321	0.0244	1.1852	0.4812	21.
9.0747	0.2246	0.0241	1.1545	0.4687	22.
8.8573	0.2175	0.0240	1.1249	0.4567	23.
8.6465	0.2107	0.0238	1.0965	0.4452	24.
8.4422	0.2043	0.0236	1.0690	0.4340	25.
8.2439	0.1983	0.0235	1.0425	0.4233	26.
8.0515	0.1925	0.0233	1.0169	0.4129	27.
7.8645	0.1869	0.0232	0.9921	0.4028	28.
7.6829	0.1817	0.0231	0.9681	0.3931	29.
7.5062	0.1766	0.0230	0.9449	0.3836	30.
7.3345	0.1718	0.0229	0.9224	0.3745	31.
7.1674	0.1671	0.0228	0.9006	0.3656	32.
7.0047	0.1627	0.0227	0.8794	0.3570	33.
6.8463	0.1584	0.0226	0.8588	0.3487	34.
6.6920	0.1543	0.0225	0.8389	0.3406	35.
6.5418	0.1503	0.0225	0.8195	0.3327	36.
6.3953	0.1465	0.0224	0.8006	0.3251	37.
6.2525	0.1428	0.0223	0.7823	0.3176	38.
6.1133	0.1392	0.0223	0.7645	0.3104	39.
5.9776	0.1358	0.0222	0.7471	0.3033	40.
5.8451	0.1324	0.0222	0.7302	0.2965	41.
5.7159	0.1292	0.0221	0.7137	0.2898	42.
5.5898	0.1261	0.0221	0.6977	0.2833	43.
5.4667	0.1231	0.0220	0.6821	0.2769	44.
5.3465	0.1202	0.0220	0.6668	0.2707	45.
5.2292	0.1173	0.0219	0.6520	0.2647	46.
5.1146	0.1146	0.0219	0.6375	0.2588	47.
5.0026	0.1119	0.0219	0.6234	0.2531	48.
4.8933	0.1093	0.0219	0.6096	0.2475	49.
4.7865	0.1068	0.0218	0.5962	0.2421	50.

TABLE B-6A.—*Projections of asset prices, rentals, and efficiencies using Box-Cox L_0-unconstrained point estimates*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
4.6821	−0.1044	−0.0218	−0.5830	−0.2367	51.
4.5800	0.1020	0.0218	0.5702	0.2315	52.
4.4803	0.0997	0.0218	0.5577	0.2264	53.
4.3828	0.0975	0.0218	0.5455	0.2215	54.
4.2875	0.0953	0.0217	0.5336	0.2166	55.
4.1943	0.0932	0.0217	0.5219	0.2119	56.
4.1932	0.0911	0.0217	0.5106	0.2073	57.
4.0141	0.0891	0.0217	0.4994	0.2028	58.
3.9269	0.0872	0.0217	0.4886	0.1984	59.
3.8417	0.0852	0.0217	0.4779	0.1940	60.
3.7583	0.0834	0.0217	0.4676	0.1898	61.
3.6767	0.0816	0.0217	0.4574	0.1857	62.
3.5969	0.0798	0.0217	0.4475	0.1817	63.
3.5188	0.0781	0.0217	0.4378	0.1777	64.
3.4424	0.0764	0.0217	0.4283	0.1739	65.
3.3676	0.0748	0.0217	0.4190	0.1701	66.
3.2944	0.0732	0.0217	0.4099	0.1664	67.
3.2228	0.0716	0.0217	0.4011	0.1628	68.
3.1528	0.0701	0.0217	0.3924	0.1593	69.
3.0842	0.0686	0.0218	0.3839	0.1559	70.
3.0170	0.0671	0.0218	0.3756	0.1525	71.
2.9513	0.0657	0.0218	0.3674	0.1492	72.
2.8869	0.0643	0.0218	0.3595	0.1459	73.
2.8240	0.0630	0.0218	0.3517	0.1428	74.
2.7623	0.0617	0.0218	0.3441	0.1397	75.
2.7019	0.0604	0.0219	0.3366	0.1367	76.
2.6428	0.0591	0.0219	0.3293	0.1337	77.
2.5850	0.0579	0.0219	0.3222	0.1308	78.
2.5283	0.0567	0.0219	0.3152	0.1280	79.
2.4728	0.0555	0.0219	0.3083	0.1252	80.
2.4185	0.0543	0.0220	0.3016	0.1225	81.
2.3653	0.0532	0.0220	0.2950	0.1198	82.
2.3132	0.0521	0.0220	0.2886	0.1172	83.
2.2622	0.0510	0.0220	0.2823	0.1146	84.
2.2123	0.0499	0.0221	0.2762	0.1121	85.
2.1634	0.0489	0.0221	0.2701	0.1097	86.
2.1155	0.0479	0.0221	0.2642	0.1073	87.
2.0686	0.0469	0.0222	0.2585	0.1049	88.
2.0226	0.0459	0.0222	0.2528	0.1026	89.
1.9776	0.0450	0.0222	0.2472	0.1004	90.
1.9336	0.0441	0.0223	0.2418	0.0982	91.
1.8905	0.0431	0.0223	0.2365	0.0960	92.
1.8482	0.0423	0.0223	0.2313	0.0939	93.
1.8068	0.0414	0.0224	0.2262	0.0918	94.
1.7663	0.0405	0.0224	0.2212	0.0898	95.
1.7266	0.0397	0.0225	0.2163	0.0878	96.
1.6878	0.0389	0.0225	0.2115	0.0859	97.
1.6497	0.0381	0.0226	0.2068	0.0840	98.
1.6124	0.0373	0.0226	0.2022	0.0821	99.
1.5759	0.0365	0.0226	0.1977	0.0803	100.

TABLE B-6B.—*Projections of asset prices, rentals, and efficiencies using point estimated from regression (15), (t = 1970)*

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
15.7005	−0.0000	−0.0000	−0.0000	−0.0000	1.
15.2853	0.4152	0.0264	1.9852	1.0000	2.
14.8811	0.4042	0.0264	1.9327	0.9736	3.
14.4876	0.3935	0.0264	1.8816	0.9478	4.
14.1044	0.3831	0.0264	1.8319	0.9227	5.
13.7315	0.3730	0.0264	1.7834	0.8983	6.
13.3684	0.3631	0.0264	1.7363	0.8746	7.
13.0148	0.3535	0.0264	1.6903	0.8515	8.
12.6707	0.3442	0.0264	1.6456	0.8289	9.
12.3356	0.3351	0.0264	1.6021	0.8070	10.
12.0094	0.3262	0.0264	1.5598	0.7857	11.
11.6918	0.3176	0.0264	1.5185	0.7649	12.
11.3826	0.3092	0.0264	1.4784	0.7447	13.
11.0816	0.3010	0.0264	1.4393	0.7250	14.
10.7886	0.2930	0.0264	1.4012	0.7058	15.
10.5033	0.2853	0.0264	1.3642	0.6872	16.
10.2256	0.2778	0.0264	1.3281	0.6690	17.
9.9551	0.2704	0.0264	1.2930	0.6513	18.
9.6919	0.2633	0.0264	1.2588	0.6341	19.
9.4356	0.2563	0.0264	1.2255	0.6173	20.
9.1861	0.2495	0.0264	1.1931	0.6010	21.
8.9432	0.2429	0.0264	1.1615	0.5851	22.
8.7067	0.2365	0.0264	1.1308	0.5696	23.
8.4764	0.2302	0.0264	1.1009	0.5545	24.
8.2523	0.2242	0.0264	1.0718	0.5399	25.
8.0341	0.2182	0.0264	1.0435	0.5256	26.
7.8216	0.2125	0.0264	1.0159	0.5117	27.
7.6148	0.2068	0.0264	0.9890	0.4982	28.
7.4134	0.2014	0.0264	0.9628	0.4850	29.
7.2174	0.1960	0.0264	0.9374	0.4722	30.
7.0265	0.1909	0.0264	0.9126	0.4597	31.
6.8407	0.1858	0.0264	0.8885	0.4475	32.
6.6598	0.1809	0.0264	0.8650	0.4357	33.
6.4837	0.1761	0.0264	0.8421	0.4242	34.
6.3122	0.1715	0.0264	0.8198	0.4130	35.
6.1453	0.1669	0.0264	0.7981	0.4020	36.
5.9828	0.1625	0.0264	0.7770	0.3914	37.
5.8246	0.1582	0.0264	0.7565	0.3811	38.
5.6706	0.1540	0.0264	0.7365	0.3710	39.
5.5206	0.1500	0.0264	0.7170	0.3612	40.
5.3746	0.1460	0.0264	0.6980	0.3516	41.
5.2325	0.1421	0.0264	0.6796	0.3423	42.
5.0941	0.1384	0.0264	0.6616	0.3333	43.
4.9594	0.1347	0.0264	0.6441	0.3245	44.
4.8283	0.1311	0.0264	0.6271	0.3159	45.
4.7006	0.1277	0.0264	0.6105	0.3075	46.
4.5763	0.1243	0.0264	0.5944	0.2994	47.
4.4553	0.1210	0.0264	0.5786	0.2915	48.
4.3375	0.1178	0.0264	0.5633	0.2838	49.
4.2228	0.1147	0.0264	0.5484	0.2763	50.

TABLE B-6B.—*Projections of asset price, rentals, and efficiencies using point estimates from regression (15)*—Continued

q_s	$q_s - q_{s-1}$	$\frac{q_s - q_{s-1}}{q_{s-1}}$	c_{s-1}	Φ_{s-1}	s
4.1111	−0.1117	−0.0264	−0.5339	−0.2690	51.
4.0024	−0.1087	−0.0264	−0.5198	−0.2618	52.
3.8965	0.1058	0.0264	0.5061	0.2549	53.
3.7935	0.1030	0.0264	0.4927	0.2482	54.
3.6932	0.1003	0.0264	0.4797	0.2416	55.
3.5955	0.0977	0.0264	0.4670	0.2352	56.
3.5004	0.0951	0.0264	0.4546	0.2290	57.
3.4076	0.0926	0.0264	0.4426	0.2230	58.
3.3178	0.0901	0.0264	0.4309	0.2171	59.
3.2300	0.0877	0.0264	0.4195	0.2113	60.
3.1446	0.0854	0.0264	0.4084	0.2057	61.
3.0615	0.0832	0.0264	0.3976	0.2003	62.
2.9805	0.0810	0.0264	0.3871	0.1950	63.
2.9017	0.0788	0.0264	0.3769	0.1898	64.
2.8249	0.0767	0.0264	0.3669	0.1848	65.
2.7502	0.0747	0.0264	0.3572	0.1799	66.
2.6775	0.0727	0.0264	0.3478	0.1752	67.
2.6067	0.0708	0.0264	0.3386	0.1705	68.
2.5378	0.0689	0.0264	0.3296	0.1660	69.
2.4707	0.0671	0.0264	0.3209	0.1616	70.
2.4053	0.0653	0.0264	0.3124	0.1574	71.
2.3417	0.0636	0.0264	0.3041	0.1532	72.
2.2798	0.0619	0.0264	0.2961	0.1492	73.
2.2195	0.0603	0.0264	0.2883	0.1452	74.
2.1608	0.0587	0.0264	0.2806	0,1414	75.
2.1037	0.0571	0.0264	0.2732	0,1376	76.
2.0481	0.0556	0.0264	0.2660	0.1340	77.
1.9939	0.0542	0.0264	0.2590	0.1304	78.
1.9412	0.0527	0.0264	0.2521	0.1270	79.
1.8898	0.0513	0.0264	0.2454	0.1236	80.
1.8399	0.0500	0.0264	0.2390	0.1204	81.
1.7912	0.0487	0.0264	0.2326	0.1172	82.
1.7438	0.0474	0.0264	0.2265	0.1141	83.
1.6977	0.0461	0.0264	0.2205	0.1111	84.
1.6528	0.0449	0.0264	0.2147	0.1081	85.
1.6091	0.0437	0.0264	0.2090	0.1053	86.
1.5666	0.0426	0.0264	0.2035	0.1025	87.
1.5251	0.0414	0.0264	0.1981	0.0998	88.
1.4848	0.0403	0.0264	0.1928	0.0971	89.
1.4456	0.0393	0.0264	0.1877	0.0946	90.
1.4073	0.0382	0.0264	0.1828	0.0921	91.
1.3701	0.0372	0.0264	0.1779	0.0896	92.
1.3339	0.0362	0.0264	0.1732	0.0873	93.
1.2986	0.0353	0.0264	0.1687	0.0850	94.
1.2643	0.0343	0.0264	0.1642	0.0827	95.
1.2308	0.0334	0.0264	0.1599	0.0805	96.
1.1983	0.0325	0.0264	0.1556	0.0784	97.
1.1666	0.0317	0.0264	0.1515	0.0763	98.
1.1357	0.0308	0.0264	0.1475	0.0743	99.
1.1057	0.0300	0.0264	0.1436	0.0723	100.

The Implications of Federal Tax Policy for Leasing

Merton Miller and Charles Upton

University of Chicago

The Implications of Federal Tax Policy for Leasing*

Merton Miller and Charles Upton

University of Chicago

Like many corporate decisions, the rent-or-buy decision is frequently justified in terms of the implications for the corporation's tax bill. The advertisements of some leasing companies continue to argue that lease payments are deductible in full, whereas the owner of a machine can only deduct its depreciation plus that part of capital cost represented by interest payments. While this is a false argument, there are important asymmetries in the tax law that do affect the decision and which will be explored below. These asymmetries lead us to the conclusion that a taxable corporation is unlikely to find it financially advantageous to buy rather than to rent, while tax-exempt institutions and individuals will probably find leasing financially unattractive.

Despite the concentration here on the financial aspects, there are other considerations that must be taken into account in making the lease-or-buy decision. For example, a short term lessee has no natural incentive to consider the impact of maintenance expenditures on the resale value of the equipment; users also often lack the specialized knowledge to determine the optimal level of maintenance expenditures. Leasing companies can address both problems by providing "free" maintenance services at the level which they, with their specialized knowledge, believe to be optimal. Arguments can be made that such an arrangement renders leasing both more and less attractive relative to buying. Other differences arise because of the different costs in legal fees and taxes to transfer ownership in a property than to transfer a lease.[1]

Leases can, of course, take a variety of forms. The only form to be considered here is a life of the property lease in which the leasing

---

* This paper is an abstract of a report, "Leasing, Buying, and the Cost of Capital Services."

[1] One could conceive of Hertz or Avis selling cars rather than renting them; the costs of searching for a buyer at the end of the day could presumably be avoided by including a repurchase agreement effective at the end of the day in the sales contract. But such a procedure would necessitate two filings of a title transfer. Another real factor mitigating against the use of sales-cum-repurchase agreements is the cost of enforcement. Failing to return a rented car is a criminal offense, and the lessor has access to police powers to recover the car; failing to honor a repurchase agreement is a civil offense for which the only remedy available to Hertz would be an expensive civil suit.

company agrees to pay the lessor a series of annual payments, $L_i(t)$, in years $t=1, \ldots, T$ for the right to use a machine of type i for its entire useful life of T years, at which time the machine has zero salvage value.[2] The lease is assumed to be noncancellable, with the lease payments treated as riskless. The lessee does have the privilege of subleasing.

Consider now the problem of a company considering whether to purchase the machine at a price π_i or to lease it. If the user company purchases the machine, its cost will be the purchase price π_i net of the present value of the future tax deductions available on the machine

$$\pi_i - \sum_{t=1}^{T} \tau^B D_i^B(t) \frac{1}{(1+r)^t} \tag{1}$$

where $D_i^B(t)$ represents the tax deductions available to the purchaser in year t on a machine of type i, τ^B represents the purchaser's tax rate, and r represents the riskless interest rate. If the user company leases, its cost will be

$$\sum_{t=1}^{T} (1 - \tau^B) L_i(t) \frac{1}{(1+r)^t} \tag{2}$$

where $L_i(t)$ is the lease payment for year t (not necessarily the same for all t).

To determine what these lease payments will be, consider the problem from the perspective of the leasing company. The after-tax receipts of the leasing company in year t of the life of the lease will be

$$L_i(t)(1 - \tau^L) + D_i^L(t)\tau^L, \tag{3}$$

where $D_i^L(t)$ represents the amount of depreciation allowed the leasing company for tax purposes, and τ^L is the leasing company's tax rate. Since both the lease payments and the depreciation deductions are assumed certain, the present value of these payments, computed by discounting at the risk-free interest rate, r, is

$$\pi_i^* = \sum_{t=1}^{T} \frac{L_i(t)(1 - \tau^L)}{(1+r)^t} + \tau^L \sum_{t=1}^{T} \frac{D_i(t)}{(1+r)^t} \tag{4}$$

Competition among leasing companies will force the lease payments to be set so that π^*_i is equal to π_i, the price of a new machine.[3]

---

[2] In practice, recent IRS regulations rule out such leases. They remain, however, useful expository devices. For a more general treatment, including short term leases in which the lessor holds claim to the uncertain residual value of the machine at the end of the lease, see "Leasing, Buying, and the Cost of Capital Services" by Merton H. Miller and Charles W. Upton *Journal of Finance*, June 1976.

[3] Equation (4) has been derived under the assumption that all corporations, including leasing companies, must finance themselves entirely with equity capital. However the results obtained here can easily be extended to the case where firms are allowed to issue debt and deduct the interest payments from the corporate income tax.

The difference between the after tax cost of leasing and owning, Δ, will be

$$\Delta = (1-\tau^B)\sum_{t=1}^{T} L_i(t)\ \frac{1}{(1+r)^t}\ - \pi_i + \sum_{t=1}^{T} \tau^B D_i^B \frac{1}{(1+r)^t}$$

$$= \left(\frac{\tau^L - \tau^B}{1-\tau^L}\right)\pi_i + \sum_{t=1}^{T} [\tau^b D_i^B(t) - \tau^L D_i^L(t) \qquad (5)$$

$$+\ \tau^B\tau^L(D_i^L(t) - D_i^B(t))]\frac{1}{(1+r)^t}$$

If both the user and the leasing company are subject to the same tax provisions, with the same tax rates (implying $\tau^L = \tau^B$) and the same deductions for depreciation and investment tax credit (implying that $D_i^L(t) = D_i^B(t)$), then $\Delta = 0$ meaning that, insofar as financial considerations are concerned, the choice between renting and buying is a matter of indifference for the user firm. But there are other cases where there are asymmetries in the tax code, and here there can be a difference between leasing and buying.

For example, the full value of the investment tax credit impounded in the $D_i(t)$'s cannot be obtained unless the affected firm has enough income from other sources. A firm undertaking a large capital expansion program could find the immediate tax credits exceeding its allowable limit; then the value of the tax credits would be lower than those of a leasing company which presumably can take full advantage of the credits. In that case $D_i^L(t) > D_i^B(t)$ and (assuming $0 < \tau^L = \tau^B = \tau < 1$)

$$\Delta = (\tau - \tau^2)\sum_{t=1}^{T} \{D_i^B(t) - D_i^L(t)\}\ \ \frac{1}{(1+r)^t}\ < 0. \qquad (6)$$

Thus, it will pay the user firm to rent when it cannot take full advantage of the investment tax credit. That way the firm can still get the full benefit of the deductions that competition among leasing companies will build into rental rates.

A different result is obtained when the user is not subject to the corporate income tax. Consider, for example, a university deciding whether to buy a computer or take a long-term lease. Since the university is tax exempt, $\tau_B = D_i^B(t) = 0$, and Δ is given by

$$\Delta = \frac{\tau^L}{1-\tau^L}\pi_i - \tau^L\sum_{t=1}^{T} D_i^L(t)\ \frac{1}{(1+r)^t} \qquad (7)$$

By substituting in equation (4), Δ can be more revealing stated as

$$\Delta = \tau^L\sum_{t=1}^{T} [L_i(t) - D_i^L(t)]\ \ \frac{1}{(1+r)^t}, \qquad (8)$$

or the present value of the leasing company's tax payments, net of any advantages from accelerated depreciation or the investment tax credit.

If the present value of these payments were negative, it would violate what we take to be the intent of these tax subsidies, namely to reduce, not reverse, the effect of the corporate income tax. The word *intent* should be emphasized since ingenious taxpayers constantly search for (and sometimes find) new ways of expanding tax exemptions and subsidies. Our proposition asserts only that such successful subversions of the intent of the law are transitory and will be eliminated by regulations or new legislation if they threaten to become widespread.[4] Similar results, of course, obtain for the case of consumer durables such as owner-occupied housing.

Summary

Economists have largely taken for granted that rental rates for capital equipment would adjust until, in equilibrium, the purely financial advantages of the two arrangements were equal. When the peculiarities of the U.S. tax laws are allowed for, a financial presumption in favor of leasing does, however, emerge. What tends to tip the scales in favor of leasing for ordinary corporations is not the way rentals and interest payments are treated for tax purposes, as has sometimes been suggested, but rather the fact that user firms may not always be able to take full advantage of the various tax subsidies to hardware that Congress bestows. Leasing companies have thus tended to become entities specialized, among other things, to the maximum utilization of these subsidies, which are in turn reflected in equilibrium rental rates. For nontaxable user firms such as universities, however, and for the special case of owner-occupied housing, there are tax-related benefits to ownership that offset or more than offset the normal presumption in favor of renting.

---

[4] The tax treatment of universities, in fact, provides some of the best illustrations of this catch-up process. Equation (8) implies that universities would find advantages in seeking out taxable corporations that own land or other capital assets and offer to share the tax savings via sale-and-leaseback arrangements. The spread of such arrangements was responsible in part for the imposition of the tax on "unrelated business income" of tax-exempt organizations. Note also in this connection that leasing companies are no longer permitted to claim the investment tax credit on any property they lease to a tax-exempt organization.

Saving and Investment Aspects of Corporate and Personal Tax Integration: A Framework for Analysis

T. Nicolaus Tideman and Warren E. Weber

Virginia Polytechnic Institute

Saving and Investment Aspects of Corporate and Personal Tax Integration: A Framework for Analysis

T. Nicolaus Tideman and Warren E. Weber

Virginia Polytechnic Institute

The possible integration of corporate and personal taxes is an important issue because under present U.S. law the return to equity is doubly taxed. The corporate income from which dividends are paid is taxed, typically at 48 percent, and then these dividends are subject to the personal income tax (except for a minimal dividend exclusion). A milder form of double taxation applies to corporate earnings that are retained. Although retained earnings are not directly subject to the personal income tax, the present value of the net income from retained earnings is capitalized into higher stock prices. If the stocks are subsequently sold, the capital gains are subject to the personal tax, but generally at rates which are only one-half of those for other income. The effective rate of double taxation on retained earnings is further reduced by virtue of the fact that tax is due only when capital gains are realized, so that the value of rate reduction is compounded by the value of deferral.

In contrast with the treatment of equity, interest on debt is deductible from the taxable income of corporations, so that return to debt is taxed only once, as personal income. This asymmetry in the tax treatment of returns to debt and equity under present U.S. tax law is sometimes criticized as "unfair" or "inequitable." Our concern in this paper, however, is not with the fairness of the tax system; an investor who does not like the tax treatment of equity capital is free to invest in bonds. Our concern is solely with the allocative inefficiencies that are induced by the asymmetrical treatment of debt and equity.

The purpose of this study is to develop a framework within which the savings and investment effects of the integration of corporate and personal taxes can be analyzed. This paper is a progress report on work just begun, and no empirical estimates are offered here.

The publication of the Report of the Royal Commission on Taxation [8] (hereafter referred to as the Carter Commission) stirred the publication of several studies of the effects of integration on savings and investment for Canada. These studies are by Helliwell [4], Reuber and Bodkin [9], and by Lynch and Witherell [5]. In addition, there are some

calculations of these effects in the summary volume of the Carter Commission report. However, for reasons which we will detail below, we do not feel that it would be appropriate to adapt any of these studies directly to determine the effects of integration on savings and investment in the United States. As a consequence we developed our own framework for such an analysis.

Throughout this analysis we assume that the integration of taxes is total. In other words, we assume that integration is achieved by converting the corporate income tax to a withholding tax. This conversion would be accomplished by specifying that any tax paid by corporations would be allocated to shareholders in proportion to their ownership and could be taken as a refundable credit against their tax liabilities. However, individuals would also be required to include their pro rata shares of pretax corporate income in their taxable personal incomes. Retained earnings would be personal income and would also be treated as increasing the base used to calculate capital gains. We will also be concerned with the changes in individual tax rates that would be required to maintain total tax revenue if integration is adopted.

This framework is somewhat different from that suggested by the Carter Commission, in that the Carter Commission proposed that integration of retained earnings be at the option of firms. A variety of other partial integration schemes might also be considered. (See Pechman [7, pp. 142–7] or McLure [6] for examples.) We expect that with relatively minor modifications our analysis can be used to predict the consequences of partial integration proposals.

The remainder of the paper proceeds as follows: In section I we present a general discussion of the potential consequence of integration and our view of consumer rationality. Section II offers a theoretical development of the manner in which integration affects saving and investment. In section III we discuss our views regarding the relevant saving function.

I. The Effects of Integration

There are four possible effects of integrating corporate and personal income taxes which should be considered. The first effect is that integration will change the debt-equity ratio of firms. The higher rate of taxation that is currently applied to equity capital induces firms to substitute debt for equity. As Tambini [10] has noted, this increase in the debt/equity ratio increases the risk of losing one's investment for both the holders of debt and the holders of equity. The higher the debt ratio, the smaller will be the fall in income, relative to investment, that would compel a firm to default on its obligations to its creditors.

If the firm becomes bankrupt, bondholders may receive partial repayment, or they may obtain control of the firm, but they will not receive the secure income stream for which they bargained. Thus risk

premiums for debt are higher at higher debt/equity ratios. Stockholders stand to lose their entire investment in bankruptcy, but they also obtain greater leverage at a higher debt/equity ratio, so that risk premiums for equity could either rise or fall when the debt/equity ratio rises.

Firms are induced to incur the additional risk of higher debt/equity ratios by their tax savings, which are private gains but not social gains. The unnecessary increase in financial risk means that owners of capital receive a return that is subjectively lower than the return that they could receive if the same level of taxes were collected in a way that did not influence the debt/equity ratio. Integration would provide such neutrality and would induce firms to reduce their debt/equity ratios. The reduced riskiness of investment would provide a utility gain for savers and would have impacts on saving described in detail later.

A second effect of integration would be on rates of return and the allocation of capital. Integration would eliminate the double taxation of equity, so the return on equity would rise sharply. This would tend to raise stock prices and interest rates. Capital would flow to the corporate sector from noncorporate enterprises and from housing, until rates of return were equalized again.

The third effect of integration would be to alter the attitude of shareholders towards retained earnings. Because corporate income can escape part of its double taxation if it is retained, the shareholders of a corporation would prefer that the corporation retain earnings rather than distribute them, other things being equal. This bias toward retained earnings would be eliminated by integration, and therefore retained earnings would presumably fall. But this would lower the level of total saving and investment in the economy only if consumers did not take account of retained earnings in making their personal saving decisions. If consumers do take account of corporate saving in making their personal saving decisions, personal saving will rise to offset any fall in corporate saving that is induced by integration, so that the fall in retained earnings will have no impact on total saving.

Eliminating the bias toward retained earnings should have a beneficial impact on the economy in another way. When retained earnings escape some of the taxes on dividends, shareholders will agree to retention even if the return at which the firm can invest those earnings is lower than the market rate of return. Integration would induce shareholders to insist that earnings be retained only if a return at least as high as the going market rate could be expected.

The fourth and final effect of integration is that personal tax rates might have to be increased because the increase in personal taxes due to integration would not fully offset the loss in corporate taxes. To the extent that marginal tax rates are changed and the marginal propensity to save differs at different income levels, saving may be affected by these changes.

II. Theoretical Development

At a general level, our approach to the effects of integration that operate through interest rates and saving is to estimate a saving (s) function of the form:

$$s = s(y, w, r_e, r_d, \hat{p}) \tag{1}$$

where y is after-tax income, w is wealth, r_e is the after-tax rate of return on equity, r_d is the after-tax rate of return on debt, and $\hat{p}$ is the expected rate of inflation.

We know that, ex-post, the change in investment (Δi) must equal the change in saving (Δs), (assuming the government deficit is unchanged), or

$$\Delta i = \Delta s \tag{2}$$

The possible change in investment in any one period is small relative to the existing capital stock. Thus under the assumption that the flow of capital services is fixed by the capital stock and that employment of labor is unaffected by integration, the nominal pretax return to capital, R, will not be affected by integration in the first period. In other words, we assume a one-period investment demand function that is perfectly elastic with respect to R, so that if capital earns a nominal pretax return of $\bar{R}$ before integration, then,

$$R = \bar{R} \tag{3}$$

after integration as well. We must determine the impact of integration on the variables that affect saving, using the condition that $R = \bar{R}$, and then investment in the short run will be whatever people want to save under the new conditions.

In the longer run, changes in saving and investment will affect R and the change in R will affect the variables that enter the savings function.

The nominal pre-tax return to capital in the corporate sector, R, may be expressed as

$$R = fq_d + (1 - f)q_e \frac{1}{1 - u} \tag{4}$$

where q_d and q_e are the nominal returns to debt and equity respectively that investors receive, f is the fraction of investment that is debt-financed (i.e., f is $D/(D + E)$, where D and E are the value of debt and equity respectively), and u is the corporate tax rate. Firms select the value of f that minimizes the return they must earn. If debt were a perfect substitute for equity, investment would be entirely debt-financed, and no corporate income taxes would be paid. This is clearly not the case.

To see the factors that enter into minimizing capital costs, differentiate R with respect to f:

$$\frac{\partial R}{\partial f} = f\frac{\partial q_d}{\partial f} + q_d + \left[(1 - f)\frac{\partial q_e}{\partial f} - q_e\right]\frac{1}{1 - u} \tag{5}$$

The returns to debt and equity may be considered as composed of basic returns, B_d and B_e, plus risk premiums, P_d and P_e, that vary with f. Therefore (5) may be replaced by:

$$\frac{\partial R}{\partial f} = f\,\frac{\partial P_d}{\partial f} + P_d + B_d + \left[(1-f)\,\frac{\partial P_e}{\partial f} - P_e - B_e\right]\frac{1}{1-u} \tag{6}$$

To proceed further, one must make some assumption about the form of the risk premium functions. One reasonably plausible form, used by Fisher [3] for the risk premium for debt, is to assume that the risk premiums are constant-elasticity functions of the equity/debt ratio. This generates zero risk premiums with full equity financing. The equity/debt ratio may be expressed as $(1 - f)/f$. Thus the constant elasticity assumption implies that

$$\frac{\partial P_d}{\partial(1-f)/f}\cdot\frac{(1-f)/f}{P_d}$$

is some constant, η_d. The partial derivative in (6), $\partial P_d/\partial f$, may be expressed as:

$$\frac{\partial P_d}{\partial f} = \frac{\partial P_d}{\partial(1-f)/f}\cdot\frac{d(1-f)/f}{df} \tag{7}$$

One may deduce from the constant elasticity assumption that

$$\frac{\partial P_d}{\partial(1-f)/f} = \eta_d P_d\,\frac{f}{1-f}, \tag{8}$$

and ordinary differentiation reveals that

$$\frac{d(1-f)/f}{df} = -f^{-2}, \tag{9}$$

so that, substituting from (8) and (9) into (7) yields

$$\frac{\partial P_d}{\partial f} = \frac{-\eta_d P_d}{f(1-f)} \tag{10}$$

A similar line of reasoning for equity produces a corresponding conclusion for η_e, the elasticity of equity risk premiums with respect to the equity/debt ratio:

$$\frac{\partial P_e}{\partial f} = \frac{-\eta_e P_e}{f(1-f)} \tag{11}$$

Substituting from (10) and (11) into (6),

$$\frac{\partial R}{\partial f} = \frac{-\eta_d P_d}{1-f} + P_d + B_d + \left[\frac{-\eta_e P_e}{f} - P_e - B_e\right]\frac{1}{1-u} \tag{12}$$

A first-order condition of equilibrium in firm financing is that (12) equal 0. We plan to use this condition to estimate η_d and η_e.

Integration would change the value of u from .48 to 0, and would therefore change the value of f that minimized R. This new value of f

would be an estimate of the debt/capital ratio under integration. We would then ask how much would have to be added to B_d and B_e to restore R to its original value. This would be an estimate of the initial rise in interest rates under integration.

The effect on households is more complex. Rational households would make their saving decisions on the basis of some overall interest rate that combined the interest rates on various types of assets. We shall assume that the interest rate that households perceive is a function of the after-tax rates of return on debt and equity, and their expectations of inflation. Integration will change the interest rate perceived by households by changing f, which will change r_e and r_d. Integration will also change the marginal tax rates on debt and equity, which will change the perceived interest rate. Changes in taxes may also change household income, which will affect saving. We intend to use a model of household saving behavior to estimate the magnitudes of these effects, and the resulting impacts on saving and investment.

III. A Model of the Saving Process

As demonstrated in the previous section, effects on savings will play an important part in determination of effects of integration of corporate and personal income taxes. In this section we present the derivation of the explicit savings function which will be used in the empirical part of our study, a version of the life-cycle hypothesis of saving.

A. The life-cycle consumption and saving model

The life-cycle model of the consumption and saving process of a consumer can be described as follows. Consider a representative consumer at economic age zero, who will die at economic age L. He is initially endowed with the amount of nonhuman wealth w_o and will receive in each period t the amount of labor (or more precisely noncapital-related) income y_t. Under the assumptions that the individual can purchase one-period bonds which promise to pay one unit of consumption at the beginning of the next period and that the individual desires to leave no bequests, the intertemporal wealth constraint of the individual is

$$w_o + \sum_{t=0}^{L} \frac{y_t}{(1+r)^t} = \sum_{t=0}^{L} \frac{c_t}{(1+r)^t} \tag{13}$$

where r is the real rate of interest and c_t is the amount of consumption which the individual purchases in period t. We will assume that consumer durables do not exist in this model. This allows us to circumvent the discussion of whether c_t is "consumption" as defined to include purchases of nondurable goods and services and the "purchases" of the *services* of durables or whether c_t is defined to include the purchases of nondurable goods and services and the purchases of durables in the period.

The objective of the representative consumer is assumed to be to maximize his utility function

$$U = U(c_0, c_1, \cdots, c_L) \tag{14}$$

subject to the intertemporal wealth constraint given above. The necessary conditions for the individual to be maximizing utility will be that

$$U_t/U_t = -1/(1+r)^{t'-t}$$
$$t' > t, t = 0, \cdots, L-1. \tag{15}$$

and that the intertemporal budget constraint be satisfied. These conditions imply that the individual will maximize his utility subject to his intertemporal wealth constraint by choosing the consumption bundle $c^* = (c_0^*, c_1^*, \cdots, c_L^*)$ such that his indifference curve is tangent to his budget constraint at this point.

The consumption function which one obtains by performing the above maximization can be written in implicit functional form as

$$c_i = c^i\left[r, w_o + \sum_{t=0}^{L} \frac{y_t}{(1+r)^t}\right] \tag{16}$$

where the second argument in the function is the total human and nonhuman wealth of the consumer at the beginning of the period.

Saving in the context of the life-cycle consumption model is the difference between the individual's bond holdings at the end and at the beginning of the current period. Letting b_t denote the bonds which the individual holds at the beginning of period t, saving in period t (s_t) is defined to be

$$s_t = b_{t+1} - b_t = rw_t + y_t - c_t. \tag{17}$$

If one is willing to concede that $rw_t + y_t$ is a reasonable definition of disposable income in the context of a life-cycle theory, then the definition of saving (17) corresponds to the conventional definition that saving is the difference between disposable income and consumption. Further, under the above considerations the savings function becomes

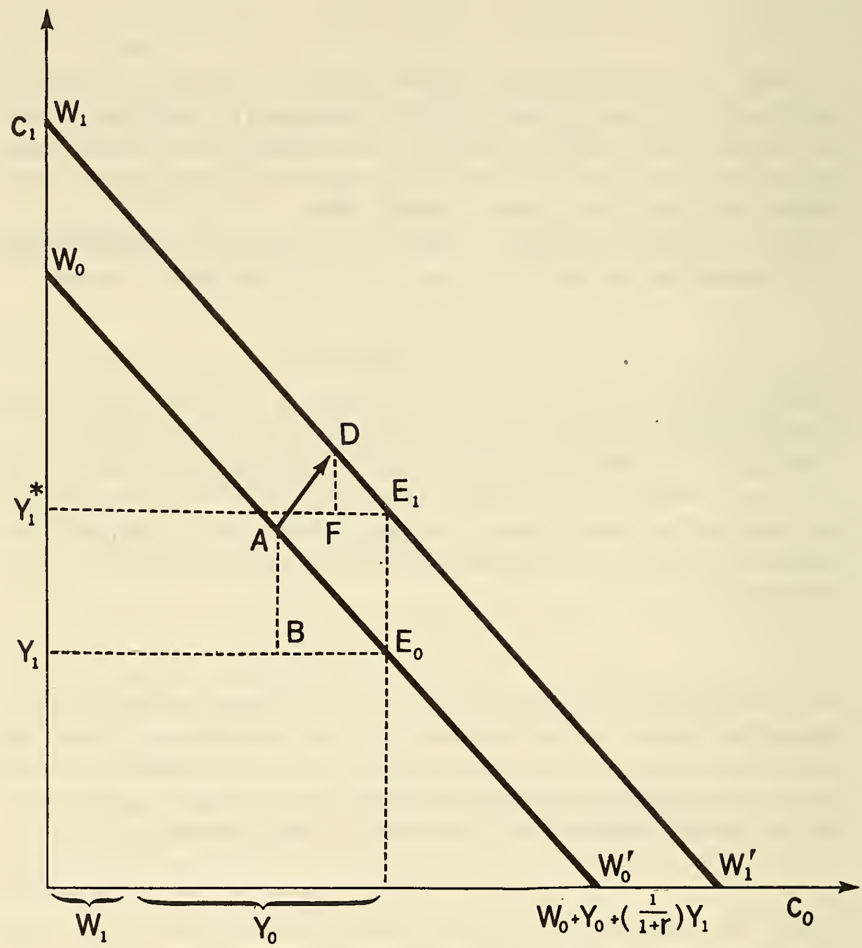
$$s_i = s^i[r, w_o, y_o, y_1, \cdots, y_L]. \tag{18}$$

Unlike the consumption function which depends only upon total human and nonhuman wealth, the savings function depends upon the exact time shape of the individual's income stream.

We now briefly discuss the effects of wealth and interest rate changes on consumption and savings.

The Effects of Total Wealth Changes. The effects on consumption of an increase in the total wealth of the consumer either through an increase in w_o, y_o, or y_1 are easily demonstrated in figure 1 which presents the two-period case. Let E_o represent the representative consumer's endowment of nonhuman wealth and labor income in the two periods. His intertemporal wealth constraint is given by the line

Figure 1



$W_o W'_o$ which is drawn through E_o with the slope $-(1 + r)$. His utility maximizing consumption bundle is given by A.

An increase in wealth will shift out the budget constraint of the individual from $W_o W'_o$ to $W_1 W'_1$. If consumption in both periods is a normal good, then the effect of the increase in wealth must be to increase consumption in both periods as we have illustrated by showing the consumer moving from A to D.

The effects of the total wealth change on saving, however, are not as easily determined. Within the framework of the life-cycle model it is not necessarily true that an increase in wealth will cause saving to increase. Suppose that the entire increase in wealth was due to an increase in future labor income from y_1 to y_1^* so that the individual's endowment increased from E_o to E_1 as illustrated in figure 1. Then, as the figure clearly illustrates, the individual's saving will decline since his disposable income has remained constant and his consumption increases from OB to OF. Thus, whereas the effects of increasing wealth on consumption are clearly unambiguous, the effects of increasing wealth on saving are not, but depend crucially upon the way in which the consumer's wealth increases.

At this point we should point out a confusion of the effects of wealth on saving which appears in the Reuber and Bodkin article on estimating the effects of integration on domestic saving. Reuber and Bodkin estimate the average increase in wealth which will occur as a result of the Carter Commission proposals. They find this to be in the neighborhood of 2 to 7 percent. They then multiply the increase in wealth by the marginal propensity to *consume* out of wealth for various income classes. This increase in consumption is then calculated to be the decrease in saving which occurs as a result of the wealth effects of Carter Commission proposals. Equation (17) implies that they should have calculated the change in saving as r times the increase in wealth minus the increase in consumption.[1]

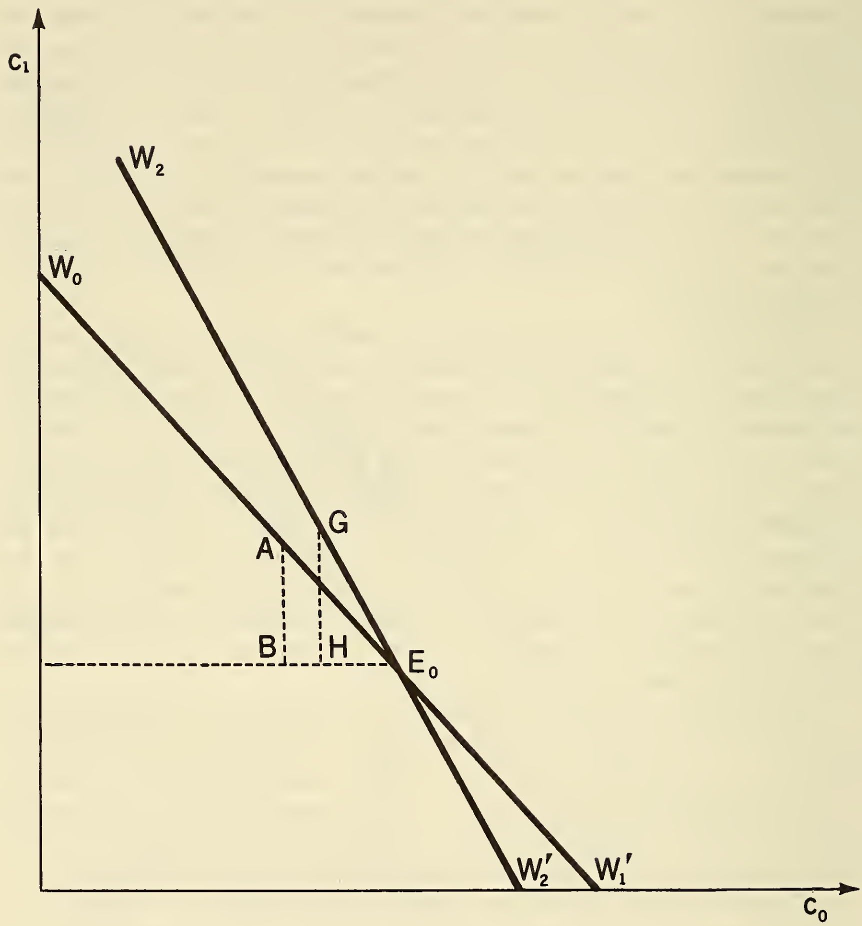
The Effects of Interest Rate Changes. The effects of an increase in the rate of interest are illustrated in figure 2 where the representative consumer's initial endowment and budget constraint have been drawn to correspond with those in figure 1. The effect of an increase in the interest rate is to pivot the initial budget constraint through E_o, so that the budget constraint after the interest rate increase is $W_2 W'_2$.

In this case the effect of the interest rate change on consumption is ambiguous. The substitution effect of the higher interest rate will cause

---

[1] The Reuber and Bodkin derivation of saving is correct if disposable income is constant when w_o changes. Yet, if disposable income is defined as $rw_o + y$, then the only ways in which this could occur would be through a fall in r (but they assume that r increases) or a fall in labor income which does not seem to be what R-B have in mind. If they had properly defined disposable income to take account of the effects of the increase in nonhuman wealth, then they would have found that the increase in wealth due to integration could have increased both consumption and savings.

Figure 2



the consumer to increase his consumption in period 1 and decrease his consumption in period 0. However, when the consumer is a lender, the increase in the interest rate gives rise to an income effect which, if current consumption is a normal good, should tend to increase future consumption. Since both the substitution and income effects work in the same direction for future consumption, the increase in the interest rate would cause the consumer to move eastward from A in response to the interest rate increase. And if the income effect outweighs the substitution effect for current consumption (as Weber [2] indicates), then the consumer will move northeastward from A to a point such as G in response to the interest rate increase.

The discussion of the effects of interest rate changes on saving is not as straightforward as the discussion of the interest rate effects on consumption. The difficulty surrounds the treatment of the rw_t term. At first the problem may seem simple as $\partial(rw_t)/r = w_t$. However, this is not necessarily correct. Bonds held coming into the current period may carry an interest rate that is unaffected by changes in the current rate. In this case, $r\bar{w}_t$ is actually rw_t and $\partial rw_t/\partial r = 0$. This is the case shown in figure 1. Alternatively, a convex combination of these two polar cases may hold. Thus, we will analyze the interest rate effects on saving as

$$\frac{\partial s_t}{\partial r} = \frac{\partial(rw_t)}{\partial r} - \frac{\partial c_t}{\partial r}. \tag{19}$$

B. the specific model

The Explicit Demand Functions. In the previous section we developed a prototype model of the consumer's lifetime allocation process for consumption. In this section we transform the implicit demand functions of the prototype model into explicit demand functions suitable for empirical estimation.

One possible approach to this problem would be to assume that (16) is either linear or log linear in its arguments. However, this solution presents the following difficulty. The classical utility maximization calculus implies certain restrictions on the partial derivatives of a set of demand functions. In our case, however, we are not able to observe the entire set of demand functions since the demands for purchases of nondurables and durables in future periods are not observable. Therefore, in the empirical analysis we would not be able to take advantage of the additional information conveyed by these restrictions.

Thus, our approach to the problem will be to assume that the representative consumer has a utility function of a specific functional form. Maximizing this utility function subject to the consumer's intertemporal wealth constraint will yield explicit demand functions for current purchase of nondurables and durables which place some restrictions on the partial derivatives of these two demand functions.

We first assume that the consumer's utility index for consumption in any age j is a monotonic transformation of a CES utility function of the form

$$U(c_0, \cdots, c_L) = \alpha \sum_{j=0}^{L} (1 + \gamma)^j c_j^{-\beta}, \tag{20}$$

where $\gamma > -1$, $-1 < \beta < \infty$ ($\beta \neq 0$), and α are parameters. The restrictions on the parameters given above plus the restriction that $\alpha\beta < 0$ assure that (20) is strictly quasi-concave with a positive marginal utility of consumption in each time period. In (20) the parameter γ is the individual's subjective rate of time preference and $\sigma = 1/(1 + \beta)$ is the Allen partial elasticity of substitution between consumption in two different time periods.

Maximizing (20) subject to the intertemporal wealth constraint (13) we find that the individual's demand for consumption in the current period is

$$c_o = g_o(\cdot)v_o, \tag{21}$$

where

$$g_o(\cdot)^{-1} = \sum_{j=0}^{L} [(1 + \gamma)(1 + r)^{-\beta}]^{\sigma j}$$

and

$$v_o = w_o + \sum_{j=0}^{L} \frac{1}{(1 + r)^j} y_j .$$

Thus v_o can be interpreted as the individual's total human and nonhuman wealth at the beginning of the period and $g_o(\cdot)$ is his marginal propensity to consume out of total wealth.

Expectations and Unobservable Variables. A final step remains before (21) can be estimated. This demand function has future labor income as an exogenous variable. Since this variable is not observable, we must relate it to observable variables in order to be able to estimate (21). We will make this relation by including a specific assumption concerning the way in which the consumer forms his expectations about future labor income as a part of our maintained hypothesis.

We assume that the representative consumer expects future labor income to grow at the constant rate ξ over the remainder of his working lifetime. Under this assumption his future labor income is related to current labor income according to

$$y_j = (1 + \xi)^j y_o \qquad j = 1, \cdots, E \tag{22}$$

where E is the age at which he retires. The parameter ξ will be estimated as a part of the empirical estimation.

Next, we consider the interest rate to be used in the empirical analysis. The representative consumer may have long-run expectations concerning interest rates which should be included in his demand

functions. However, these expectations may or may not be sensitive to changes in current market interest rates, and they may not be solely related to the current level of an arbitrary market interest rate. Therefore, to take account of these possibilities we redefine the interest rate used in empirical analysis to be

$$1 + r = \theta_1 + \theta_2 \hat{r} - \theta_3 \hat{\rho} \tag{23}$$

where θ_1, θ_2 and θ_3 are parameters, $\hat{r}$ is an expected nominal interest rate and ρ is the expected rate of inflation. The parameters θ_1, θ_2, and θ_3 are discussed in more detail below.

However, (23) has an unobservable variables problem since the individual's expectations of inflation and nominal interest rates cannot be observed directly. Thus, we must include some assumptions concerning the way in which the individual forms his expectations as a part of our maintained hypothesis.

We first consider the case of the individual's expectations concerning the rate of inflation. One possibility, which has been used a great deal in the literature, is to assume that the consumer uses an adaptive expectations mechanism to form his expectations. Suraj Gupta [1] has shown that the adaptive expectations framework can be written as

$$\hat{\rho} = \mu [\log_e(p_i) - \log_e(\hat{p}_i)] \tag{24}$$

where p_i and $\hat{p}_i$ are the price of consumption goods which the consumer observed at age i and the price of consumption goods which he expected to hold at age i, respectively. This expectations framework states that if the current price level is above the "normal" price level which the consumer expected to prevail in this period, then he will expect the price level to continue to increase in the future. Gupta goes on to show that (24) can be written as

$$\hat{\rho} = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j \rho_{i-j} , \tag{25}$$

where $\lambda = e^{-\mu}$ and $\rho^n{}_{i-j}$ is the inflation rate which the individual observed at age $i - j$. Thus, under the adaptive expectations scheme the individual forms his expectation of the rate of inflation which will occur in the future by taking a weighted average of all past inflation rates.

However, Gupta proposes a second expectations scheme; namely,

$$\bar{\rho} = \mu' \; [\log_e(\hat{p}_{i+1}) - \log_e(p_i)] \cdot \tag{26}$$

This expectations framework states that if the current price level is above the normal price level which the consumer expects to prevail in the future, then the consumer expects a negative rate of inflation as the price level returns to its normal level.

There is, of course, no a priori way to choose between these two hypotheses. However, Gupta is able to show that

$$\bar{\rho} = \hat{\rho} [\mu' \; (\mu - 1)/\mu] \cdot \tag{27}$$

indicating that these two expectations hypotheses will be negatively related if $0 < \mu < 1$, the generally expected case. However, our formulation (23) allows either of these cases to occur. If $\theta_3 > 0$, then this formulation argues that the consumer is forming his expectations according to (24). However, if $\theta_3 < 0$, then it argues that the consumer is forming his expectations according to (26). The formulation (23) also has the advantage of allowing us to test whether changes in inflation rates have an effect on consumer expenditures by testing whether or not θ_3 is significantly different from zero.

Our assumptions concerning the way in which the representative consumer forms his expectations concerning nominal interests will take account of both past experience and the fact that the consumer can hold either debt or equity in his portfolio and that debt and equity may not be perfect substitutes. With these considerations in mind we include as a part of our maintained hypothesis the assumption that the individual forms his expectations concerning nominal interest rates according to

$$\hat{r} = [\phi \hat{r}_1{}^{-\omega} + (1 - \phi)\hat{r}_2{}^{-\omega}]^{-1/\omega}, \tag{28}$$

where $\hat{r}_k = [(1 - \lambda') \sum_{j=0}^{\infty} (\lambda')^j r_{i-j}^k]$, $k = 1, 2$, and r_{i-j}^2 is a particular market interest rate on debt, and r_{i-j}^1 is a particular rate of return on equity which the individual observed at age $i - j$.

Thus, according to (28) the representative consumer first forms his expectations concerning future nominal yields on debt and equity. He then combines these expectations, using a CES index, to obtain his expectation of the nominal rate of return he can earn. Thus, $1/(1 + \omega)$ is his perceived elasticity of substitution between debt and equity and ϕ is a distribution parameter. Both ω and ϕ will be estimated as a part of the empirical analysis.

Further, in (23) we can now see that θ_2 affords the same advantages with respect to nominal yields that θ_3 did for inflation rates. It allows us both to test whether or not interest rate expectations are influenced by changes in current interest market rates and to discriminate between the competing expectations hypotheses (24) and (26). In addition, the constant θ_1, allows the consumer to have a non-zero long-run expected interest rate even if his expectation is not affected by changes in current nominal interest rates or inflation rates.

If we substitute (22), (23), (25), and (28) into (21), we would obtain the individual's demand functions for nondurables and durables in terms of only observable variables. However, in the resultant demand functions we would encounter an "identification" problem. This problem is that in these demand functions the effects of one or more of the parameters in the model cannot be distinguished empirically. Consequently, it is necessary to reparametrize the model as follows. Define the vector of parameters $z = (z_1, \cdots, z_4)$ to be:

$$\begin{aligned} z_1 &= (1 + \gamma)^{-1/\beta}\theta_1 & z_3 &= -(1 + \gamma)^{-1/\beta}\theta_3 \\ z_2 &= (1 + \gamma)^{-1/\beta}\theta_2 & z_4 &= [(1 + \gamma)^{-1/\beta}(1 + \xi)]^{-1} \end{aligned} \tag{29}$$

Then in terms of this new notation the individual's explicit demand function for current consumption in terms of only observable variables is:

$$c_o = g_o^e(\cdot)v_o^e \tag{21'}$$

where $g_o^e(\cdot)$ and v_o^e denote $g_o(\cdot)$ and v_o after substituting (22), (23), (25) and (28).

We hope that in the near future we will be able to offer empirical estimates of the concepts discussed in this prospectus of our work.

References

1. Gupta, S. "The Portfolio Balance Theory of the Expected Rate of Change of Prices." *Review of Economic Studies* 37 (April 1970): 187–203.
2. Weber, W. "The Effect of Interest Rates on Aggregate Consumption." *American Economic Review* 60 (1970): 591–600.
3. Fisher, L. "Determinants of Risk Premiums on Corporate Bonds." *Journal of Political Economy* 67 (1959): 217–37.
4. Helliwell, J. "Simulating the Macroeconomic Effects of the Carter Proposals." *Canadian Journal of Economics* 1 (Supplement, 1968): 233–54.
5. Lynch, P., and Witherell, W. "The Carter Commission and the Savings Behavior of Canadian Corporations." *National Tax Journal* 22 (1969): 57–65.
6. McLure, C. "Integration of the Personal and Corporate Income Taxes: The Missing Element in Recent Tax Reform Proposals." *Harvard Law Review* (1975).
7. Pechman, J. *Federal Tax Policies*. Norton: 1971.
8. *Report of the Royal Commission on Taxation*. Ottawa: Queen's Printer, 1966.
9. Reuber, G., and Bodkin, R. "On Estimating the Effect of the Carter Commission's Recommendations on Domestic Savings." *Canadian Journal of Economics* 1 (Supplement, 1968): 206–32.
10. Tambini, L. "Financial Policy and the Corporation Income Tax." In *The Taxation of Income from Capital*. Ed. by Harberger and Bailey. Washington, D.C.: The Brookings Institution, 1969.

Notes on the Tax Treatment of Human Capital

Michael J. Boskin

Stanford University and National Bureau of Economic Research

Notes on the Tax Treatment of Human Capital

Michael J. Boskin

Stanford University and National Bureau of Economic Research

Over the last decade and a half, economists have rediscovered the concept of human capital.[1] The analysis of both the discrepancy between the growth of output and inputs in industrialized economies and the distribution of earnings has led economists to focus on the acquisition of knowledge and skills by human beings. It is now widely recognized that such human capital investments as expenditures on education, job-training, migration, and health are an important feature of advanced economies.

While human capital has played a central role in labor economics for some time, and in growth accounting from time to time,[2] it only rarely has crept into the mainstream of public finance. Most analyses of tax incidence and of the efficiency properties of alternative tax devices have simply ignored human capital. Further, most analyses of human capital have simply ignored taxes. When labor economists have attempted to discuss the tax treatment of human capital, or when public financiers have attempted to incorporate human capital into the analysis of a problem in taxation, the result frequently has been an unsupported *assumption* that the current tax treatment of human capital discourages its accumulation. For example, in his widely heralded presidential address, T. W. Schultz asserts:[3]

> Our tax laws everywhere discriminate against human capital. Although the stock of such capital has become large and even though it is obvious that human capital, like other forms of reproducible capital, depreciates, becomes obsolete and entails maintenance, our tax laws are all but blind on these matters.

Further, virtually every writer on the modern theory of optimal income taxation[4] defends the assumption of an elasticity of substitution between the consumption of goods and leisure much larger than empirical studies of labor supply would suggest by invoking tax disincentives to human capital accumulation.

---

[1] This rediscovery is frequently associated with T. W. Schultz' presidential address [1961] and Becker [1964]; Kiker [1966] traces the human capital concept back to Petty.

[2] See Griliches [1970], Becker [1967], and Mincer [1969].

[3] Schultz [1961], p. 17. However, Becker [1964] and Goode [1962] point out the tax-free nature of foregone earnings.

[4] See Mirrlees [1971] and Sheshinski [1972] as examples.

While the issue is basically an empirical one, some light can be shed on the efficacy of such statements by carefully examining the ways in which taxes affect human capital accumulation. Section 1 presents a preliminary attempt at such a clarification.

If the current tax system (or any proposed alternative) does discourage human capital accumulation, the overall economic implications are potentially enormous. Kendrick [1975] estimates that the aggregate annual investment in human capital is on the same order of magnitude as conventionally measured savings. Any significant tax-induced decrease in human capital accumulation might result in a large decrease in output and/or redistribution of income. Section 2 outlines a simple general equilibrium model with two capital goods—physical and human—and the saving corresponding to each, to begin to deal with these issues.

Once human capital is viewed as an alternative source of wealth and hence human capital investment as a source of current saving (resources withdrawn from current consumption to help increase future output[5]), the old issue of the differential tax treatment of alternative types of capital arises.[6] Sensible tax policy with respect to the taxation of either physical or human capital must take into account the tax treatment of the alternative asset. Section 3 outlines some points of departure for such an analysis.

1. Does the Current Tax Treatment Discourage Human Capital Investment?

As noted above, when the effects of the current tax treatment of human capital are discussed, the assumption appears to be that a large disincentive to such investment exists. Certainly this has never been documented empirically. Nor do adherents to this view identify the real culprit in the situation. Is it the taxation of the returns to human investment at a positive rate? Is it the progressive rate structure of the personal income tax? Is it the failure to allow educational expense deductions? Is it the income effect of the tax combined with differential public and private marginal propensities to invest in human capital?

While the effects of the personal income tax on human capital investment depend upon all of these details, we begin by focusing on what we believe to be the single most important feature of the relation between human capital investment and the tax system, namely that *the bulk of such investments are financed out of foregone earnings which are not taxed*. The failure to appreciate this basic feature of human investment is, I believe, a source of much of the confusion on the effects of taxes on human investment.

---

[5] Recognized quite some time ago by Abramovitz [1956].

[6] Discussed in detail in the static case by Harberger [1966].

From Kendrick [1975], we note that roughly eighty percent of human capital investment consists of education and informal on-the-job training; the remaining twenty percent consists of approximately equal amounts of mobility and health expenditures. Further, Mincer [1962] estimates that roughly 40 percent of combined education and training costs are accounted for by on-the-job training. Virtually all of job training costs are foregone earnings as are perhaps three-fourths of higher education costs and some fraction of medical and migration costs. Thus, well over one-half of human capital investment costs consist of foregone earnings.[7]

Let us take on-the-job training as an example. In the early part of the typical work life, it appears that a substantial fraction of time spent on the job is devoted to training as opposed to directly productive work. While the distinction between work and investment is very difficult to make in practice, an interesting analysis by Heckman [1973] indirectly infers that the percentage of time spent training may be as high as 30 or 40 percent in the early years of labor market experience and declines toward zero over perhaps 20 years or so. Hence, a typical young worker earning \$10,000 per year may be directly working only two-thirds of the time, being paid at an annual rate of \$15,000 and buying back one-third of his time for training (acquisition of skills) for \$5,000.

How is this \$5,000 human capital investment taxed? The worker's true income is \$15,000; at a flat tax rate of t, with no depreciation of the human capital investment, the worker pays a tax of \$15,000t. Since the \$5,000 in human investment is financed out of foregone earnings which are tax-free, the worker's actual tax payment is \$10,000t. The failure to tax foregone earnings is thus equivalent to an immediate write-off of the investment cost.[8]

In the absence of an income tax, the worker would engage in on-the-job training up to the point where the incremental investment cost just matched the present value of expected future returns. In our example, the \$5,000 cost must be matched by at least a present value of \$5,000 in expected future returns from the investment. The imposition of a tax at a flat rate t on the income from the investment reduces the net return by one-third; the instantaneous write-off reduces the tax liability by one-third. The present value of the depreciation deduction equals the cost of the investment and if the training was a profitable investment with no tax, it is still profitable in the presence of the tax-free foregone earnings.

More formally, the prospective investor in job training purchases an asset—skill, knowledge, etc.—costing C and yielding an incremental income stream Y_t. The present discounted value of the job training before the imposition of an income tax is just

[7] Of course, this percentage also may be influenced by tax considerations.

[8] See E. C. Brown [1948] for a discussion of the neutrality of immediate depreciation (plus loss offsets) of physical capital.

$$\int_{t = t_o}^{t = 65} Y_t e^{-rt}\, dt - C \tag{1.1}$$

where r is the rate of interest. When the asset is purchased with foregone earnings, the *net* return to investing in the training (or other human capital investment so financed) is simply

$$(1 - t)\left(\int_{t = t_o}^{t = 65} Y_t e^{-rt} - C\right). \tag{1.2}$$

If the investment was worth undertaking in the no tax situation, it is still worth undertaking and a flat rate income tax which does not include foregone earnings in the tax base does not discourage human capital investments financed out of foregone earnings.

The major human capital investment cost which is not tax exempt is the direct cost of education, i.e. tuition, books and related expenses. It is these expenses which have received the most attention in the public finance literature. The argument has been that such expenditures are a valid cost of earning income and should be deductible either when made or depreciated throughout the working life.[9] While true economic depreciation of educational expenses would be nondistortionary (since under true economic depreciation the differential equation describing the value of human capital is independent of the tax rate, the value of the investment would not be affected by the tax) it is not the only way to achieve neutrality. Indeed, any tax which between its interest deductibility and depreciation allowances yields a deduction whose present value equals the investment cost is neutral.[10] While I would be the last to argue that capital markets work perfectly, particularly in financing human investment, a substantial fraction of higher education expenses are financed by borrowing and at least the interest on this debt is deductible against future income.

In analyzing when tax depreciation of educational expenses would be neutral it is important to note that many students investing in education have little other income and hence would not benefit from immediate write-off of out-of-pocket educational expenses. Unless they are allowed to carry such a write-off forward for a considerable period, the present value of the depreciation allowance will fall short of the present value of the tax liability on the return to the investment and hence will discourage investment in education.

The progressive rate structure of the personal income tax acts in an analogous manner and not just on educational investments. Any human capital investment which increases future earnings enough to drive the taxpayer into a higher tax bracket (after accounting for

---

[9] See Goode [1962] for a discussion of these issues. Frequently ignored is the lack of taxation on the human capital gain during education; this tax is postponed until the income stream is realized.

[10] This is demonstrated most effectively in Stiglitz [1975] for the analogous physical investment case.

income averaging provisions) may decrease the ratio of the present value of the depreciation allowance to the present value of the incremental tax liability. Investments which are profitable at the current tax rate may not be so when account is taken of the increased future tax rate. In the on-the-job training example noted above, the worker will require a before-tax expected present value of increased earnings of \$3,333/$(1 - t')$ where t' is the new (presumably higher than one-third) marginal tax rate. If the new rate is 40 percent, he requires an increase of \$5,555 in present value terms, or about 11 percent more than with the unchanged tax rate.

Note also that the instantaneous depreciation implicit in foregone earnings favors longer-lived human investment, e.g. general, rather than job-specific, training.

The final source of tax distortions in human investment decisions stems from the income effect of the tax. Since the tax revenue transfers resources from the private to the public sector, the issue hinges on differential marginal propensities to invest in human capital publicly and privately. There is a substantial amount of public human capital investment, but separating the marginal from the average propensity to invest in human captial is not easy; nor is it easy to determine to what extent the private sector adjusts its own human capital investment decisions to the perceived public investment.

In summary, the current progressive rate structure of the personal income tax probably creates a disincentive to accumulate human capital; this disincentive is perhaps most severe for secondary workers in two-earner families whose incremental incomes from human investment may generate a large increase in marginal tax rates. The lack of an educational expenditure depreciation allowance probably biases investments away from education to job training. While the extent of these distortions is primarily an empirical matter—which forms the bulk of the research yet to be conducted—recall that a substantial fraction of human investment is exempt from tax.

Finally, we note that the optimal investment decisions of households deal simultaneously with human investment and conventional saving. A full examination of the effects of the tax system on human investment requires an analysis of the tax treatment of physical capital and the potential substitution of physical and human capital. Indeed, intelligent tax policy with respect to the depreciation of physical capital should account for the special, and perhaps inevitable, tax treatment of human capital. A flat rate tax may not be neutral in such a setting, and tax increases may lead to a substitution of human for non-human capital.

2. Tax Incidence in an Economy with Human Capital Accumulation

Until quite recently, the typical analysis of the effects of taxes on the distribution of income has been made in a static context. Fixed stocks

of capital and labor may be mobile across sectors in response to after-tax return differentials in such models, but the problem of the growth of factor supplies has been relatively ignored. A series of recent papers has refocused attention on tax-induced changes in saving, capital accumulation, and the long-run distribution of income.[11] Most of these studies have adopted rather simplistic savings functions. In perhaps the most important of these papers, Feldstein [1974a] has generalized the savings behavior to allow both differential propensities to save out of wages, profits and government revenue, and a potentially interest-elastic savings rate. In [1974b], he demonstrates that a significant fraction of a capital income tax may be shifted to labor via a decreased capital-labor (and hence wage/rental) ratio. Perhaps the most surprising—although in retrospect quite understandable—result from his work is that in the long run the elasticity of the supply of labor is totally irrelevant in determining the incidence of the tax. This occurs because of the usual assumption of a constant returns-to-scale technology under which only the rate of growth of the labor force, not its size at any point in time, affects the wage/rental ratio.

While these models of long-run tax incidence have not yet been totally assimilated in the teaching and practice of public finance, I believe it is important to extend such models to account for the second—and quantitatively equally as important—type of capital accumulation in advanced economies: investment embodied in the knowledge and skills of the labor force. In models designed to examine the long-run incidence of a tax, we would do well to adopt a more general view of the supply of labor, defining it not just as total person-hours of work but in its envelope sense, subsuming human capital investments.

From this perspective, the rate of growth of the effective, or quality-corrected labor force, may be affected by taxes. Indeed, a tax which lowers the after-tax rental rate on human capital, such as an income or payroll tax, given the rate of interest and the price of any purchased inputs in human capital production, will decrease human capital investment[12] unless an appropriate depreciation policy is followed.[13] This in turn will drive up the ratio of physical to human capital and the ratio of unit rental rates of men to machines. In symmetry with the result of Feldstein quoted above, taxes on human capital, i.e. taxes on earnings, may be shifted in part to owners of physical capital. While I am still in the process of experimenting with alternative forms of representing this phenomenon in a simple general equilibrium setting, let us sketch out some of the basic consideration. This is done most readily by taking the Feldstein model as our point of departure and making some simple additions.

---

[11] See Diamond [1970], Sato [1967], Kryzyaniak [1967], and Feldstein [1974a] and [1974b].

[12] See Ben-Porath [1970].

[13] See section 1 above.

Following Griliches [1970], we define a constant return-to-scale production function of physical capital, K, and quality-corrected labor, EN, where N is the number of workers and E is a labor-augmenting quality multiplier:

$$Y = F(K, EN) \tag{1}$$

Physical capital accumulation, convential saving, and human capital accumulation follow:

$$\dot{K} = sY - \delta K \tag{2}$$

$$\dot{E} = h\frac{Y}{N} - \delta' E \tag{3}$$

where δ and δ' are depreciation rates and s and h represent saving rates, for physical and human capital.[14]

The population grows exogeneously at rate n:

$$\frac{\dot{N}}{N} = n \tag{4}$$

Letting $L = EN$, and defining capital accumulation net of depreciation, we have

$$Y = F(K, L) \tag{1'}$$

$$\dot{K} = sY \text{ and} \tag{2'}$$

$$\dot{L} = hY + nL \tag{3'}$$

Factors are paid their respective marginal products:

$$F_k = r \tag{4}$$

$$F_L = w\,(1 + \mu) \tag{5}$$

where r and w are after-tax factor returns and μ is the rate of tax on earnings.

Following Feldstein [1974a and 1974b], we allow saving to respond to its rate of return; however, each type of saving will depend upon the returns to each type of savings:

$$s = s\,(r, w) \tag{6}$$

$$h = h\,(r, w) \tag{7}$$

Where r and w are returns to physical and human capital, respectively.[15]

---

[14] Conlisk [1970] uses this model to discuss the residual in growth accounting.

[15] We allow our notation to slip here, confusing rates of return and rental prices. An additional unit of human capital earning w at each point of time from t to T has a marginal rate of return of $w\,(1 - e^{-r\,(T-t)})$, where r is the interest rate.

Again, following Feldstein, we also allow short-run labor supply to respond to its return:

$$L = 1(w) \cdot N \tag{8}$$

Finally, the equilibrium growth path requires $\frac{\dot{L}}{L} = \frac{\dot{K}}{K}$, so

$$sLY = hKY + nKL. \tag{9}$$

This system of equations can be transformed (via substitution and total differentiation) into a system of linear equations in which dw and dr, the change in the returns to human and physical capital, are a function of the tax rate and saving and human capital accumulation propensities, their rate-of-return sensitivities and the parameters of the production technology, i.e.

$$dw = f\,(s, h, s_r, s_w, h_r, h_w, \sigma, \alpha, \mu)$$
$$dr = g\,(s, h, s_r, s_w, h_r, h_w, \sigma, \alpha, \mu)$$

where subscripts denote partial derivatives, σ is the elasticity of substitution in production and α labor's share in production.

The incidence of the tax may be inferred from the change in the returns to physical and human capital. The important point to note is that the "long-run" supply of labor and the sensitivity of human capital to its return do affect the incidence of the tax.

It is also important to note that even if human capital accumulation is own rate-of-return inelastic, or if tax policy appropriately neutralizes the direct effect of the tax on the rate of return to human investment, a more general savings behavior allowing differential public and private propensities to invest in human capital would suffice to render human capital accumulation important in the analysis of long-run tax incidence.

Let me conclude this section by noting the woefully inadequate empirical information upon which to approximate long-run tax incidence. While we have a fairly decent handle on the parameters of the production function, we have very little information usable in obtaining a rough guide to the conventional saving and human capital accumulation equations. We have virtually no information on the latter (indeed, an estimate of the annual investment can be obtained only indirectly); on the former, a renewed interest has emerged, but inclusion of the interest rate in consumption functions or savings equations is the exception, rather than the rule. Worse yet, use of the nominal rather than the real interest rate quite likely biases the result toward a zero interest elasticity.[16]

Thus, an improved set of estimates of savings functions and a set of estimates (if any) of human capital accumulation equations, as well as a

[16] See Feldstein [1970].

better grasp on government saving and human investment, are essential to shed some light on long-run tax incidence.

3. Some Brief Comments on Optimal Capital Accumulation

In section 1 above, I focused on some of the ways the tax system directly affected the accumulation of human capital; in section 2, I introduced physical capital as well and outlined a model of long -run tax incidence. I return now to the question of efficient capital accumulation when I account simultaneously for physical and human capital accumulation.

Consider the problem of maximizing social welfare measured as the discounted sum of individual utilities:

$$V = \int_0^\infty C^{-\rho t} P_t U(C_t)\, dt \tag{10}$$

subject to the constraints

$$\dot{K} = sF(K, L)$$
$$\dot{L} = hF(K, L) + nL$$

where $C_t = \dfrac{(1 - s_t - h_t) \quad F(K_t, L_t)}{N}$

and $S_t, h_t \geqslant 0$
$S_t + h_t < 1.$

Given initial stocks of physical and human capital, we may apply the maximum principle to this problem by defining the present value of the Hamiltonian as:

$$\begin{aligned}\phi(K_t, L_t, S_t, h_t, q_t, v_t, t) = e^{-\rho t} \{P_t U(c_t) \\ + q_t s_t F(K_t, L_t) + V_t (h_t F(K_t, L_t) + n_L)\end{aligned} \tag{11}$$

Where $e^{-\rho t} q_t$ and $e^{-\rho t} V_t$ are the auxiliary variables associated with the differential equations defining capital accumulation. Necessary and sufficient conditions for a maximum are the following:

$$\begin{aligned}\dot{K} &= s_t F(K_t, L_t) \qquad \text{(with } K_0 > 0\text{)}\\ \dot{h} &= h_t F(K_t, L_t) + nL\\ \dot{q}_t &= [\rho - s_t F_k] q_t - h_t F_k v_t - (1 - s_t - h_t) U_c F_k)\\ \dot{v}_t &= [\rho - h_t F_L] v_t - s_t F_L q_t - (1 - s_t - h_t) U_c F_L\\ q_t &= U_c\\ V_t &= U_c\end{aligned} \tag{12}$$

Where subscripts denote partial differentiation.[17] Substituting and rearranging, we have:

$$\frac{\dot{C}_t}{C_t} = \frac{F_k - \rho}{\sigma}$$
$$\frac{\dot{C}_t}{C_t} = \frac{F_L - \rho}{\sigma}$$

[17] I spare the reader the laborious proof of this proposition.

Where $\sigma = -c\, U_{cc}/U_c$, the elasticity of marginal utility. It is thus obvious that optimality requires $F_k = F_L$, i.e. the marginal efficiency of physical and human capital must be equal. This result hardly should be surprising; it is precisely analogous to the usual efficiency rule of equalizing gross-of-tax rates of return on physical capital in all uses. Again in analogy with the usual treatment of the physical capital stock, owners of capital will respond to net-of-tax rates of return in making their investment choices. It is thus inappropriate solely to examine the tax treatment of physical capital, or of human capital, in isolation. The tax treatment of each must be examined simultaneously.

While a detailed analysis of this proposition is beyond the scope of these notes a few general remarks will serve to illustrate some of its implications. First, it is obvious that our tax system through exclusion, preferences, maximum rates and other devices often imposes different rates of tax on income from physical and human capital accruing to the same taxpaying unit in the same year. The same taxpaying unit also faces widely different rates on the two types of income given the usual lifecycle patterns of the two types of saving and the progressive rate structure and limited averaging possibilities in the current personal income tax. While there may well be other social objectives served by such differentation, it does distort the composition and timing of investment choices.

Second, there may well be inherent constraints in the tax treatment of human capital which have important implications for the tax treatment of physical capital. For example, liquidity and enforceability constraints may make it extremely costly to attempt to include foregone earnings in the tax base,[18] i.e. it is likely that a large fraction of human capital investments must be tax-exempt and thus treated as if instantaneously depreciated. Even if we achieve the objective of uniform tax treatment of income from physical capital in all sectors of the economy (via whatever combination of rate structure, depreciation allowance and other devices is necessary), if the tax system uniformly lowered the rate of return of all types of physical capital relative to human capital, we would be underinvesting in physical capital. Indeed if it is the case that the tax system discriminates against physical capital relative to human capital (I suspect it does but this is a difficult proposition to establish given the wide variation in effective tax rates on alternative types of physical and human capital), a strong case could be made for liberalization of the tax treatment of physical capital.[19] In any event, I plan to attempt to say something empirical on this point in future research.

---

[18] See Boskin (1975) for a discussion of the inability to tax foregone earnings due to household work.

[19] Efficient allocation of the total capital stock may thus directly conflict with short-run income distribution objectives. Seymour Fiekowsky points out to me that inflation exacerbates the tax distortion, since physical and human capital typically have different proportions of tax recovery of costs via depreciation.

Finally, let me conclude with a proviso:

The entire analysis has been conducted in the usual closed economy framework. In an open economy, tax policies may be disciplined sharply by the (actual or potential) international movement of human, as well as financial, capital.

References

Abramovitz, M., "Resources and Output Trends in the United States since 1870," *Amer. Econ. Rev.*, May 1956.

Becker, G., *Human Capital*, Columbia University Press for NBER, 1964.

Becker, G. *Human Capital and the Personal Distribution of Income*, Ann Arbor, 1967.

Ben-Porath, Y., "The Production of Human Capital Over Time," in W. Hansen, ed., *Education, Income and Human Capital*, NBER, 1970.

Boskin, M., "Efficiency Aspects of the Differential Tax Treatment of Market and Household Economic Activity," *Jour. Pub. Econ.*, January 1975.

Brown, E. C., "Business-Income Taxation and Investment Incentives," Ch. IV in Pt. III of *Income Employment and Public Policy, Essays in Honor of Alvin Hansen*, 1948.

Conlisk, J., "Comment" in W. Hansen, ed., *Education, Income and Human Capital*, 1970.

Diamond, P., "Incidence of an Interest Income Tax," *Jour. Econ. Theory*, September 1970.

Feldstein, M., "Tax Incidence with Growth and Variable Factor Supply," *Quar. Jour-Econ.*, November 1974.

Feldstein, M., "Incidence of a Capital Income Tax in a Growing Economy with Variable Savings Rates," *Rev. Econ. Stud.*, October 1974.

Feldstein, M., "Inflation, Specification Bias and the Impact of Interest Rates" *Jour. Pol. Econ.*, November 1970.

Goode, R., "Educational Expenditures and the Income Tax," in *Economics of Higher Education*, S. Mushlin, ed., 1962.

Griliches Z., "Notes on the Role of Education in Production Functions and Growth Accounting," in W. Hansen, ed., *Education, Income and Human Capital*, 1970.

Harberger, A., "Efficiency Aspects of Taxes on Income from Capital," in M. Kryzyaniak, ed., *The Corporation Income Tax*, 1966.

Heckman, J. "Estimates of a Human Capital Production Function Embedded in a Life Cycle Model of Labor Supply," mimeo, 1973.

Kendrick, J. *The Formation and Stocks of Total Capital*, NBER, forthcoming, 1975.

Kiker, B., "The Historical Roots of the Concept of Human Capital," *Jour. Pol. Econ.*, October 1966.

Kryzyaniak, M., "The Long-run Burden of a General Tax on Profits in a Neoclassical World," *Public Finance*, 1967.

Mincer, J., "On-the-Job Training: Costs Returns and Some Implications," *Jour. Pol. Econ.*, October 1962.

Mincer, J., "The Personal Distribution of Income. A Survey with Special Reference to the Human Capital Approach," *Jour. Econ. Lit.*, 1969.

Mirrlees, J., "A Exploration in the Theory of Optimal Income Taxation," *Rev. Econ. Studies.*, April 1971.

Sato, K., "Taxation and Neoclassical Growth," *Public Finance*, 1967.

Schultz, T. W. "Investment in Human Capital," *Amer. Econ. Rev.*, 51, March 1961.

Sheshinski, E., "An Example of the Income Tax Schedule which is Optimal for the Maxi-Min Criterion," IMSSS Tech. Report No. 74, 1972.

Stiglitz, J., "The Corporation Tax," Tech. Report No. 162, IMSSS, Stanford, 1975.

The Intergenerational Transmission of Wealth and Terminal Capital Gains Taxation

Part I

James D. Smith and Stephen D. Franklin

Pennsylvania State University

Taxing the Intergenerational Transmissions of Wealth: A Simulation Experiment

Part II

James D. Smith and Stephen D. Franklin
Pennsylvania State University

Guy H. Orcutt
Yale University

The Intergenerational Transmission of Wealth and Terminal Capital Gains Taxation

Part I

James D. Smith and Stephen D. Franklin

Pennsylvania State University

A Note on Presentation

Each major simulation task is presented in a section consisting of three segments. The first is a short introduction to the nature of the task. The second segment presents the simulation logic in the general form and sequence of equations which implement each operating characteristic. The third segment presents the statistical estimation of the operating characteristics. The reader wishing a quick understanding of how the model will operate may skip the third segment of each task, or refer to it only for purposes of decerning the coefficients for a few functions which were so large or complex that the values are not shown in the second part.

The intergenerational transmission of wealth and its taxation is itself a rather large piece of work and is presented in Part II.

A schematic representation of the sequencing of the events associated with asset accumulation and the devolution of wealth is presented in Chart 1. The processes take place in two computer modules: a saving and taxation module, and a wealth devolution module. Prior to this stage in the simulation algorithm, the population has been subjected to several other modules which simulate general population characteristics, property and transfer income and education. The process related to wealth accumulation, taxation of income taxes and wealth devolution are as follows:

(Numbers 1 through 12 are presented in Part I.)

1. Accrued capital gains on corporate stock
2. Capital gains from sale of corporate stock (+, −)
3. Capital gains from sale of other assets (+, −)
4. Purchases (net) of residential housing (+, −)
5. Federal income taxation
6. Savings, total (+, −)
7. Saving in the form of earning assets (+, −)
8. Saving in the form on nonearning assets (other than housing) (+, −)

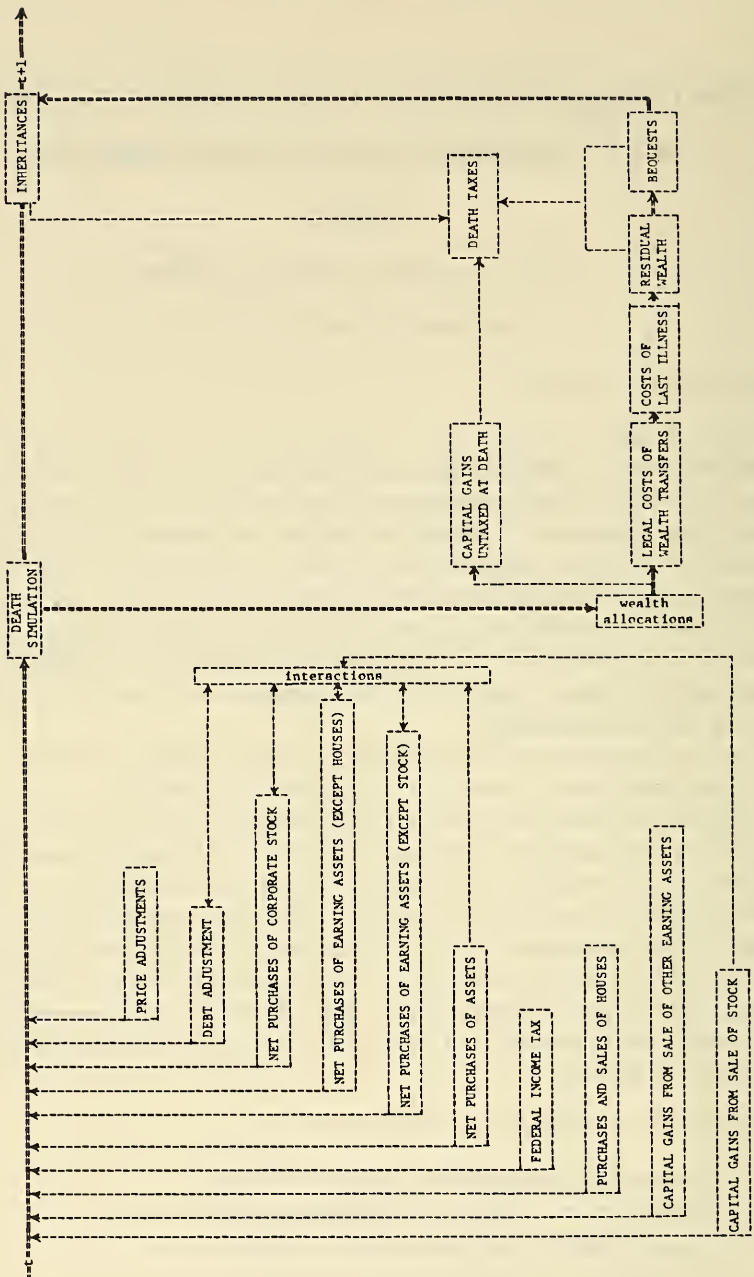


Chart 1.—*Simulation Logic for Portion of Model Related to the Generation of Capital Gains, Income Taxation, Savings, and the Intergenerational Transmission of Wealth*

9. Saving in the form of corporate stock (+, −)
10. Saving in the form of debt (+, −)
11. Capital gains accured to corporate stock during year (+, −)
12. Capital gains accruing to other assets during year (+, -)

(Numbers 13 through 17 are presented in Part II.)

13. Intrafamily allocation of wealth
14. Medical costs of last illness
15. Legal costs of transferring wealth
16. Death taxes (estate, inheritance and/or succession)
17. Bequests of wealth to persons and charities

One slight exception should be noted. The growing of a population with a full set of kinship linkages has not been implemented, though the computer mechanism required to maintain the addressing system for the kinship linkages is complete. Implementation was delayed since it would not be needed until later. The delay will permit taking advantage of running efficiencies resulting from MASH system modifications (now completed).

Simulation of the accrual, transmission and taxation of capital gains at death will be done within The Urban Institute Microanalytic Simulation Model. (For some applications, MASH, the computer software usually employed with the Microanalytic Model, will not be used. In its place, less elaborate and, consequently, less costly software will be used.) In the process of running the Microanalytic Model a number of events—births, deaths, marriages, divorces, income from labor, transfer income, education and aging—will be simulated year by year. These events will shape the characteristics of the population whose wealth and capital gains are simulated, but no feedback to these events from the wealth simulation will occur.[1]

This paper deals only with processes directly related to the simulation of wealth transmission, capital gains accrual and taxation which were completed or upon which substantial progress was made in the first year. The number of processes which one must take into account in a closed microsystem in which capital grows and is taxed is rather large if the system is to be realistic enough and flexible enough to lend itself to policy experiments. It is best then to start by laying out the full set of processes which are simulated in that portion of the model dealing with wealth and its taxation.

I. Introduction

The work reported here was undertaken in fulfillment of contract TOS–74–19 between the Office of Tax Analysis of the U.S. Treasury and The Urban Institute. Guy H. Orcutt was the principal investigator

[1] For a complete description of the basic operating characteristics see Orcutt, Caldwell, Smith and Wertheimer, "Policy Exploration through Microanalytic Simulation," Urban Institute Working Paper 509–12, May 20, 1975.

and significant contributions were made to the effort by Stephen D. Franklin and James D. Smith.

The work is the first part of a two part effort to simulate the intergenerational transmission of wealth and the untaxed capital gains embodied therein at death. Direct evidence on the magnitude of accrued capital gains passing at death does not exist. However, estimates of about $2 billion have been made of the tax loss (Eilbott).

The original research strategy was for The Urban Institute to adapt its microsimulation model to accept: (a) special population files, (b) a time series of accrued capital gains and (c) a set of policy models embodying alternative death taxes. Items a, b, and c were to be provided by the Treasury. The Urban Institute was also to provide for the dynamic generation of family saving and the growing of a population with a full set of kinship linkages so that wealth could be transferred along familial lines consistent with observed patterns.

Pressure on Treasury resources in the first year resulted in a lower level of Treasury participation than was anticipated. Fortunately, work by Smith (not a party to the original contract) was proceeding independently along lines very similar to that started for the Treasury. By enlarging the scope of Smith's work and focusing parts of it directly on the needs of the Treasury project, all of the initially specified goals of the Treasury and The Urban Institute staffs were completed. In addition, some work to be undertaken in the second year's effort was also completed.

II. Accrued Capital Gains

Tangible assets depreciate and their value in one year is a function of their value in the preceding year: $A_{i,t} = A_{i,t-1}(1 - d)$, where d is an appropriate depreciation rate. Individual assets may also appreciate in real terms, that is, their exchange value vis-a-vis all other assets, increase. The real value of a tangible asset i in year t is then: $A_{i,t} = A_{i,t-1}(1 + r - d)$, where r is the rate of change in the exchange rate between asset i and all other assets. In the work undertaken for the Treasury, r is the rate of change in the market value of an asset, because simulation of capital gains taxes which distinguish between real and monetary changes are not contemplated. In the case of financial assets depreciation can be ignored.

The model is limited to four types of assets: residential housing, other nonearning assets, corporate stock and other earning assets. Corporate stock and housing have been singled out because of their importance in tax policy.

Accrued Gains on Corporate Stock

Turning first to the treatment of corporate stock, the accrued capital gain in any year is determined by

$$\text{ACGAINSTOCK}_t = \text{STOCKVAL}_{t-1}(r_1 + E) + \text{ACGAINSTOCK}_{t-1} - \text{STOCKSAVE}_{t-1} - \text{GAINSTOCK}_{t-1}$$

where STOCKVAL is the value of corporate stock at the beginning of the preceding year, r_1 is the rate of change in stock market values, E is a random draw from the distribution of residuals of rates of change in the value of individual portfolios around the market rate of change. The rate of change in the market value of corporate stock has historically been around 7 percent per year, r_1 is an experimental variable which can be set to reflect the historical trend, other rates, or modified to capture more complex market behavior, such as cyclical movement. The variance of individuals' portfolio performances around the market performance will initially be estimated as the sampling distribution of the performance of the sample portfolios of 15 securities (drawn with probabilities of selection proportional to the value of the issue outstanding to the value of all issues in the market). The distribution of residuals of the sample portfolio performance from market performance will then be used to draw random normal deviates to be added to r_1 in generating the annual accrued gains assigned to individual familes.

Accrued Gains on Earning Assets and Housing

Accrued capital gains on other earning assets and on housing will also be tracked in the model. In both cases the assignment of the annual gain accrual is done without a stochastic term. Thus for these two assets the value of the accrued gain in year t will be:

$$\mathrm{ACGAINEARNASSET}_t = \mathrm{ACGAINEARNASSET}_{t-1} - \mathrm{HOUSESAVE}_{t-1} - \mathrm{HOUSEGAIN}_{t-1}$$

$$\mathrm{ACGAINEARNASSET}_t = \mathrm{ACGAINEARNASSET}_{t-1} + \mathrm{EARNASSETVAL}_{t-1}(r_3) - \mathrm{EARNASSETSAVE}_{t-1} - \mathrm{EARNASSETGAIN}_{t-1}$$

At the present time we are ignoring any capital gains or losses taking place on other nonearning assets. These consist largely of consumer durables, demand deposits, automobiles and personal property.

III. Realized Capital Gains on Corporate Stock

The realization of a capital gain on corporate stock is an elective act by stockholders. Empirical work on who takes capital gains on stock is not abundant. Work by Barlow, Brazer and Morgan[2] and by David and Miller[3] both found a persistence of realization of gains to particular individuals. Our own analysis of merged observations for the same families in the Survey of Financial Characteristics of Consumers and the Survey of Changes in the Financial Characteristics of Consumers

---

[2] Barlow, Brazer and Morgan, *Economic Behavior of the Affluent*, The Brookings Institution, 1966.

[3] Martin David and Roger Miller, "Capital Gains and Individual Income—Evidence of realization and Persistence," in James D. Smith, *The Personal Distribution of Income and Wealth*, National Bureau of Economic Research, 1975.

revealed that the most important predictor of the occurrence of a capital gain or loss in 1963 was a capital gain or loss in 1962.

Simulation of Realized Capital Gains on Corporate Stock

In the simulation model the first task of imputing a capital gain to a family unit is the assignment of a probability of realizing a capital gain or loss from sales of corporate stock. This probability is a function of whether or not a gain or loss occurred last year, the education and age of head, the ratio of value of family's stock to mean value of family stockholdings, wages as a percentage of gross income and the percentile rank of debt.

$$P(\text{GAINSTOCK}_{kt} \neq 0) = F_1(\text{GAINSTOCK}_{k,t-1}, \text{ED}_{kt}, \text{AGE}_{kt}, \text{STOCKRATIO}_{k,t-1}, \text{WAGES}_{kt}/\text{GROSSING}_{kt}, \text{DETRANK}_{kt}$$

If RANDOM 1 $< P(\text{GAINSTOCK}_{kt} \neq 0)$, capital gain is computed according to:

$$\text{GAINSTOCK}_{kt} = [-0.026 + 0.337 \cdot (\text{ASSETRANK}_{k,t-1}/1000) - 0.0074 \cdot (\text{STOCKRATIO}_{k,t-1}/10000) + \text{ERROR1}] \cdot \text{ASSETS}_{k,t-1}$$

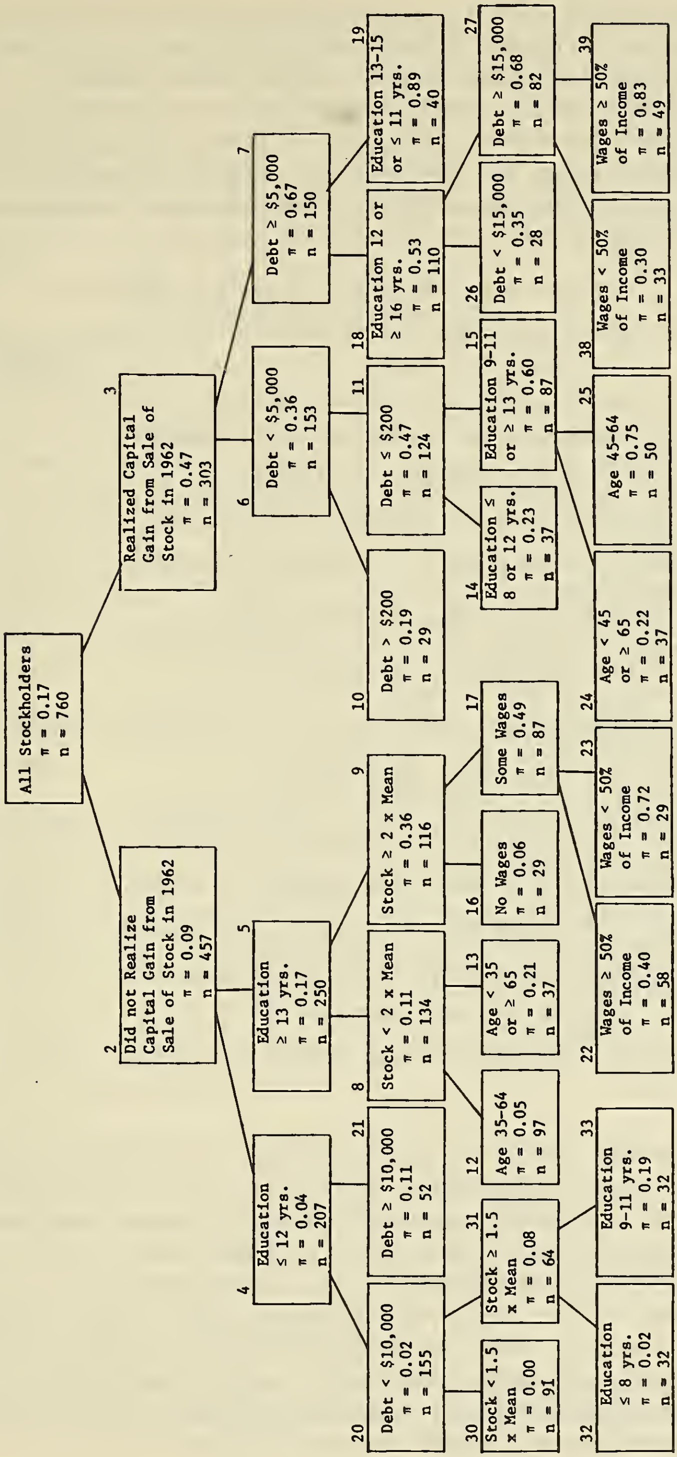
ERROR1 is a function of age of head, rank of gross income, rank of assets and wages as a percentage of gross income estimated in the AID analysis.

$$\text{ERROR1} = F_2(\text{AGE}_{kt}, \text{GROSSINCRANK}_{kt}, \text{ASSETRANK}_{k,t-1}, \text{WAGES}_{kt}/\text{GROSSING}_{kt})$$

Estimation of Operating Characteristics for Realized Capital Gains on Stock

The first step in the process was to estimate the probability that a family would take a capital gain. AID was used to break the population down into groups (based upon characteristics which would be available in the simulation) with differing probabilities of taking a capital gain on corporate stock. The results of the AID analysis are shown in chart 2 and summarized in table 1. In the chart the branching sequence shows the order of importance of variance reduction of each variable, the most important variable appearing earliest. Thus it will be seen in the chart that the best single predictor (of those available to the program) was whether the family took a realized capital gain in the preceding year. In the table the final groups with all their built in interactions are shown in ascending order of the value of the probability of taking a gain. It will be seen that the probability of realizing a gain if a family had not realized a gain in the preceding year, had a head with less than 13 years of education, debts less than $10,000 and corporate stock worth less than $1,500 was about .01. On the other hand, families who did realize a gain last year, had debt over $14,999 and a head with a high school

Chart 2.—*Probability of Realized Capital Gain from Sale of Stock*



17 final groups account for 41.9% of variance

diploma or more than 3 years of college, and whose wage income was greater than 50 percent of their total income had an .83 probability of taking a realized capital gain. (A slightly higher probability, .89, was estimated for families who had a gain last year, debt in excess of $4999 and a head with 13 to 15, or less than 12 years of education. The rather unexpected values of the education variable are explained by the splitting off of those that either finished high school or finished college, thus leaving the college and high school dropouts.)

TABLE 1.—*Probabilities of capital gain from sale of stock*
[Summary from AID tree in Chart 2]

	Probability equals
If:	
NO GAIN LAST YEAR AND ED <13 AND DEBT <10000 AND STK <1.5 * M	0.001
NO GAIN LAST YEAR AND ED <9 AND DEBT <10000 AND STK >1.5 * M	.020
NO GAIN LAST YEAR ED >12 AND STK <2 * M AND AGE =: 35, 64:	.050
NO GAIN LAST YEAR AND ED >12 AND STK >2 * M AND WAGES = 0	.060
NO GAIN LAST YEAR AND ED <13 AND DEBT >9999	.110
NO GAIN LAST YEAR AND ED = :9, 12 AND DEBT <10000 AND STK > 1.5 * M	
GAIN LAST YEAR AND DEBT <5000 AND DEBT >199	.190
NO GAIN LAST YEAR AND ED >12 AND STK <2 * M AND (AGE <35 OR >64)	.210
GAIN LAST YEAR AND DEBT <200 AND (ED =: 9, 11: OR >12) AND (AGE <45 OR >64)	.220
GAIN LAST YEAR AND DEBT <200 AND (ED <9 or = 12)	.230
GAIN LAST YEAR AND DEBT >14999 AND (ED = 12 OR >15) AND WAGES <0.5 * Y	.300
GAIN LAST YEAR AND DEBT >5000 AND (ED = 12 OR >15) AND DEBT <15000	.350
NO GAIN LAST YEAR AND ED >12 AND STK >2 * M AND WAGES >0.5 * Y	.400
NO GAIN LAST YEAR AND ED >12 AND STK >2 * M AND WAGES >0.5 * Y	.720
GAIN LAST YEAR AND DEBT <200 AND (ED = : 9, 11: OR >12) AND AGE =: 45, 64:	.750
GAIN LAST YEAR AND DEBT >14999 AND (ED = 12 OR >15) AND WAGES >0.5 * Y	.830
GAIN LAST YEAR AND DEBT >4999 AND (ED = : 13, 15: OR <12)	.890

The second step in the estimation was to regress the value of the realized gain (for those taking gains) as a ratio to total assets on the family rank (0–100) in ascending order of total assets, and the ratio of family stock values to the mean value of all families' stock. As will be seen in table 2, we are able to explain a pathetically small (1.7 percent) proportion of the variance of the ratio, capital gain on stock/total assets. However, when the predicted ratios are converted to dollar values and compared to the observed values of capital gains, we find that we explain about 17 percent of the variance of the dollar values. The regression statistics are presented in table 2.

TABLE 2.—*Regression with dependent variable ratio of capital gain from sale of stock to assets*

Variable	Coefficient	t = Ratio
1. Constant	−0.0260	−1.70
2. Rank of assets/1,000	0.3370	2.13
3. Stock/(mean × 10,000)	−0.0074	−0.25

The estimated standard deviation of the dependent variable about the regression line is 0.0237 with 258 degrees of freedom.

$R^2 = 1.7\%$.

The predicted ratios explain 17.3% of variance in capital gain.

Finally, we take the residuals of the ratios from the regression equation and subject them to an AID analysis. The AID results are shown in the form of an AID tree in chart 3. It will be seen there that the variables which contribute to a reduction of variance in the residuals of the original regression are age of head, level of assets, gross income and share of gross income represented by wages. The six final groups in chart 3 explain about 5 percent of the variance of the residuals of the ratios, which would amount to about 40 percent of the variance in the value of capital gains. Thus, by the combined use of regression and AID approaches we squeezed out about 65 percent of the variance in the value of capital gains for families taking capital gains in the SFCC files.

IV. Realized Capital Gains on Assets Other than Corporate Stock and Housing

In addition to the realization of capital gains on corporate stock discussed above, and the capital gains on houses, which are generated by the process of buying and selling houses and price changes, realized capital gains are also generated on earning assets other than corporate stock. These gains are essentially on commercial assets. As with corporate stock, we have used the SFCC files as an empirical base.

The Simulation of Realized Capital Gains on Earning Assets other than Corporate Stock

The simulation model implements these estimates as follows:

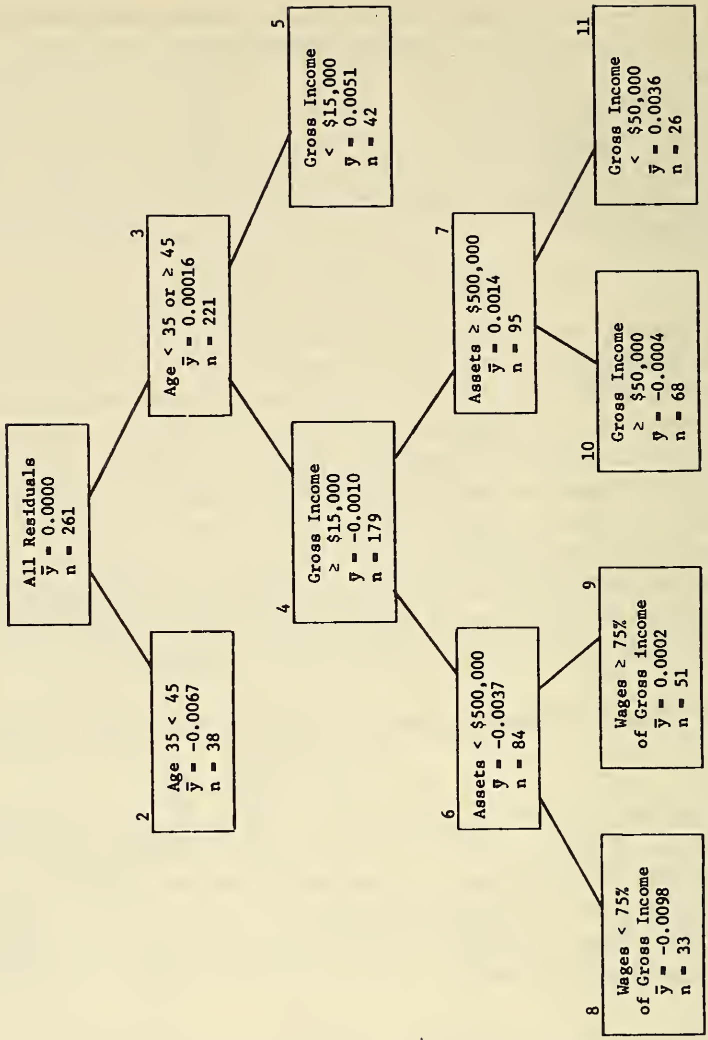
$$P(\text{GAINEARNING}_{kt} \neq 0) = F_3(\text{GAINEARNING}_{k,t-1}, \text{ASSETRANK}_{k,t-1}, \text{DEBTRANK}_{kt}, \text{GROSSINCRANK}_{kt}, \text{ED}_{kt}, \text{AGE}_{kt}, \text{URBAN}_{kt}\text{size of area}), \text{WAGES}_{kt}/\text{GROSSING}_{kt}, \text{EARNASSETRATIO}_{k,t-1})$$

and if RANDOM2 $< P(\text{GAINEARNING}_{kt} \neq 0)$

$$\text{GAINEARNING}_{kt} = [0.0807 - 0.5501 \cdot (\text{ASSETRANK}_{k,t-1}/1000) + 0.5991 \cdot (\text{EARNTRATIO}_{k,t-1}/10000) + 0.028 \cdot \Delta\ \text{GROSSING}_{kt}\ (\%\ \text{change})] \cdot \text{ASSETS}_{k,t-1}$$

Chart 3.—*Residual Ratio of Capital Gain (From Sale of Stock) to Assets*

$R^2 = 5.0\%$



Estimation of Operating Characteristics for Realized Capital Gains on Assets other than Corporate Stock and Housing

Realized capital gains on assets other than corporate stock and housing were estimated using the SFCC merged files by first estimating the probability of a gain or loss using AID and then estimating the expected gain (+, −) for those families taking a gain or loss. An error term was not estimated as was done for capital gains from corporate stock. The regression used to estimate the value of capital gains or losses had as its dependent variable the ratio of the capital gain to total family assets. The R^2 for the regression in this form is only 1.6, but when the predicted ratios are converted to dollar values, the latter explain 30 percent of the variance of capital gains. The AID tree, its summary and the regression results are shown in chart 4 and tables 3 and 4.

TABLE 3.—*Probabilities of capital gain from sale of assets other than stock*
[Summary for AID tree in Chart 4]

	Probability equals
If:	
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT <500 ..	0.005
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >499 AND ED =: 12, 15: AND (AGE <25 OR >44)	.005
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >499 AND (ED <12 OR >15) AND (AGE <25 OR >64)	.010
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >499 AND ED =: 12, 15: AND AGE =: 25, 44: AND ASSETS >2 * M AND DEBT >50000	.010
NO GAIN LAST YEAR AND ASSETS <25000	.013
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >499 AND ED =: 12, 15: AND AGE =: 25, 44: AND ASSETS >2 * M	.038
GAIN LAST YEAR AND DEBT <5000	.051
GAIN LAST YEAR AND DEBT >4999 AND WAGES >0.75 * Y ...	.085
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >4999 AND (ED <12 OR >15) AND AGE =: 25, 64: AND Y <15000 AND (AREA = RURAL OR SMSA)	.103
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >499 AND ED = : 12, 15: AND AGE = : 25, 44: AND ASSETS >2 * M.	.170
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >499 AND (ED <12 OR >15) AND AGE = : 25, 64: AND Y <15000 AND AREA = URBAN. NONSMSA	.229
GAIN LAST YEAR AND DEBT >4999 AND WAGES <0.75 * Y ...	.366
NO GAIN LAST YEAR AND ASSETS >24999 AND DEBT >49999 AND (ED <12 OR >15) AND AGE =: 25, 64: AND Y >14999	.428

TABLE 4.—*Regression with dependent variable ratio of capital gain from sale of earning assets other than stock to assets*

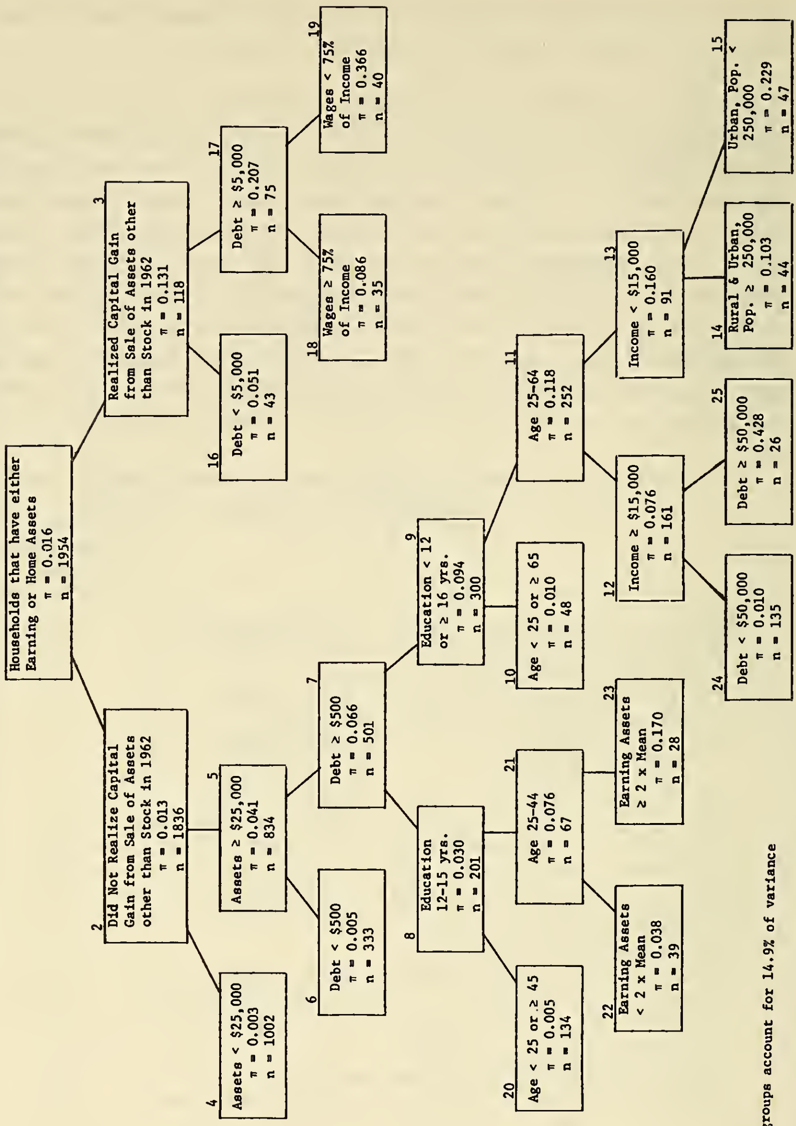
Variable	Coefficient	t = Ratio
1. Constant	0.0807	0.74
2. Rank of assets/1,000	−0.5501	−0.47
3. Earning assets/(mean × 10,000)	0.5991	0.58
4. Percent change in gross income	0.0280	0.84

The estimated standard deviation of the dependent variable about the regression line is 0.0983 with 67 degrees of freedom.

R^2 = 1.6%.

The predicted ratios explain 30.4% of variance in capital gain.

Chart 4.—*Probability of Capital Gain From Sale of Assets Other Than Stock*



13 final groups account for 14.9% of variance

V. Housing Transactions and Capital Gains on Housing

Because preferential tax treatment is afforded homeowners, transactions involving residential housing have been treated separately from other assets in the simulation model. Three types of transactions are permitted families in a given simulation year:

1. Buying a house if another one is not owned during the year.
2. Selling a home.
3. Buying and selling a home during the year.

The Simulation of Housing Transactions and Housing Capital Gains

If the family unit owns a home the probability of selling the home is found. As noted above probability is a function of age and education of head, wages as a percentage of gross income, size of area, family size, the ranks of gross income, assets and debt and the ratio of home value to mean home value.

$$P(SELLHOME_{kt}) = F_4(AGE_{kt}, ED_{kt}, WAGES_{kt}/GROSSING_{kt}, URBAN_{kt}, FAMSIZ_{kt}, GROSSINCRANK_{kt}, ASSETRANK_{k,t-1}, DEBTRANK_{kt}, HOMERATIO_{k,t-1})$$

The event of selling the home occurs if RANDOM3 < $P(SELLHOME_{kt})$, in which case the capital gain or loss is computed and the probability of buying a home is found. The probability of buying a home conditional on having sold a home in the same year is a function of the age of head, the ranks of gross income, assets and debt, number of children and size of area.

$$P(BUYHOME_{kt} | SELLHOME_{kt}) = F_5(AGE_{kt}, GROSSINCRANK_{kt}, ASSETRANK_{k,t-1}, DEBTRANK_{kt}, NUMKIDS_{kt}, URBAN_{kt})$$

$$\text{If } RANDOM4 < P(BUYHOME_{kt} | SELLHOME_{kt})$$

the family unit buys a home the value of which is calculated according to:

$$HOUSEVAL_{kt} = (0.365 + 0.0088 \cdot ASSETRANK_{k,t-1} + 0.0073 \cdot DEBTRANK_{kt} - 0.2075 \cdot HOMERATIO_{k,t-1}) \cdot HOUSVAL_{k,t-1}$$

Currently, downpayment on the purchase of a home is set to 25 percent of the price. The remainder becomes the mortgage debt for the family unit. The rate of interest for home mortgages is obtained from the macromodel and is stored as part of the family's permanent record. Repayment period is assumed to be 25 years and the yearly mortgage payment is calculated according to:

$$MORTPAY_{kt} = [MORTDEBT \cdot INTRATE_{kt} \cdot (1 + INTRATE_{kt})^{25}] / [1 + INTRATE_{kt})^{25} - 1]$$

If the family unit did not own a home at the start of the simulated

year, the probability of buying a home is found and is a function of age and education of head, rank of gross income, rank of debt, wages as a percentage of gross income, and family size.

$$P(\text{BUYHOME}_{kt} \mid \text{NONOWNER}_{k,t-1}) = F_6(\text{AGE}_{kt}, \text{ED}_{kt}, \text{GROSSINCRANK}_{kt}, \text{DEBTRANK}_{kt}, \text{WAGES}_{kt}/\text{GROSSING}_{kt}, \text{FAMSIZ}_{kt})$$

If $\text{RANDOM5} < P(\text{BUYHOME}_{kt} \mid \text{NONOWNER}_{k,t-1})$

the unit buys a home, the cost of which is computed by:

$$\text{HOUSEVAL}_{kt} = (1.8073 - 0.0079 \cdot \text{ASSETRANK}_{k,t-1}) \cdot \text{GROSSING}_{kt}$$

Yearly mortage payment is calculated as before.

Net changes in home value and mortgage debt are computed and stored in common with the saving module.

Functions F_1 through F_6 are given in tables 10 through 17.

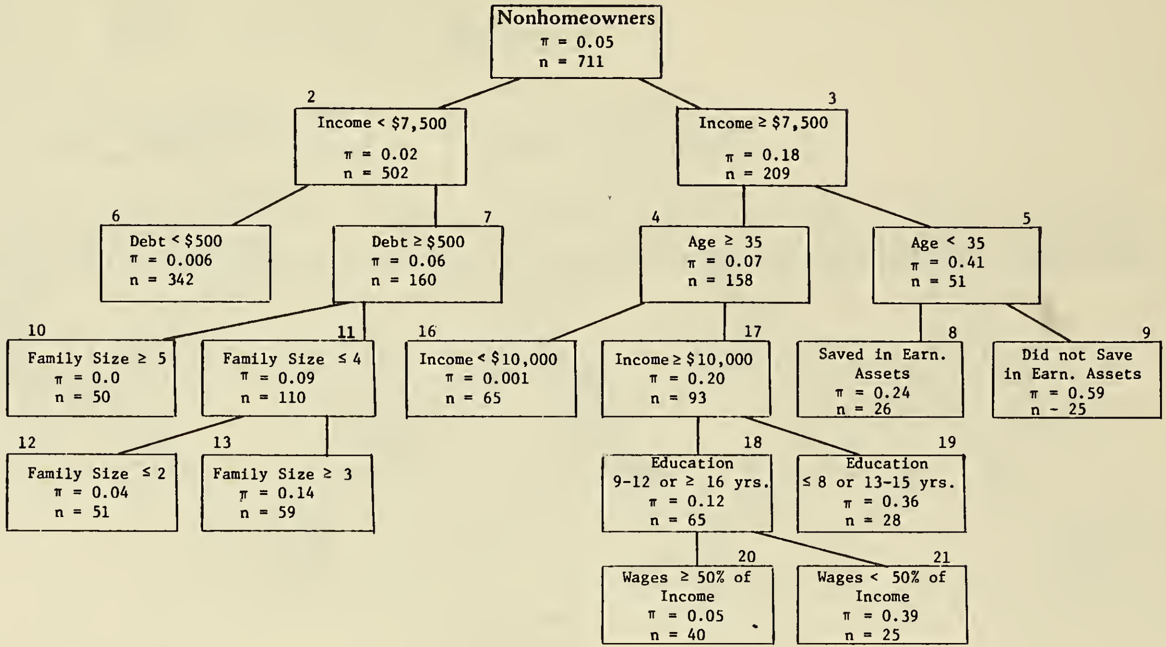
Estimation of Operating Characteristics for Housing Transactions

Three probabilities were first estimated: the probability that a family would purchase a house if it did not own one, the probability that a family would sell a house, and the conditional probability that a family would buy a house if it sold one. Each of these probabilities was estimated using AID. The AID trees are shown in charts 5 through 7 and the results are summarized in tables 4 through 6. For nonowners, most of whom never owned a home, the probability of purchase for low income families appears to depend on family size. Those with small families and large families have lower probabilities of purchase than those with family sizes in the 3 to 4 range. Presumably, the former feel less need to own or are unable to find small enough houses to economically match their small family sizes, while those with five or more persons find the demands on their income so great that home ownership is constrained. See chart 5 and table 5.

TABLE 5.—*Probabilities of buying home (nonhomeowners)*
[Summary of AID tree in Chart 5]

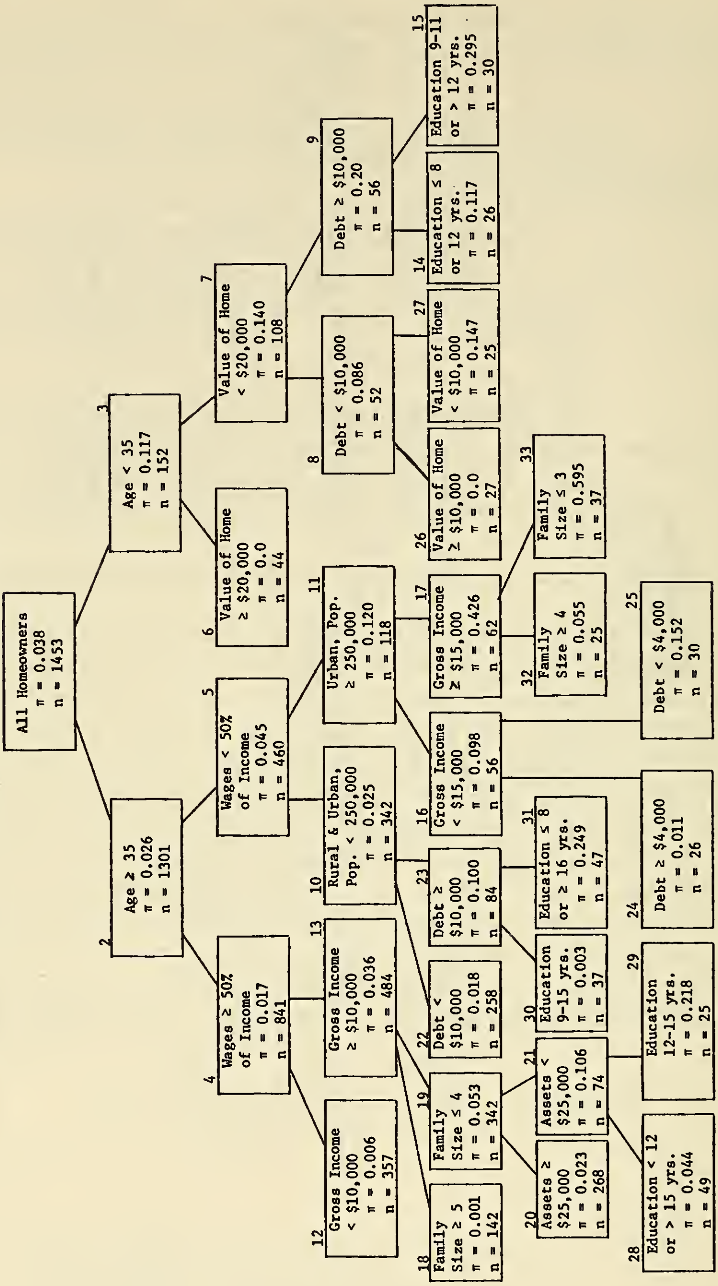
	Probability equals
If:	
Y <7500 AND DEBT >499 AND FAMSIZE >4	0.001
Y <7500 AND DEBT <500	.006
Y <7500 AND DEBT >499 AND FAMSIZE <3	.040
Y >9999 AND AGE >34 AND (ED =: 9, 12: OR >15) AND WAGES >0.5 * Y	.050
Y =: 7500, 9999: AND AGE >34	.070
Y <7500 AND DEBT >499 AND FAMSIZE =: 3, 4:	.140
Y >7499 AND AGE <35 AND ESAVE >100	.240
Y >9999 AND AGE >34 AND (ED <9 OR =: 13, 15)	.360
Y >9999 AND AGE >34 AND (ED =: 9, 12: OR >15) AND WAGES <0.5 * Y	.390
Y >7499 AND AGE <35 AND ESAVE <100	.500

Chart 5. – *Probability of Buying Home (Nonhomeowners)*



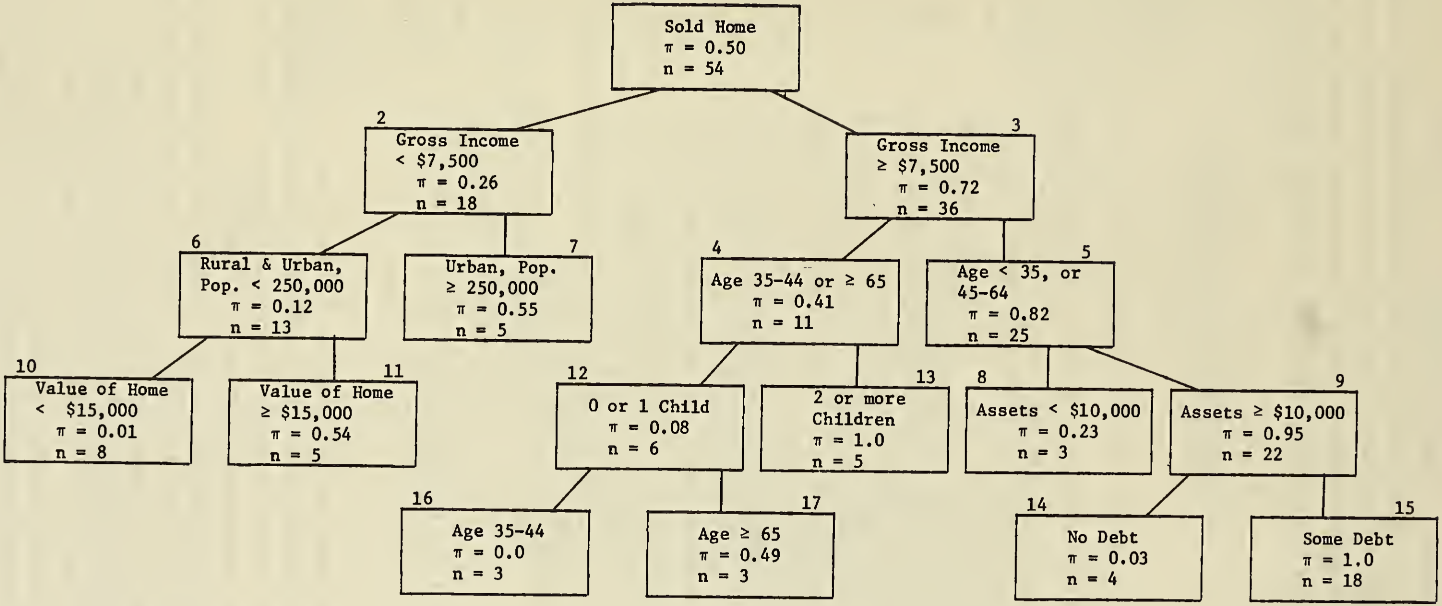
10 final groups account for 28.8% of variance

Chart 6.—*Probability of Selling Home*



17 final groups account for 14.0% of variance

Chart 7.—*Conditional Probability of Buying Home Given Sale*



9 final groups account for 70.8% of variance

The probability of selling a home was sensitive to the age and education of the family head, the size of income and debt of the family and the value of the house. The tendency was for families with older heads, less expensive houses and fewer persons to have higher probabilities of selling a house, given that they owned one. See chart 6 and table 6.

TABLE 6.—*Probabilities of selling home*
[Summary of AID tree in Chart 6]

	Probability equals
If:	
AGE >34 AND WAGES >0.5 * Y AND Y >9999 AND FAMSIZE >4	0.001
AGE <35 AND HOMVAL >19999	.001
AGE <35 AND HOMVAL <20000 AND DEBT <10000 AND HOMVAL >10000	.001
AGE >34 AND WAGES <0.5 * Y AND (AREA = RURAL OR URBAN, NONSMSA) AND DEBT >9999 AND ED =: 9, 15:	.003
AGE >34 AND WAGES >0.5 * Y AND Y <10000	.006
AGE >34 AND WAGES <0.5 * Y AND AREA = URBAN, SMSA AND Y <15000 AND DEBT >3999	.011
AGE >35 AND WAGES <0.5 * Y AND (AREA = RURAL OR URBAN, NONSMSA) AND DEBT <10000	.018
AGE >34 AND WAGES >0.5 * Y AND Y >9999 AND FAMSIZE <5 AND ASSETS >24999	.023
AGE >34 AND WAGES >0.5 * Y AND Y >9999 AND FAMSIZE <5 AND ASSETS <25000 AND (ED <12 OR >15)	.044
AGE >34 AND WAGES <0.5 * Y AND AREA = URBAN, SMSA AND Y >14999 AND FAMSIZE >3	.055
AGE <35 AND HOMVAL <20000 AND DEBT >10000 AND (ED <9 OR = 12)	.117
AGE <35 AND HOMVAL <10000 AND DEBT <10000	.147
AGE >34 AND WAGES <0.5 * Y AND AREA = URBAN, SMSA AND Y <15000 AND DEBT <4000	.152
AGE >34 AND WAGES >0.5 * Y AND Y >9999 AND FAMSIZE <5 AND ASSETS <25000 AND ED =: 12, 15:	.218
AGE >34 AND WAGES <0.5 * Y AND (AREA = RURAL OR URBAN, NONSMSA) AND DEBT >9999 AND (ED <9 OR >15)	.249
AGE <35 AND HOMVAL <20000 AND DEBT >10000 AND (ED =: 9, 11: OR >12)	.295
AGE >34 AND WAGES <0.5 * Y AND AREA = URBAN, SMSA AND Y >14999 AND FAMSIZE <4	.595

The conditional probability that a family buys a house given that it sold one in the same year was found to be a function of family income, age of head, and value of assets, with younger ages higher incomes and assets being associated with higher probabilities.

In all of the estimates of housing transaction probabilities and values we are hampered by the relatively small sample of families who are found to have such transactions in a one year period when we start with a total sample of only about 2,500 cases in the SFCC files.

To assign the value of a purchased home we required an estimate of the ratio of purchase price to some other variable in the model which was being moved along each simulation year. For homes purchased when one was sold, we estimated the ratio of the purchase price to the

TABLE 7.—*Probabilities of buying home, conditional on sale of home*
[Summary of AID Tree in Chart 7]

	Probability equals
If:	
Y <7500 AND (AREA = RURAL OR URBAN, NONSMSA) AND HOMVAL <15000	0.010
Y >7499 AND (AGE <35 OR AGE =: 45, 64: AND ASSETS >9999 AND DEBT = 0	.030
Y >7499 AND (AGE <35 OR AGE = : 45, 64: AND ASSETS <10000	.230
Y >7499 AND AGE =: 35, 44: AND NUMKIDS =: 0, 1:	
Y >7499 AND AGE >64 AND NUMKIDS =: 0, 1:	.490
Y <7500 AND (AREA = RURAL OR URBAN, NONSMSA) AND HOMVAL >14999	.540
Y <7500 AND AREA = URBAN, SMSA	.550
Y >7499 AND (AGE =: 35, 44: OR >64) AND NUMKIDS >1	1.000
Y >7499 AND (AGE <35 OR AGE =: 45, 64: AND ASSETS >9999 AND DEBT >0	1.000

price of the house sold. That ratio was estimated by regressing it on the family ranks (0–100) of assets and of debt, and the ratio of the value of the home sold to the mean value of all homes. The regression does a fairly decent job of predicting the value of the home sold. About 71 percent of the variance in the purchase price was explained.

In the case of homes purchased when none was sold, a regression of the ratio of cost of purchased home to total income was run on rank (0–100) of family assets. The regression explained about 64 percent of the variance in purchased home prices. However, the rather low constant term 1.8 and the negative sign of the coefficient on rank of total income suggest that the SFCC may be an inappropriate sample.[4] The regression statistics are shown in tables 8 and 9.

TABLE 8.—*Regression with dependent variable ratio of cost of purchased home to gross income*

Variable	Coefficient	t Ratio
1. Constant	1.8073	10.49
2. Rank of assets	−0.0079	−2.87

The estimated standard deviation of the dependent variable about the regression line is 0.6931 with 64 degrees of freedom.

R^2 = 11.4%.

The predicted ratios explain 64.1% of variance in cost of purchased home.

VI. Federal Income Tax

The model includes an income tax which captures the main features of the 1972 federal statute. For instance, account is taken of the distinctions between married filers (all married filers are assumed to file joint returns), single, and head of household filers. Exemptions are

[4] These results are after the original regressions had been studied and appropriate adjustments had been made to the data to damp the heterogenous distribution of residuals, and the regressions rerun.

TABLE 9.—*Regression with dependent variable ratio of cost of purchased home to value of sold home*

Variable	Coefficient	t Ratio
1. Constant	0.3650	0.68
2. Rank of assets	0.0088	1.19
3. Rank of debt	0.0073	2.13
4. Value of sold home/mean home value	−0.2075	−1.96

The estimated standard deviation of the dependent variable about the regression line is 0.5283 with 27 degrees of freedom.

$R^2 = 20.0\%$.

The predicted ratios explain 70.8% of variance in cost of purchased home.

provided for each family member and additional exemptions are provided for heads and wives over age 64. Deductions are calculated according to a formula specified in consideration of the law and the ratio of deductions to adjusted gross income over $8,000 in 1972. The published statutory income tax rates are converted to six functions (two for each marital status).

The income tax is computed in simulation for each family filer, or unattached individual by the following steps.

1. Determination of adjusted gross income (AGI).

   AGI = WAGES + PROPINC + DIVIDEND + GAINEARN + GAINSTOCK − ADJUST

   AGI is roughly equivalent to the concept used by the IRS. ADJUST is a variable (currently set to 0) which can be used to model various tax provisions such as sick pay and moving expenses.

2. Determination of taxable income.

   EXEMPT is a variable which can be set to represent the value of personal exemptions under the law. It is currently set at $750. FAMSIZE is the total family size of each filer unit. H is a dummy variable set to 1 if the family head is 65 or older and otherwise to 0. W is set to 1 if the wife of the family head is 65 or older and otherwise to 0.

   DDUCT is the value of claimed deductions. It is determined by the equation DDUCT = 1300 + Z(AGI − 8667), where Z is the ratio of deductions to AGI. At present Z = 0.2434, based on calculations from the 1972 returns of persons with AGI in excess of $8000.

3. Levying tax.
   INCTAX = TXING[(INCTXRT1) (MS1) (LEVEL1) + (INCTXRT2) (MS1) (LEVEL2) + (INCTXRT3) (MS2) (LEV-

   EL) + (INCTXRT4) (MS2) (LEVEL2) + (INCTXRT5) (MS3) (LEVEL1) + (INCTXRT6) (MS3) (LEVEL2)]

MS1 = 1 if filer is married otherwise 0.
MS2 = 1 if filer is unmarried and living with others, otherwise 0.
MS3 = 1 if filer is single, otherwise 0.
LEVEL1 = 1 is filer's income is over $8000, otherwise 0.
LEVEL2 = 1 if filer's income is $8000 or less, otherwise 0.
INCTXRT1 = −16.7088 + 13.1770 (Log TAXING/1000)
INCRXRT2 = 14.1906 + 1.1187 (Log TAXING/1000)
INCTXRT3 = −13.3524 + 13.0228 (Log TAXING/1000)
INCTXRT4 = 14.4187 + 1.6405 (Log TAXING/1000)
INCTXRT5 = −9.8655 + 12.8403 (Log TAXING/1000)
INCTXRT6 = 14.7284 + 2.0641 (Log TAXING/1000)

4. Disposable income determination.
DISPOSINC = AGI − INCTAX

VII. Saving Behavior

The simulation requires generating not only families' total savings each year but also the composition of their portfolios. The latter requires that, at least implicitly, saving be done in the form of the specific assets identified in the model. Saving (+, −) then, is permitted in five specific forms:

1. Change in the value of mortgage debt.
2. Net purchases of earning assets other than corporate stock.
3. Net purchases of nonearning assets except housing.
4. Net purchases of corporate stock.
5. Change in the value of nonmortgage debt.

Saving in earning assets, nonearning assets except houses, corporate stock and nonmortgage debt are constrained to sum to an independently assigned value of total saving in a two-step process. First, the sum of net purchases of earning assets (except stock), nonearning assets (except houses) and corporate stock is subtracted from total savings. If the remainder is greater than 0, nonmortgage debt is reduced by the difference up to the value of nonmortgage debt. If the difference is greater than nonmortgage debt, the net purchase of the three other assets are reduced proportionately to set their sum equal to total savings. This procedure permits us to take advantage of the greater precision with which total saving can be estimated compared to saving in a specific form and yet permits us to use our weaker information to move portfolio composition through time.

The Simulation of Saving

Saving is done at the family level in the following forms:

1. Saving in mortgage debt.
2. Total saving; saving in all forms of assets and debts.
3. Saving in earning assets other than corporate stock.

4. Saving in nonearning assets.
5. Saving in corporate stock.
6. Saving in nonmortgage debt.

The amount of savings in each form, including saving in houses, which is generated in another operating characteristic, was given a chance when the functions used in the model were being estimated, to predict the amounts of saving in other forms. In the model the forms listed above are generated in the order shown and each output is used to generate subsequent outputs.

Mortgage Debt

Saving in the form of mortgage acquisition and liquidation at the purchase or sale of houses is simulated in another operating characteristic. Additional saving in the form of mortgage debt results from mortgage payments made to principle.

$$\text{MORTSAVE}_{kt} = \text{MORTSAVE}_{kt} + \text{MORTPAY}_{kt} - (\text{MORTDEBT}_{kt} \cdot \text{INTRATE}_{kt})$$

Total Saving

Total saving, saving in the forms of earning assets, nonearning assets and corporate stock are all simulated with the same logic. (Chart 8)

The first task is to assign a probability of saving at least \$500.[5] This probability is a function of the percentile ranks of the unit's disposable income, debt and assets, age and education of head, and wages as a percent of gross income.

$$P(\text{TOTALSAVE}_{kt} \geq 500) = F_1(\text{DISPINCRANK}_{kt}, \text{DEBTRANK}_{k,t-1}, \text{ASSETRANK}_{k,t-1}, \text{AGE}_{kt}, \text{ED}_{kt}, \text{WAGES}_{kt}/\text{GROSSINC}_{kt}).$$

A random number is selected and the unit saves at least \$500 if

$$\text{RANDOM1} < P(\text{TOTALSAVE}_{kt} \geq \$500)$$

Otherwise, the conditional probability of dissaving at least \$500 is found. This probability is a function of the percentile ranks of the unit's disposable income, debt and assets, age and education of head, and family size.

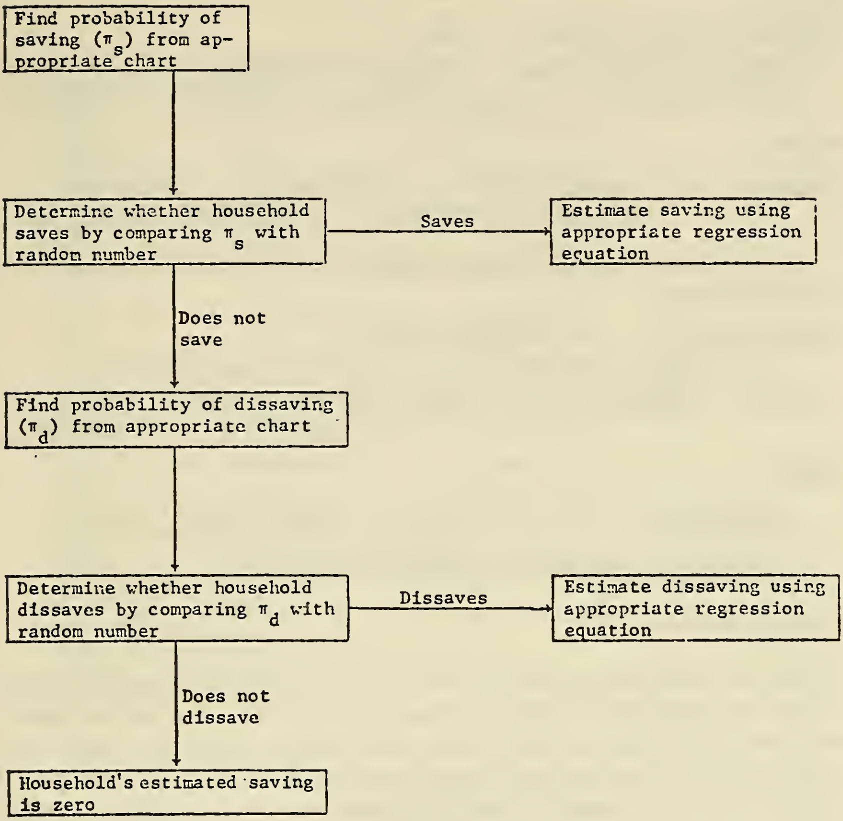
$$P(\text{TOTALSAVE}_{kt} \leq -500 | \text{TOTALSAVE}_{kt} < 500) = F_2(\text{DISPINCRANK}_{kt}, \text{DEBTRANK}_{kt}, \text{ASSETRANK}_{m,t-1}, \text{AGE}_{kt}, \text{ED}_{kt}, \text{FAMSIZ}_{kt})$$

Another random number is selected and the unit dissaves at least \$500 if

$$\text{RANDOM2} < P(\text{TOTALSAVE}_{kt} \leq -500 | \text{TOTALSAVE}_{kt} < 500)$$

[5] 1963 dollars.

Chart 8.—*Calculation of Total Saving*



If the unit does not save or dissave at least \$500, saving for that unit is set to zero.

For units randomly chosen to save at least \$500, saving is calculated as:

$$\text{TOTALSAVE}_{kt} = [0.4371 - 0.0136 \cdot (\text{WAGES}_{kt}/\text{GROSSINC}_{kt})^* - 3.5996 \cdot \text{DISPINCRANK}_{kt} + 29.2830 \cdot \text{DISPINCRANK}_{kt} \cdot \text{ASSETRANK}_{k,t-1} + \text{ERROR1}] \cdot \text{DISPINC}_{kt}$$

where

$$\text{ERROR1} = F_{11}(\text{ASSETRANK}_{k,t-1}, \text{DEBTRANK}_{kt}, \text{HOUSEVAL}_{kt}, \text{WAGES}_{kt}/\text{GROSSINC}_{kt}, \text{NUMKIDS}_{kt}, \text{AGE}_{kt}, \text{ED}_{kt})$$

For the family units randomly chosen to dissave at least \$500, saving is calculated as:

$$\text{TOTALSAVE}_{kt} = -1/[-5.0116 + 0.3741 \cdot (\text{WAGES}_{kt}/\text{GROSSINC}_{kt})^{**} + 0.2767 \cdot \text{ASSETRANK}_{k,t-1} + 0.0750 \cdot \text{DEBTRANK}_{kt} + 0.1003 \cdot \text{DISPINCRANK}_{kt} + \text{ERROR2}] \cdot (\text{ASSETS}_{k,t-1} + \text{DISPINC}_{kt})$$

where

$$\text{ERROR2} = F_{22}(\text{ASSETRANK}_{k,t-1}, \text{DEBTRANK}_{kt}, \text{AGE}_{kt})$$

F_1 through F_{22} are shown in tables 10 through 13.

Earning Assets

The probability of saving at least \$100[6] in the form of earning assets other than corporate stock is a function of total saving by the family unit (output #2), the ranks of disposable income and debt, wages as a percentage of gross income, education of head, family size, number of children, ratio of family's earning assets to mean value of earning assets, and home ownership status.

$$P(\text{EARNINGSAVE}_{kt} \geq 100) = F_3(\text{TOTALSAVE}_{kt}, \text{DISPINCRANK}_{kt}, \text{DEBTRANK}_{kt}, \text{WAGES}_{kt}/\text{GROSSINC}_{kt}, \text{ED}_{kt}, \text{FAMSIZ}_{kt}, \text{NUMKIDS}_{kt}, \text{EARNRATIO}_{k,t-1}, \text{HOUSEVAL}_{kt})$$

The family unit saves at least \$100 in the form of earning assets other than corporate stock if

$$\text{RANDOM3} < P(\text{EARNINGSAVE}_{kt} \geq 100)$$

---

* Coded as follows:

0 = 0
1 – (0,0125)
2 = (0.25,0.50)
3 = (0.50,0.75)
4 = (0.75,1)
5 = ≥ 1

** Recoded (see above).

[6] 1963 dollars.

TABLE 10.—*Probabilities of saving at least $500*
[Function F_1]

	Probability equals
If:	
YRANK* <26.7 and DEBTRANK <47.2	0.06
YRANK <26.7 and DEBTRANK = [47.2, 64.3]	.23
YRANK <26.7 and DEBTRANK >64.3	.51
YRANK >26.7 and DEBTRANK >55.2 and ASSETRANK >20.9 ED<9	.03
YRANK >26.7 and DEBTRANK <40.5 and ASSETRANK <20.9 and ED>8	.18
YRANK >26.7 and DEBTRANK = [40.5, 55.2] and ASSETRANK <20.9 and ED>8	.55
YRANK >26.7 and DEBTRANK <55.2 and ASSETRANK = [49.6, 77.9] and YRANK <66.2 and WAGES/GROSSINC <0.75	.29
YRANK >26.7 and DEBTRANK <55.2 and ASSETRANK = [49.6, 77.9] and YRANK <66.2 and WAGES/GROSSINC >0.75	.49
YRANK >26.7 and DEBTRANK <55.2 and ASSETRANK >77.9 and YRANK <66.2 and (AGE = [25, 44] or >64)	.50
YRANK >26.7 and DEBTRANK <55.2 and ASSETRANK <66.2 and (AGE < 25 or = [45, 64])	.78
YRANK >66.2 and DEBTRANK >55.2 and ASSETRANK <20.9 and WAGES = 0	.49
YRANK >66.2 and DEBTRANK <55.2 and ASSETRANK >20.9 and WAGES >0	.67
YRANK >26.7 and DEBTRANK >74.7 and YRANK <46.0	.47
YRANK >26.7 and DEBTRANK = [55.2, 74.7] and YRANK <46.0 and (AGE <25 or = [35, 54]	.58
YRANK >26.7 and DEBTRANK = [55.2, 74.7] and YRANK >46.0 and (AGE = [25, 34] or >54)	.84
YRANK >46.0 and DEBTRANK >55.2	.75

* Rank of disposable income.

TABLE 11
[Function F_{11}]

	Error equals
If:	
DEBTRANK <99.8 and ASSETRANK <99.7 and NUMKIDS ≥3	−0.033
DEBTRANK <99.8 and ASSETRANK <99.7 and NUMKIDS <3 and HOUSEVAL >0 and ED ≥16	−0.042
DEBTRANK <99.8 and ASSETRANK <77.9 and NUMKIDS <3 and HOUSEVAL >0 and ED <16 and [AGE <44 or ≥ 65]	−0.042
DEBTRANK <99.8 and ASSETRANK <77.9 and NUMKIDS <3 and HOUSEVAL >0 and ED <16 and AGE = [45, 64]	0.012
DEBTRANK <99.8 and ASSETRANK = [77.9, 99.8] and NUMKIDS <3 and ED <16 and WAGES/GROSSINC <0.75 and AGE <65	−0.041
DEBTRANK <99.8 and ASSETRANK = [77.9, 99.8] and NUMKIDS <3 and ED <16 and WAGES/GROSSINC <0.75 and AGE ≥65	0.048
DEBTRANK <99.8 and ASSETRANK = [77.9, 99.8] and NUMKIDS <3 and ED <16 and WAGES/GROSSINC ≥0.75	.052
DEBTRANK <99.8 and ASSETRANK <99.7 and NUMKIDS <3 and HOUSEVAL = 0	.034
DEBTRANK <99.8 and ASSETRANK ≥99.8	.054
DEBTRANK <99.8 and ASSETRANK <99.8	.116
DEBTRANK ≥99.8	.171

TABLE 12.—*Probabilities of dissaving at least \$500, conditional on saving <\$500*
[Function F_2]

	Probability equals
If:	
ASSETRANK <37.8 and (ED <13 or >15) and (AGE <25 or >34)	0.06
ASSETRANK <37.8 and (ED <13 or >15) and AGE = [25, 34] and YRANK* >46.0	.06
ASSETRANK <37.8 and (ED <13 or ≳15) and AGE = [25, 34] and YRANK <46.0	.23
ASSETRANK <37.8 and ED = [13, 15]	.32
ASSETRANK = [37.8, 77.9] and FAMSIZE = [2, 4] and (ED <9 or = [12, 15])	.13
ASSETRANK = [37.8, 77.9] and FAMSIZE = [2, 4] and (ED = [9, 11] or >15)	.28
ASSETRANK = [37.8, 49.6] and FAMSIZE = 1	.21
ASSETRANK = [49.6, 77.9] and FAMSIZE = 1	.40
ASSETRANK = [37.8, 77.9] and FAMSIZE >4 and DEBTRANK <55.2	.25
ASSETRANK = [37.8, 77.9] and FAMSIZE >4 and DEBTRANK >55.2	.45
ASSETRANK >77.9 and FAMSIZE = 1	.23
ASSETRANK = [77.9, 91.7] and FAMSIZE >1 and DEBTRANK <86.5 and AGE <55	.29
ASSETRANK = [77.9, 91.7] and FAMSIZE >1 and DEBTRANK <86.5 and AGE >54	.61
ASSETRANK = [77.9, 91.7] and FAMSIZE >1 and DEBTRANK >86.5	.77
ASSETRANK >91.7 and FAMSIZE >1 and ED = [12, 15]	.67
ASSETRANK >91.7 and FAMSIZE >1 and (ED <12 or >15	.92

TABLE 13
[Function F_{22}]

	Error equals
If:	
ASSETRANK <98.7 and DEBTRANK ≥95.0	−13.77
ASSETRANK <98.7 and DEBTRANK <95.0 and AGE <45	−6.14
ASSETRANK <98.7 and DEBTRANK <95.0 and AGE ≥45	3.29
ASSETRANK ≥98.7 and DEBT >0	−9.29
ASSETRANK ≥98.7 and DEBT = 0	29.91

* Rank of disposable income.

and saving in the form of earning assets is calculated as:

$$\begin{aligned} EARNSAVE_{kt} = {} & [0.2728 - 0.03008 \cdot (WAGES_{kt}/GROSSINC_{kt})^{*} \\ & -1.2573 \cdot (DEBTRANT_{kt}/1000) + 0.0632 \cdot (EARNRATIO_{k,t-1}/100 \\ & +0.1424 \cdot (DISPINCRANK_{kt} \\ & \cdot ASSETRANK_{k,t-1}/100)] \cdot DISPINC_{kt} \end{aligned}$$

Otherwise, the family's probability of dissaving at least \$100 in the form of earning assets, conditional on having saved less than \$100, is found as a function of total saving by the family, rank of disposable income, wages as a percentage of gross income, family size, number of children,

* Recoded (see page 222).

size of area, ratio of family's earning assets to mean value of earning assets, and home ownership status.

$$P(\text{EARNINGSAVE}_{kt} < -100 \mid \text{EARNINGSAVE}_{kt} < 100) = F_4(\text{TOTALSAVE}_{kt}, \text{DISPINCRANK}_{kt}, \text{WAGES}_{kt}/\text{GROSSINC}_{kt}, \text{FAMSIZ}_{kt}, \text{NUMKIDS}_{kt}, \text{URBAN}_{kt}, \text{EARNRATIO}_{k,t-1}, \text{HOUSEVAL}_{kt})$$

If $\text{RANDOM4} < P(\text{EARNSAVE}_{kt} < -100 \mid \text{EARNSAVE}_{kt} < 100)$

$$\text{EARNSAVE}_{kt} = -[0.1234 + 0.2667 \cdot (\text{ASSETRANK}_{k,t-1}/1000) - 0.7697 \cdot (\text{DISPINCRANK}_{kt}/1000)] \cdot (\text{DISPINC}_{kt} + \text{EARNASSETS}_{k,t-1})$$

Zero saving in the form of earning assets other than corporate stock is imputed to all other units.

F_3 and F_4 are shown in tables 14 and 15.

Nonearning Assets

The probability of saving at least \$100[7] in the form of nonearning assets other than homes is a function of total saving and saving in the form of earning assets by the family unit, the ranks of disposable income, debt and assets, age and education of head, wages as a percentage of gross income, family size, home ownership status, and the ratio of the family's nonearning assets to the mean value of nonearning assets.

$$P(\text{NONEARNSAVE}_{kt} > 100) = F_5(\text{TOTALSAVE}_{kt}, \text{EARNSAVE}_{kt}, \text{DISPINCRANK}_{kt}, \text{DEBTRANK}_{kt}, \text{ASSETRANK}_{k,t-1}, \text{AGE}_{kt}, \text{ED}_{kt}, \text{WAGES}_{kt}/\text{GROSSINC}_{kt}, \text{FAMSIZ}_{kt}, \text{HOUSEVAL}_{kt}, \text{NONEARNRATIO}_{k,t-1})$$

If $\text{RANDOM5} < P(\text{NONEARNSAVE}_{kt} > 100)$

$$\text{NONEARNSAVE}_{kt} = [0.2889 = 0.1902 \cdot (\text{DEBTRANK}_{kt}/1000) - 1.5324 \cdot (\text{DISPINCRANK}_{kt}/1000) + 3.1675 \cdot (\text{NONEARNRATIO}_{k,t-1}/100) - 0.2956 \cdot (\text{DISPINCRANK}_{kt} \cdot \text{NONEARNRATIO}_{k,t-1}/1000)] \cdot \text{DISPINC}_{kt}$$

Otherwise, the probability of dissaving at least \$100 in the form of nonearning assets other than houses is found as a function of total family saving and saving in the form of earning assets, the ranks of disposable income and debt, the ratio of family's nonearning assets to mean value of nonearning assets, wages as a percentage of gross income, family size, number of children, education of head, and size of area.

[7] 1963 dollars.

TABLE 14.—*Probabilities of saving at least $100 in form of earning assets*
[Function F_3]

	Probability equals
If:	
TOTALSAVE <500 and YRANK* <46.0 and EARNRATIO <0.25 and (ED <9 or >12)	0.01
TOTALSAVE <500 and YRANK <46.0 and EARNRATIO <0.25 and ED = [9, 12]	.12
TOTALSAVE <500 and YRANK <46.0 and EARNRATIO >0.25 and HOUSEVAL = 0	.09
TOTALSAVE < −500 and YRANK <46.0 and EARNRATIO >0.25 and HOUSEVAL >0	.09
TOTALSAVE = [500, 500] and YRANK <46.0 and EARNRATIO >0.25 and HOUSEVAL >0	.39
TOTALSAVE <500 and YRANK >56.0 and WAGES/GROSSINC = 1	.09
TOTALSAVE <500 and YRANK = [46.0, 95.9] and WAGES/GROSSINC <1 and (ED = [9, 11] or = [13, 15])	.18
TOTALSAVE <500 and YRANK >95.9 and WAGES/GROSSINC <1 and (ED = [9, 11] or = [13, 15])	.79
TOTALSAVE <500 and YRANK >46.0 and WAGES/GROSSINC <1 and (ED <9 or = 11 or >15) and FAMSIZE >5	.15
TOTALSAVE <500 and YRANK >46.0 and WAGES/GROSSINC <1 and (ED <9 or = 11 or >15) and FAMSIZE <4	.34
TOTALSAVE <500 and YRANK >46.0 and WAGES/GROSSINC <1 and (ED <9 or = 11 or >15) and FAMSIZE = 4	.52
TOTALSAVE >500 and EARNASSETS = 0 and NUMKIDS >2	.09
TOTALSAVE >500 and EARNASSETS = 0 and NUMKIDS <3	.29
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK >47.2 and YRANK <46.0	.35
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK >47.2 and YRANK >46.0 and ED <9 and NUMKIDS = 0	.29
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK >47.2 and YRANK >46.0 and ED = [9, 11] and NUMKIDS = 0	.50
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK >47.2 and YRANK >46.0 and ED <12 and NUMKIDS = 0	.71
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK >74.7 and YRANK = [46.0, 66.2] and ED >11	.37
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK = [47.2, 74.7] and YRANK = [46.0, 66.2] and ED >11	.66
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK >47.2 and YRANK >86.0 and ED >11	.62
TOTALSAVE >500 and EARNASSETS 0 and DEBTRANK >47.2 and YRANK = [66.2, 86.0] and ED >11	.77
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK <47.2 and YRANK <46.0	.60
TOTALSAVE >500 and EARNASSETS >0 and DEBTRANK <47.2 and YRANK >46.0	.82

* Rank of disposable income.

$$P(\mathrm{NONEARNSAVE}_{kt} < -100 \mid \mathrm{NONEARNSAVE}_{kt} < 100) = F_6(\mathrm{TOTALSAVE}_{kt}, \mathrm{EARNSAVE}_{kt}, \mathrm{DISPINCRANK}_{kt}, \mathrm{DEBTRANK}_{kt}, \mathrm{NONEARNRATIO}_{k,\,t-1}, \mathrm{WAGES}_{kt}/\mathrm{GROSSINC}_{kt}, \mathrm{FAMSIZ}_{kt}, \mathrm{NUMKIDS}_{kt}, \mathrm{ED}_{kt}, \mathrm{URBAN}_{kt})$$

If $\mathrm{RANDOM6} < P(\mathrm{NONEARNSAVE}_{kt} < -100 \mid \mathrm{NONEARNSAVE}_{kt} < 100)$

$$\mathrm{NONEARNSAVE}_{kt} = -[0.1429 + 2.662 \cdot (\mathrm{ASSETRANK}_{k,\,t-1}/1000) - 2.9001 \cdot (\mathrm{DISPINCRANK}_{kt}/1000)] \cdot \mathrm{DISPINC}_{kt}$$

TABLE 15.—*Probabilities of dissaving at least $100 in form of earning assets, conditional on saving <$100 in form of earning assets*
[Function F_4]

	Probability equals
If:	
TOTALSAVE >−500 and YRANK* <66.2 and (URBAN =1 or 3) and EARNRATIO >0.25 and HOUSEVAL =0	0.08
TOTALSAVE >−500 and YRANK <66.2 and (URBAN =1 or 3) and EARNRATIO >0.25 and HOUSEVAL >0	.22
TOTALSAVE >−500 and YRANK <66.2 and (URBAN =1 or 3) and EARNRATIO <0.25 and AGE >54	.15
TOTALSAVE >−500 and YRANK <66.2 and (URBAN =1 or 3) and EARNRATIO <0.25 and AGE <55	.34
TOTALSAVE >−500 and YRANK <66.2 and URBAN =2 and FAMSIZE >2 and (ED <9 or =[13, 15])	.09
TOTALSAVE >−500 and YRANK <66.2 and URBAN =2 and FAMSIZE >2 and (ED =[9, 12] or >15])	.36
TOTALSAVE >−500 and YRANK <66.2 and URBAN =2 and FAMSIZE <3	.51
TOTALSAVE >−500 and YRANK >66.2 and WAGES/GROSSINC <0.5 and ED =[9, 12]	.02
TOTALSAVE >−500 and YRANK >66.2 and WAGES/GROSSINC <0.5 and (ED <9 or >12) and FAMSIZE >3	.25
TOTALSAVE >−500 and YRANK >66.2 and WAGES/GROSSINC <0.5 and (ED <9 or >12) and FAMSIZE <4	.71
TOTALSAVE >−500 and YRANK =[66.2, 95.9] and WAGES/GROSSINC >0.5 and FAMSIZE >4 and ED =[9, 15]	.55
TOTALSAVE >−500 and YRANK =[66.2, 95.9] and WAGES/GROSSINC >0.5 and FAMSIZE >4 AND ED = [9, 15]	.27
TOTALSAVE >−500 and YRANK >95.9 and WAGES/GROSSINC >0.5 and FAMSIZE >4	.83
TOTALSAVE >−500 and YRANK >66.2 and WAGES/GROSSINC >0.5 and FAMSIZE >5 and (URBAN =1 or 3) and NUMKIDS =0	.38
TOTALSAVE >−500 and YRANK >66.2 and WAGES/GROSSINC >0.5 and FAMSIZE <5 and (URBAN −1 or 3) and NUMKIDS >0	.69
TOTALSAVE >−500 and YRANK >66.2 and WAGES/GROSSINC >0.5 and FAMSIZE <5 and URBAN =2	.81
TOTALSAVE <−500 and NUMKIDS >2	.47
TOTALSAVE <−500 and NUMKIDS <3 and ED =[9, 12] and (URBAN =1 or 3)	.52
TOTALSAVE <−500 and NUMKIDS <3 and ED =[9, 12] and URBAN =2	.82
TOTALSAVE <−500 and NUMKIDS <3 and (ED <9 or >12)	.89

* Rank of disposable income.

Zero saving in the form of nonearning assets other than homes is imputed to all other family units.

F_5 and F_6 are shown in tables 16 and 17.

Corporate Stock

The probability of saving in the form of corporate stock is a function of total family saving and saving in the form of other earning assets and nonearning assets, the ratio of family's stock to mean value of corporate stock, status concerning realization of capital gains from stock this year, rank of disposable income, age and education of head, family size and number of children.

TABLE 16.—*Probabilities of saving at least $100 in form of nonearning assets*
[Function F_8]

	Probability equals
If:	
TOTALSAVE <500 and YRANK* <26.7 and DEBTRANK <55.2 and ASSETRANK <49.6	0.04
TOTALSAVE <500 and YRANK <26.7 and DEBTRANK <55.2 and ASSETRANK >49.6	.18
TOTALSAVE <500 and YRANK <26.7 and DEBTRANK >55.2	.33
TOTALSAVE <500 and YRANK >26.7 and AGE >44 and EARNSAVE >−100 and DEBTRANK <86.5	.17
TOTALSAVE <500 and YRANK >26.7 and AGE >44 and EARNSAVE >−100 and DEBTRANK >86.5	.48
TOTALSAVE <500 and YRANK >26.7 and AGE >44 and EARNSAVE <−100 and (ED <12 or = [13, 15])	.26
TOTALSAVE <500 and YRANK >26.7 and AGE >44 and EARNSAVE <−100 and (ED = 12 or >15)	.71
TOTALSAVE <500 and YRANK = [26.7, 95.9] and AGE <45 and DEBTRANK <55.2 and ED = [9, 12]	.18
TOTALSAVE <500 and YRANK = [26.7, 95.9] and AGE <45 and DEBTRANK <55.2 and (ED <9 or >12)	.42
TOTALSAVE <500 and YRANK = [26.7, 95.9] and AGE <45 and DEBTRANK >55.2 and EARNSAVE <−100	.27
TOTALSAVE <500 and YRANK = [26.7, 95.9] and AGE <45 and DEBTRANK >55.2 and EARNSAVE >−100	.57
TOTALSAVE <500 and YRANK >95.9 and AGE <45	.69
TOTALSAVE <500 and FAMSIZE = 1	9.33
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK <37.8	0.46
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK >37.8 and HOUSEVAL >0 and YRANK <86.0 and ED >15 and WAGES/GROSSINC <0.75 and NONEARNRATIO >1	.24
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK >37.8 and HOUSEVAL >0 and YRANK <86.0 and ED >15 and WAGES/GROSSINC <0.75 and NONEARNRATIO <1	.48
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK >37.8 and HOUSEVAL >0 and YRANK <86.0 and ED >15 and WAGES/GROSSINC >0.75 and EARNSAVE >−100	.52
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK >37.8 and HOUSEVAL >0 and YRANK <86.0 and ED >15 and WAGES/GROSSINC >0.75 and EARNSAVE <−100	.75
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK >37.8 and HOUSEVAL >0 and YRANK <86.0 and ED <16	.74
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK >37.8 and HOUSEVAL >0 and YRANK >86.0	.65
TOTALSAVE >500 and FAMSIZE >1 and ASSETRANK >37.8 and HOUSEVAL = 0	.76

* Rank of disposable income.

$$P(\text{STOCKSAVE}_{kt} > 0) = F_7(\text{TOTALSAVE}_{kt}, \text{EARNSAVE}_{kt}, \text{NONEARNSAVE}_{kt}, \text{STOCKRATIO}_{k,\,t-1}, \text{GAINSTOCK}_{kt}, \text{DISPINCRANK}_{kt}, \text{AGE}_{kt}, \text{ED}_{kt}, \text{FAMSIZ}_{kt}, \text{NUMKIDS}_{kt})$$

If RANDOM7 $< P(\text{STOCKSAVE}_{kt} > 0)$

$$\begin{aligned}\text{STOCKSAVE}_{kt} = {} & [0.5760 - 4.3567 \cdot (\text{ASSETRANK}_{k,\,t-1}/1000) \\ & -8.8165 \cdot \text{DISPINCRANK}_{kt}/1000 + 1.5321 \cdot (\text{STOCKRATIO}_{k,\,t-1}/100) \\ & +0.9028 \cdot (\text{DISPINCRANK}_{kt} \cdot \text{ASSETRANK}_{k,\,t-1}/10000) \\ & -0.1522 \cdot (\text{DISPINCRANK}_{kt} \cdot \text{STOCKRATIO}_{k,\,t-1}/1000)] \\ & \cdot \text{DISPINC}_{kt}\end{aligned}$$

Otherwise, the probability of dissaving in the form of corporate stock is found as a function of whether or not capital gains or losses from

TABLE 17.—*Probabilities of dissaving at least $100 in form of nonearning assets, conditional on saving <$100 in form of nonearning assets*
[Function F_6]

	Probability equals
If:	
NONEARNRATIO <1.5 and DEBT >0 and FAMSIZE >1 and YRANK* <46.0 and (ED <9 or =12)	0.01
NONEARNRATIO <1.5 and DEBT >0 and FAMSIZE >1 and YRANK <46.0 and (ED = [9, 11] or >12)	.13
NONEARNRATIO <1.5 and DEBT >0 and FAMSIZE =1	.29
NONEARNRATIO <1.5 and DEBT >0 and FAMSIZE >1 and YRANK >46.0 and WAGES/GROSSINC =1	.06
NONEARNRATIO <1.5 and DEBT >0 and FAMSIZE >1 and YRANK >46.0 and WAGES/GROSSINC <1 and ED = [9, 12]	.09
NONEARNRATIO <1.5 and DEBT >0 and FAMSIZE >1 and YRANK >46.0 and WAGES/GROSSINC <1 and (ED <9 or >12) and TOTALSAVE >500	.14
NONEARNRATIO <1.5 and DEBT >0 and FAMSIZE >1 and YRANK >46.0 and WAGES/GROSSINC <1 and (ED <9 or >12) and TOTALSAVE <500	.33
NONEARNRATIO <1.5 and DEBT =0 and NUMKIDS >0	.07
NONEARNRATIO <1.5 and DEBT =0 and NUMKIDS =0 and URBAN =2	.24
NONEARNRATIO <1.5 and DEBT =0 and NUMKIDS =0 and (URBAN =1 or 3) and YRANK >26.7	.34
NONEARNRATIO <1.5 and DEBT =0 and NUMKIDS =0 and (URBAN =1 or 3) and YRANK <26.7	.59
NONEARNRATIO >1.5 and TOTALSAVE >−500 and EARNSAVE = [−100, 100]	.22
NONEARNRATIO >1.5 and TOTALSAVE >−500 and (EARNSAVE <−100 or >100) and WAGES/GROSSINC >0.75 and DEBTRANK >64.3	.43
NONEARNRATIO >1.5 and TOTALSAVE >−500 and (EARNSAVE <−100 or >100) and WAGES/GROSSINC >0.75 and DEBTRANK <64.3	.23
NONEARNRATIO >1.5 and TOTALSAVE >−500 and (EARNSAVE <−100 or >100) and WAGES/GROSSINC <0.75 and FAMSIZE >2	.49
NONEARNRATIO >1.5 and TOTALSAVE >−500 and (EARNSAVE <−100 or >100) and WAGES/GROSSINC <0.75 and FAMSIZE <3	.94
NONEARNRATIO >1.5 and TOTALSAVE <−500 and URBAN =2	.56
NONEARNRATIO >1.5 and TOTALSAVE <−500 and (URBAN =1 or 3)	.91

* Rank of disposable income.

sales of stock were realized, total saving by the family, saving in the form of earning assets, ranks of disposable income and debt, family size, number of children, and education of head.

$$P(\text{STOCKSAVE}_{kt} < 0 \mid \text{STOCKSAVE}_{kt} \leq 0) = F_8(\text{GAINSTOCK}_{kt}, \text{TOTALSAVE}_{kt}, \text{EARNSAVE}_{kt}, \text{DISPINCRANK}_{kt}, \text{DEBTRANK}_{kt}, \text{FAMSIZ}_{kt}, \text{NUMKIDS}_{kt}, \text{ED}_{kt})$$

$$\text{If RANDOM8} < P(\text{STOCKSAVE}_{kt} < 0 \mid \text{STOCKSAVE}_{kt} \leq 0)$$

$$\text{STOCKSAVE}_{kt} = -[0.5261 - 0.0752 \cdot (\text{WAGES}_{kt}/\text{GROSSINC}_{kt})^{*} + 9.7208 \cdot (\text{ASSETRANK}_{k,\,t-1}/1000) - 9.7101 \cdot (\text{DISPINCRANK}_{kt}/1000)] \cdot \text{DISPINC}_{kt}$$

* Recoded (see page 222).

TABLE 18.—*Probabilities of saving in form of corporate stock*
[Function F_7]

	Probability equals
If:	
STOCK =0 and EARNSAVE = [−100, 100]	0.002
STOCK =0 and (EARNSAVE <−100 or >100) and (ED <13 or >15) and YRANK* <95.9	.024
STOCK =0 and (EARNSAVE <−100 or >100) and (ED <13 or >15) and YRANK >95.9	.15
STOCK =0 and (EARNSAVE <−100 or >100) and ED = [13, 15] and YRANK <66.2	.001
STOCK =0 and (EARNSAVE <−100 or >100) and ED = [13, 15] and YRANK >66.2 and NUMKIDS >1	.05
STOCK =0 and (EARNSAVE <−100 or >100) and ED = [13, 15] and YRANK >66.2 and NUMKIDS <2	.25
STOCK >0 and TOTALSAVE <500 and YRANK <66.2	.05
STOCK >0 and TOTALSAVE <500 and YRANK = [66.2, 98.7] and (ED = [9, 12] or >15) and AGE >44	.05
STOCK >0 and TOTALSAVE <500 and YRANK >98.7 and (ED = [9, 12] or >15) and AGE >44	.44
STOCK >0 and TOTALSAVE <500 and YRANK >66.2 and (ED = [9, 12] or >15) and AGE <45	.39
STOCK >0 and TOTALSAVE <500 and YRANK >66.2 and (ED <9 or = [13, 15])	.55
STOCK >0 and TOTALSAVE >500 and STOCKRATIO <0.25 and (ED = [9, 12] or >15)	.11
STOCK >0 and TOTALSAVE >500 and STOCKRATIO <0.25 and (ED 9 or = [13, 15])	.40
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE >64 and YRANK <98.7	.19
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE >64 and YRANK >98.7	.84
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE <65 and NONEARNSAVE <−100 and GAINSTOCK =0	.10
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE <65 and NONEARNSAVE >100 and GAINSTOCK =0 and FAMSIZE >2	.31
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE <65 and NONEARNSAVE >100 and GAINSTOCK =0 and FAMSIZE <3	.53
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE <65 and (NONEARNSAVE <−100 or >100) and GAINSTOCK =0 and FAMSIZE <5 and (URBAN = 1 or 3)	.35
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE <65 and (NONEARNSAVE <−100 or >100) and GAINSTOCK =0 and FAMSIZE <5 and URBAN =2	.76
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE <65 and (NONEARNSAVE <−100 or >100) and GAINSTOCK =0 and FAMSIZE >4	.85
STOCKRATIO >0.25 and TOTALSAVE >500 and AGE <65 and NONEARNSAVE = [−100, 100]	.65

* Rank of disposable income.

Zero saving in the form of corporate stock is imputed to all other family units.

F_7 and F_8 are shown in tables 18 and 19.

Nonmortgage Debt

Saving in the form of nonmortgage debt is calculated as the residual of total saving and the sum of saving in each other form with the following constraint: If the residual is larger than current nonmortgage debt, saving in this form is set equal to nonmortgage debt and the

TABLE 19.—*Probabilities of dissaving in form of corporate stock, conditional on not saving in form of corporate stock*
[Function F_8]

	Probability equals
If:	
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE <4 and YRANK* <66.2	0.013
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE <4 and YRANK >66.2 and EARNSAVE <100	.002
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE <4 and YRANK >66.2 and EARNSAVE >100 and (ED <12 or >15)	.04
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE <4 and YRANK >66.2 and EARNSAVE >100 and ED = [12, 15]	.24
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE >4 and ED <16	.04
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE =4 and ED <16	.26
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE >3 and ED >15 and NUMKIDS <3	.19
GAINSTOCK =0 and TOTALSAVE > −500 and FAMSIZE >3 and ED >15 and NUMKIDS >2	.56
GAINSTOCK =0 and TOTALSAVE < −500 and (ED <9 or = [12, 15])	.34
GAINSTOCK = 0 and TOTALSAVE < −500 and (ED = [9, 11] or >15)	.71
GAINSTOCK =0 and DEBTRANK <55.2	.54
GAINSTOCK =0 and DEBTRANK >55.2	.98

* Rank of disposable income.

remainder of the residual is added to saving in the forms of earning assets, nonearning assets and corporate stock in shares proportionate to the current values of these assets. Hence, saving in specific forms must sum to a separate estimate of total saving.

Estimates of saving operating characteristics

The data used for the estimation are from *The Survey of Financial Characteristics of Consumers* (SFCC) which covered the assets and debts of consumers as of December 31, 1962 and from *The Survey of Changes in Family Finances* (SCFF) covering the saving and income of the SFCC respondents during 1963. At times we have used a merge form of these two files, created by combining observations for families interviewed in both surveys.

Saving was computed as the algebraic sum of saving in each net worth component plus saving in the form of retirement plans. Disposable income was estimated as gross income less estimated Federal income tax liability. Of 2164 observations in the SCFF file, 99 were excluded from the analysis because the households were estimated to have saved over 90 percent of their income. It was assumed that either survey measurement errors or inaccurate estimates of tax liabilities were responsible for these large saving/income ratio estimates, often over 100 percent. In addition 18 observations were excluded because the respondents reported zero or negative income.

Analysis Strategy

The analysis was designed to estimate relationships independent of the level of saving as well as the levels of the major determinants of saving (e.g., income, assets and debt). Examination of the affect of using predictor variables transformed to their relative positions in their respective distributions was also undertaken. Two values of relative position within a distribution were proposed—(1) the distance from the mean measured in standard deviations, and (2) the percent of the population having less than the value. The first measure was abandoned due to the fact that the standard deviation of a variable within a sample is very sensitive to extreme cases.

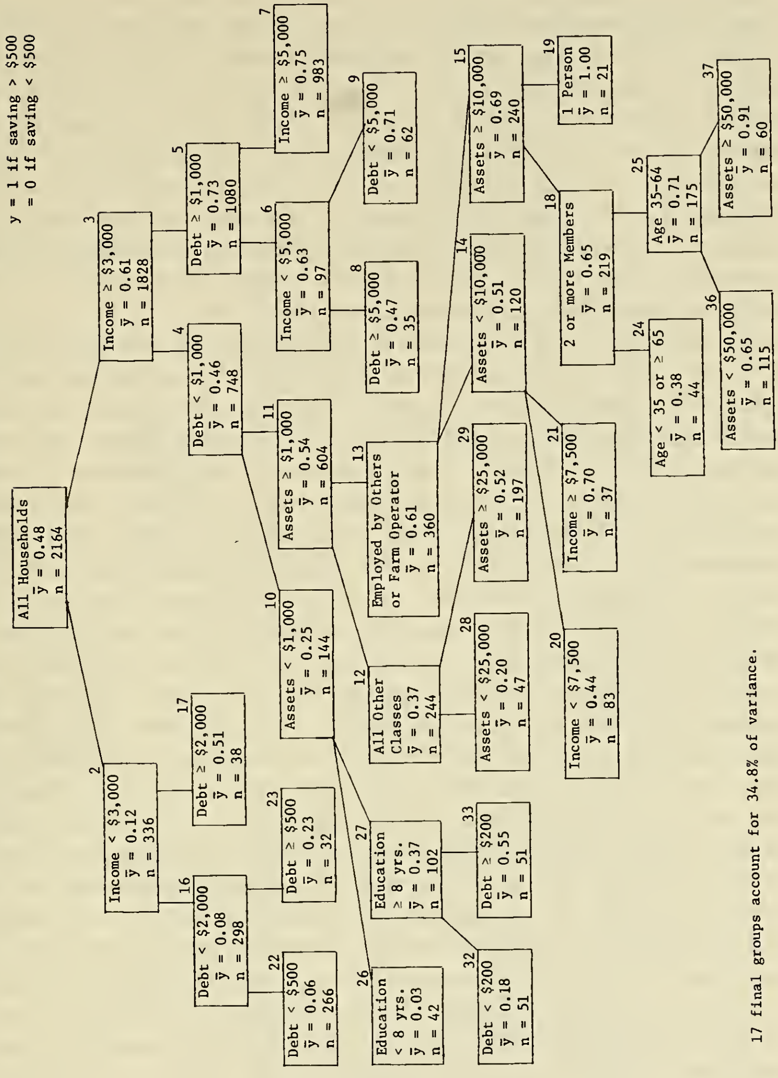
Traditional regression analysis failed to provide a simple model with much predictive power. However, it was discovered that separate regressions using savers and dissavers were much more satisfactory in every respect than an overall regression. It was felt at this point that a simulation algorithm should be developed to take advantage of this fact by allowing the algorithm to stochastically classify households as saving or dissaving on the basis of probabilities estimated from the SCFF data.

Three savings classes with arbitrary boundaries were decided upon—(1) substantial savers (saving >$500), (2) substantial dissavers (dissaving >$500), and (3) small savers or dissavers (saving or dissaving ≤$500). Automatic Interaction Detector (AID) was used to generate probability assumptions for the model in the following way. A dependent variable was created with value 1 for households saving >$500 and 0 for others. AID was then used to split the data on a set of predictors so as to maximize reduction in variance of this dummy variable. The result is the AID trees shown in chart 9. A group mean can be considered the probability that a unit saves more than $500 given that the unit has characteristics that associate it with that group.

Not surprisingly, of all groups created by splits on income or assets, those with the higher income or asset levels have the larger mean or probability of saving. Also, of the five splits on debt, only once does the group with the lower debt have the larger mean (see groups 8 and 9), indicating that households starting the year with higher debt were more likely to save. The estimated saving probabilities ranged from 0.03 for households with disposable income of at least $3,000 and assets and debt less than $1,000, whose head has less than 9 years of education (group 26); to 1.0 for households with disposable income over $3,000, debt less than $1,000, assets over $10,000, consisting of one person who is employed by others (group 19).

A second dependent variable was created for units saving less than $500. This variable is 1 if the unit dissaved more than $500 and 0 otherwise. Another AID analysis produced the AID tree shown in chart 10. In this case a group mean can be considered the probability that a unit will dissave more than $500 given that it is a member of that group and has saved less than $500. Among households that saved less than

Chart 9.—*Probability of Saving > $500*



$500, dissaving was most likely for those having higher assets, higher debt, or more family members. In addition the self-employed had a greater chance of dissaving over $500. The fraction of substantial dissavers among households within final AID groups ranged from 0.06 (groups 14) to 0.96 (group 33).

The saving simulation model will use these AID results to classify each household in its population as having one of the three types of saving behavior mentioned above. In the simulation model, however, the ranks of assets, debt and income are used in place of the values shown in the AID charts. For example, an AID group defined as

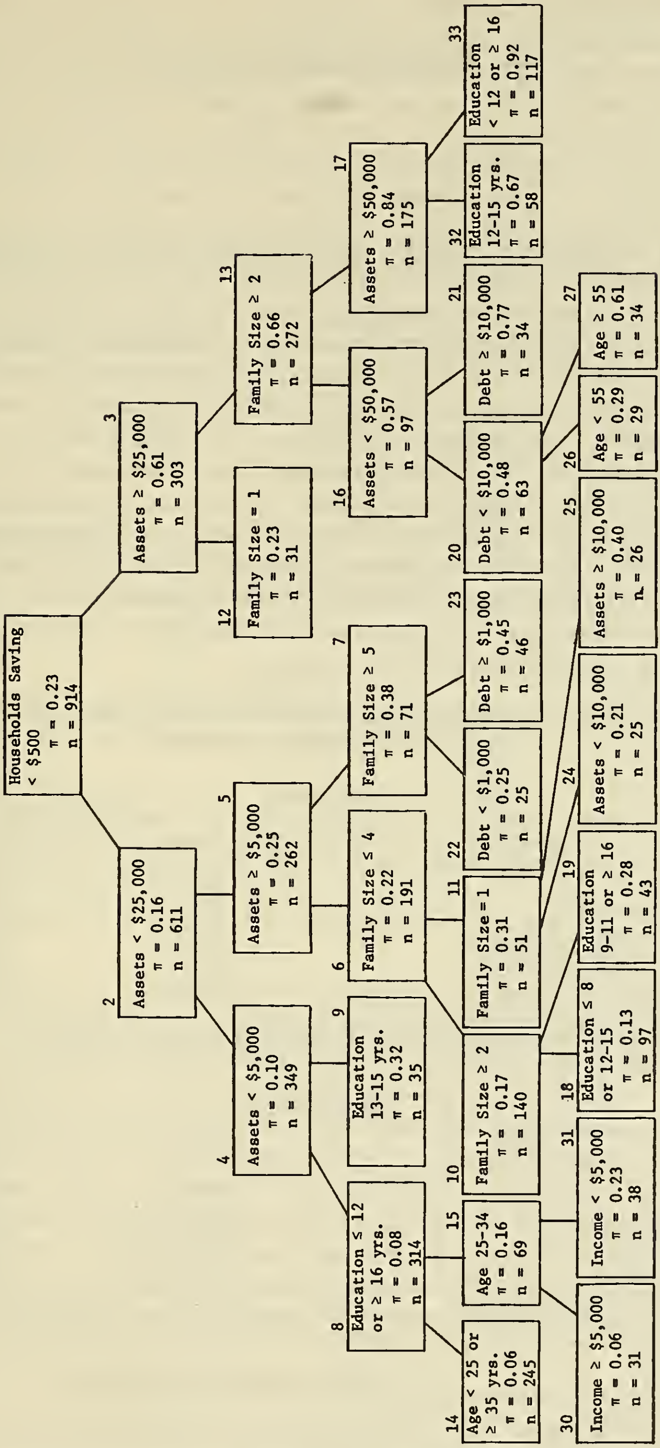
households having assets less than \$25,000 and disposable income less than \$7,500 would be redefined as households with assets within the bottom 77.85 percent and disposable income in the bottom 66.22 percent of the model population's distributions of assets and disposable income respectively. Each household will first be classified as a substantial saver or not a substantial saver by comparing a random variable from a uniform distribution on the interval [0, 1] with the appropriate AID group mean (chart 9). Those households that are not substantial savers are then classified substantial dissavers or small savers and dissavers by comparing another random uniform variable with the appropriate AID group mean (chart 10).

The strategy for the remainder of the analysis is to use regression and AID jointly to predict saving or dissaving. The technique described below was first used to assign wealth to the initial simulation population. Regression equations were estimated to predict the ratio of saving to income for households saving more than \$500 and to predict the ratio of income plus assets to dissaving for households dissaving more than \$500. Residuals from each regression were then used as dependent variables in AID analyses. The simulation model will predict saving or dissaving using the appropriate regression equation plus a random error term from a distribution which should approximate the distributions of errors in the appropriate final AID group. Most of these error distributions within AID groups were found to be approximately normal by examining normal probability plots. The rest were skewed to the right and could be "normalized" by trimming the upper tail. For units associated with an AID group that does not require "trimming," the error term that is added to the regression prediction is a random normal deviate from a distribution with mean and standard deviation of the residuals in that AID group. Error terms generated for units associated with AID groups having skewed distributions of residuals will be either (1) a random normal deviate from a distribution with mean and standard deviation of the "trimmed" distribution or (2) the mean of the extreme residuals. In this case the mean outlier is used as an error term with probability equal to the sum of the sampling weights associated with those outliers divided by the total sum of weights for the group.

Analysis for Households Saving >\$500

There were 1,151 households in the SCFF file whose saving was over \$500 and not more than 90 percent of estimated disposable income for 1963. These observations were used in regression analysis with ratio of saving to income as the dependent variable and predictors—wages as a percent of gross income coded into 5 classes and the percentile ranks of assets, debt and disposable income. In addition, the predictive power of the three variables—assets, debt and income—was compared with that of the percentile ranks of these variables.

Chart 10.—*Probability of Dissaving at Least $500*



16 final groups account for 27.9% of variance

Residuals from the model

$$\frac{S}{Y} = \text{linear function of predictors}$$

where

$$S = \text{saving}$$
$$Y = \text{income}$$

showed increasing dispersion as the predicted value increased. To obtain predicted values that have constant variance about the regression line, the same model was fit using weights of the squared inverse of the original predicted value. Results from regressions using actual amounts and percentile ranks of assets, debt and income as predictors, including an asset-income interaction are shown in table 20. Assets and debt become nonsignificant variables in ranked form but income appears to be a more powerful predictor when transformed to ranks. A surprising result is the reversal in signs of the income and asset-income coefficients. Using ranked variables, the results imply that the share of income saved decreases as income rises and increases as the product of income and assets rises. Just the opposite is implied when the actual amounts of income and assets are used as predictors. In both models the coefficient of assets is positive.

TABLE 20.—*Regressions with dependent variable ratio of saving to disposable income for households saving greater than $500*

EQUATION 1

Variable	Coefficient	t-Ratio
1. Constant	0.4375	10.65
2. Wages/gross income[1]	−0.0161	−4.17
3. Percentile rank of assets/1000	0.0659	0.10
4. Percentile rank of debt/1000	−0.2355	−1.36
5. Percentile rank of income/1000	−3.2833	−6.46
6. (Variable 3) × (Variable 5)	27.2192	3.37

NOTE.—The predicted ratios account for 56.2 percent of the variance in saving.
R^2 = 18.6%.

EQUATION 2

Variable	Coefficient	t-Ratio
1. Constant	0.3412	20.79
2. Wages/gross income[1]	−0.0253	−6.86
3. Assets/100,000	0.0061	2.70
4. Debt/100,000	0.0560	3.03
5. Income/100,000	0.0769	1.97
6. (Variable 3) × (Variable 5)	−0.0047	−4.47

[1] Coded as follows: 0 = (0), 1 = (>0, <0.25), 2 = (≥0.25, <0.50) 3 = (>0.50, <0.75), 4 = (≥0.75, <1), 5 = (1).

NOTE.—The predicted ratios account for 63.6 percent of the variance in saving.
R^2 = 7.7%.

Table 21 shows results from fitting both models without the debt and asset terms. Eliminating these variables from either equation results in

only slight reduction in the amount of variance explained in the saving/income ratios or in actual saving, demonstrating that their importance in either model is minimal. Although the ranked predictors can explain more of the variance in the saving/income ratios, it is not necessarily true that the predicted ratios from this model explain more of the variance in actual saving than predicted ratios from the other model. In fact, equation 3 for which R^2 was 18.5 percent accounts for 56.1 percent of the variance in saving while equation 4 with R^2 of 6.6 percent accounts for 63.6 percent of the variance in saving.

TABLE 21.—*Regressions excluding terms for assets and debt with dependent variable ratio of saving to disposable income for households saving more than $500*

EQUATION 3

Variable	Coefficient	t-Ratio
1. Constant	0.4371	18.89
2. Wages/gross income[1]	−0.0136	−3.65
3. Percentile rank of income	−3.5996	−7.47
4. (Variable 3) × (Percentile rank of assets)	29.2830	8.07

NOTE.—The predicted ratios account for 56.1 percent of the variance in saving.
R^2 = 18.5%.

EQUATION 4

Variable	Coefficient	t-Ratio
1. Constant	0.3508	21.63
2. Wages/gross income[1]	−0.0265	−7.19
3. Disposable income/100,000	0.0961	3.16
4. (Variable 3) × (Assets/100,000)	−0.0009	−3.89

[1] Coded as follows: 0 = (0), 1 = (>0, <0.25), 2 = (≥0.25, <0.50) 3 = (>0.50, <0.75), 4 = (≥0.75, <1), 5 = (1).

NOTE.—The predicted ratios account for 63.6 percent of the variance in saving.
R^2 = 6.6%.

Residuals from equation 3 were analyzed by AID with results shown in chart 17. Assets, debt, number of children, housing status, education of head, age of head, and wages as a percent of income were used to split the residuals into 11 final groups accounting for 7.3 percent of the variance. Other predictors which were available were income, family size, urbanization and employment status. The AID results indicate that within certain subsets of the population, households with the following characteristics have larger saving/income ratios—households with 2 or fewer children, households not owning home, households whose head does not have a college degree (education of 15 years or less) and households with wages at least 50 percent of gross income.

Analysis for Households Dissaving >$500

Three hundred and twenty-six households in the SCFF file dissaved at least $500. After experimentation with several candidate dependent variables, including functions of the ratio of dissaving to income, to

assets, and to the sum of income and assets, the selected form was the ratio of dissaving to the sum of income and assets. Weighted lease squares were again used to fit models that produce predicted values with constant variance about the regression lines. The predictors for this estimation were the same used previously and the regression results are compared in table 22. Equation 5 was fit without an asset-income interaction term and equation 6 was fit without wages/gross income because those variables were statistically nonsignificant. Ranked predictors explain a much greater portion of the variance in the dependent variable and also explain more of the variance in dissaving. Debt appears to be more important in determining dissaving than it was in determining saving but its effect on dissaving as a ranked predictor is opposite from its effect when the actual value is used.

TABLE 22.—*Regressions with dependent variable ratio of assets plus income to dissaving for households dissaving over $500*

EQUATION 5

Variable	Coefficient	t-Ratio
1. Constant	−5.0116	−6.76
2. Wages/gross income[1]	0.3741	4.63
3. Percentile rank of assets	0.2767	10.44
4. Percentile rank of debt	0.0750	4.37
5. Percentile rank of income	0.1003	6.87

NOTE.—The estimated standard deviation of the dependent variable about the regression line = 0.7161 with 305 degrees of freedom. The predicted ratios account for 48.3 percent of the variance in dissaving.

R^2 = 83.7%.

EQUATION 6

Variable	Coefficient	t-Ratio
1. Constant	17.8574	9.94
2. Assets/10,000	0.4960	6.60
3. Debt/10,000	−0.7118	−3.34
4. Income/10,000	3.4516	3.81
5. (Variable 2) × (Variable 4)	−0.0319	−4.70

[1] Coded as follows: 0 = (0), 1 = (>0, <0.25), 2 = (≥0.25, <0.50) 3 = (>0.50, <0.75), 4 = (≥0.75, <1), 5 = (1).

NOTE.—The estimated standard deviation of the dependent variable about the regression line = 0.8966 with 307 degrees of freedom. The predicted ratios account for 32.3 percent of the variance in dissaving.

R^2 = 16.8%.

Again the residuals from equation 5 were analyzed by AID. Results are shown in chart 18. Assets, debt and age were able to explain 10.7 percent of the variance in the regression residuals. Negative residuals were more likely for households that had higher debt indicating that the effect of the percentile rank of debt was overstated by the regression model. Very few splits were generated by AID due to the relatively small number of observations available with the stipulation that the minimum group size was 25.

TABLE 23-A.—*Percentage distributions of households saving more than $500 by size of saving/income ratio*

Ratio	Percent of units		
	SCFF	SCFF[1]	Simulation
0 <0.10	9.22	9.70	6.69
0.10 <0.15	16.62	17.50	12.73
0.15 <0.20	16.09	16.95	14.03
0.20 <0.25	12.44	13.11	14.03
0.25 <0.30	9.89	10.42	13.54
0.30 <0.40	14.30	15.06	17.13
0.40 <0.50	7.82	8.23	8.63
0.50 <0.70	6.68	7.03	10.59
≥0.70	6.93	1.99	2.64
Percent of all units saving <$500	46.89		47.05

[1] Excluding 99 cases where ratio was greater than 0.90.

TABLE 23-B.—*Percentage distributions of households dissaving more than $500 by size of dissaving/(assets + income) ratio*

Ratio	Percent of units	
	SCFF	Simulation
0 <0.010	1.93	1.86
0.010 <0.025	17.06	18.35
0.025 <0.04	15.59	18.76
0.04 <0.07	16.97	20.83
0.07 <0.10	10.01	12.91
1.10 <0.15	9.49	10.54
0.15 <0.25	16.01	6.93
0.25 <0.40	7.84	3.77
≥0.40	5.10	6.04
Percent of all units dissaving $500	12.97	11.55

TABLE 24.—*Percentage distributions of households and saving by level of disposable income*

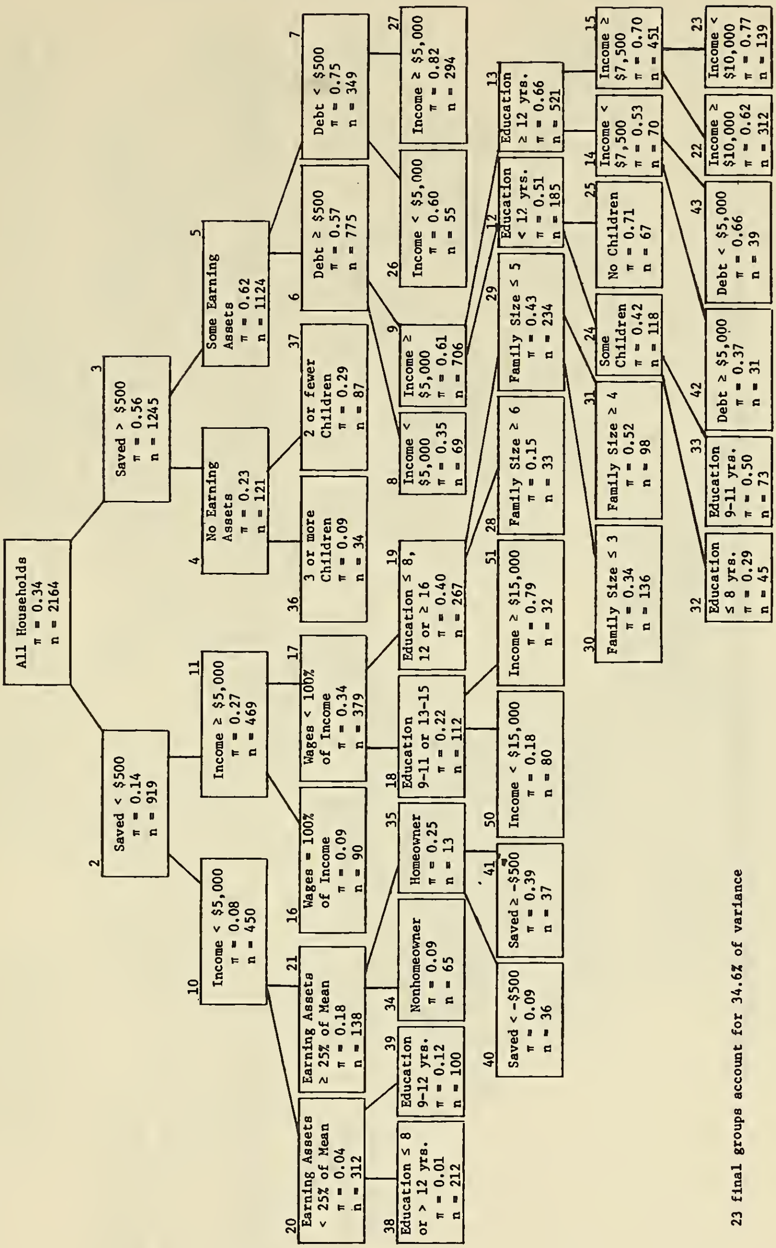
Income	SCFF		SCFF[1]		Simulation	
	Percent of units	Percent of saving	Percent of units	Percent of saving	Percent of units	Percent of saving
<1,000	7.73	−3.90	7.65	−7.73	13.72	−7.94
1,000 <2,000	10.02	−3.32	10.05	−7.17	10.20	−1.79
2,000 <3,000	8.73	1.08	8.93	1.75	10.78	−0.35
3,000 <4,000	10.57	4.23	10.61	3.55	9.13	3.77
4,000 <5,000	8.63	6.98	8.59	7.66	9.70	10.60
5,000 <6,000	7.82	9.15	7.85	10.25	10.08	13.61
6,000 <7,500	12.69	13.39	12.86	21.24	14.02	24.19
7,500 <10,000	19.89	28.06	19.89	34.38	14.45	29.37
10,000 <15,000	9.88	27.23	9.67	28.78	5.63	13.25
≥15,000	4.07	17.10	3.90	7.30	2.30	15.28
Total saving	$49.0 billion		$27.7 billion		$34.9 billion	

[1] Excluding 99 cases where saving/income ratio was greater than 0.90.

Some Simulation Results

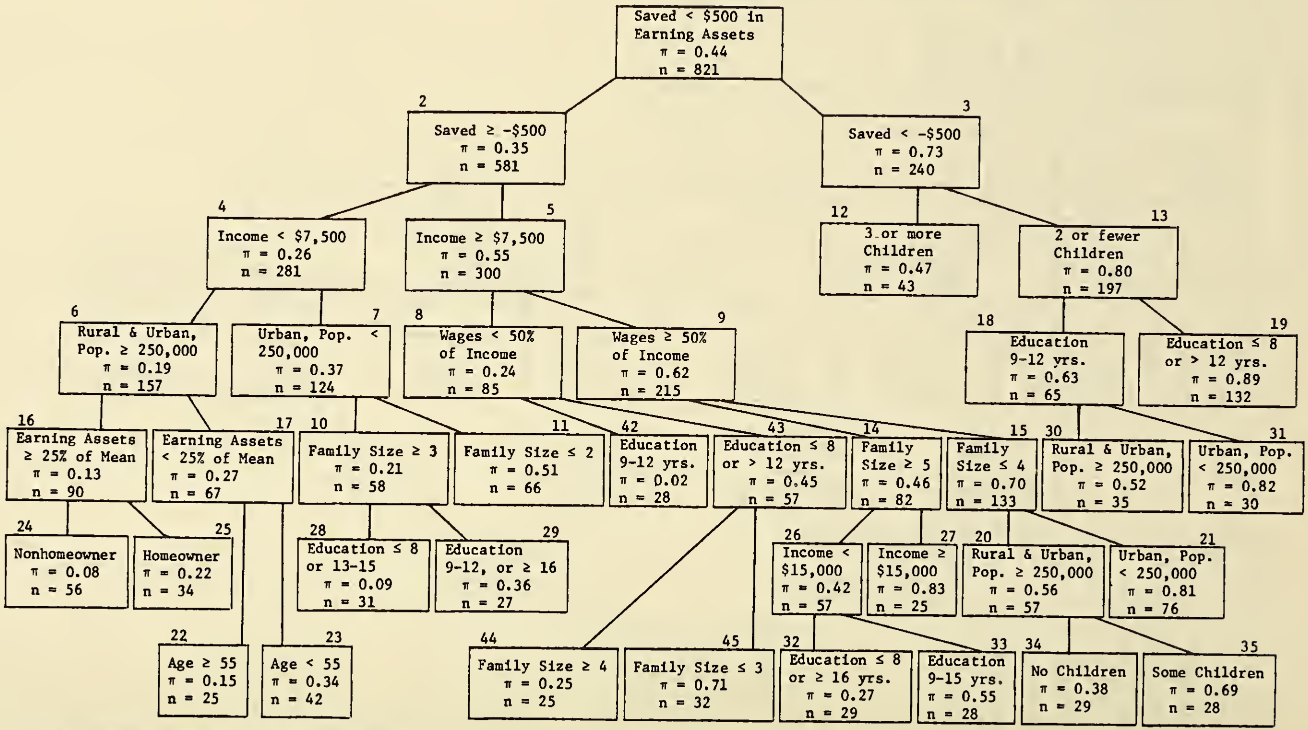
Household saving was simulated for one year using a population consisting of a random sample of 6008 households from the *1960 Census Public Use Sample* to which assets and debt had been assigned previously. This simulation model used earlier AID results which differ slightly from those presented here but the final version of the model should produce output similar to that presented below. Table 23 compares the percentage distributions of households by size of saving/income ratios and dissaving/(assets + income) ratios from the SCFF and the simulation population. A comparison of the percentage distribution of households and saving by level of income is presented in Table 24.

Chart 11.—*Probability of Saving at Least $100 in Form of Earning Assets*



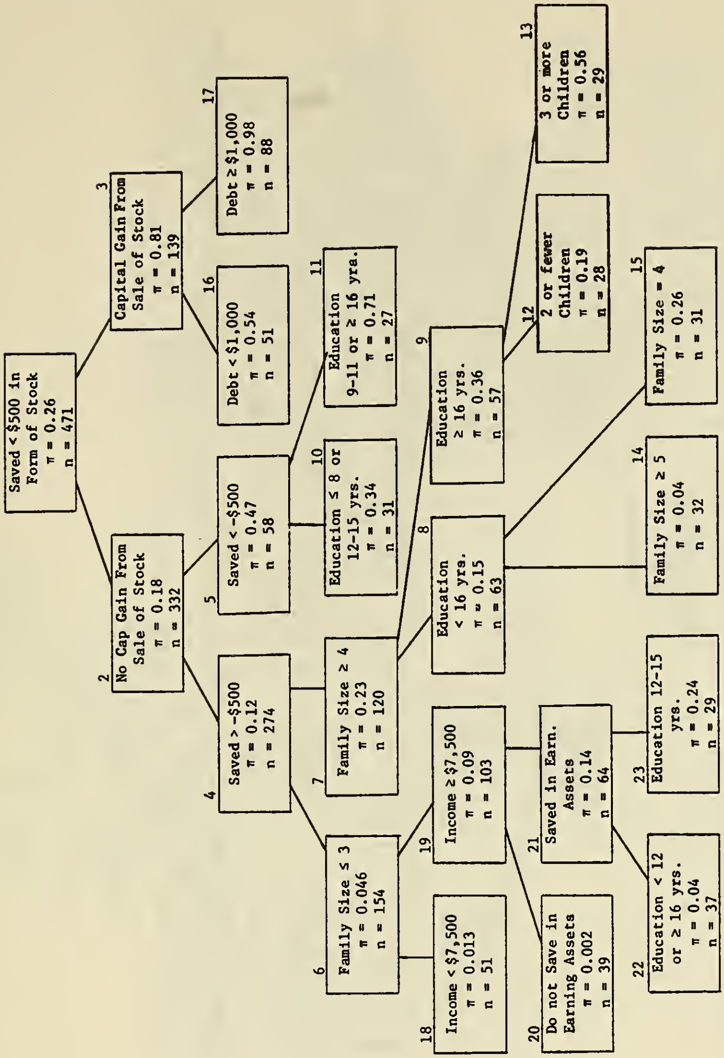
23 final groups account for 34.6% of variance

Chart 12.—*Conditional Probability of Dissaving at Least $100 in Earning Assets*



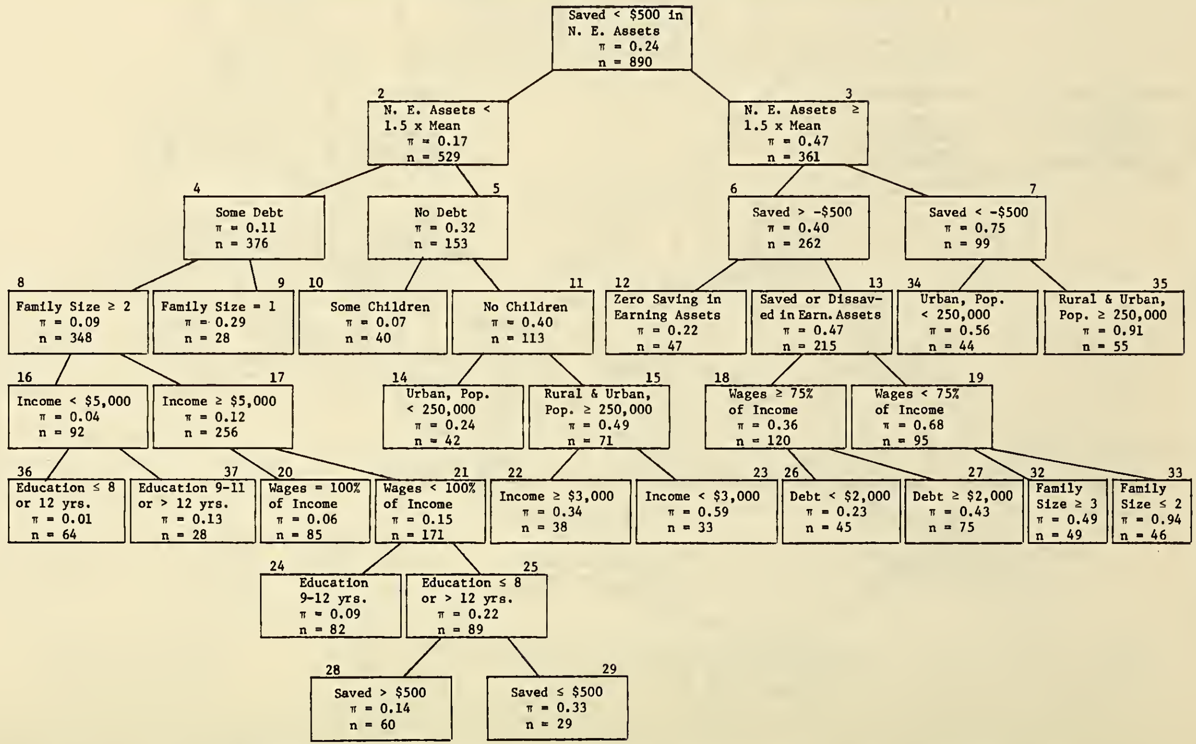
21 final groups account for 32.4% of variance

Chart 13.—Conditional Probability of Dissaving in Form of Corporate Stock



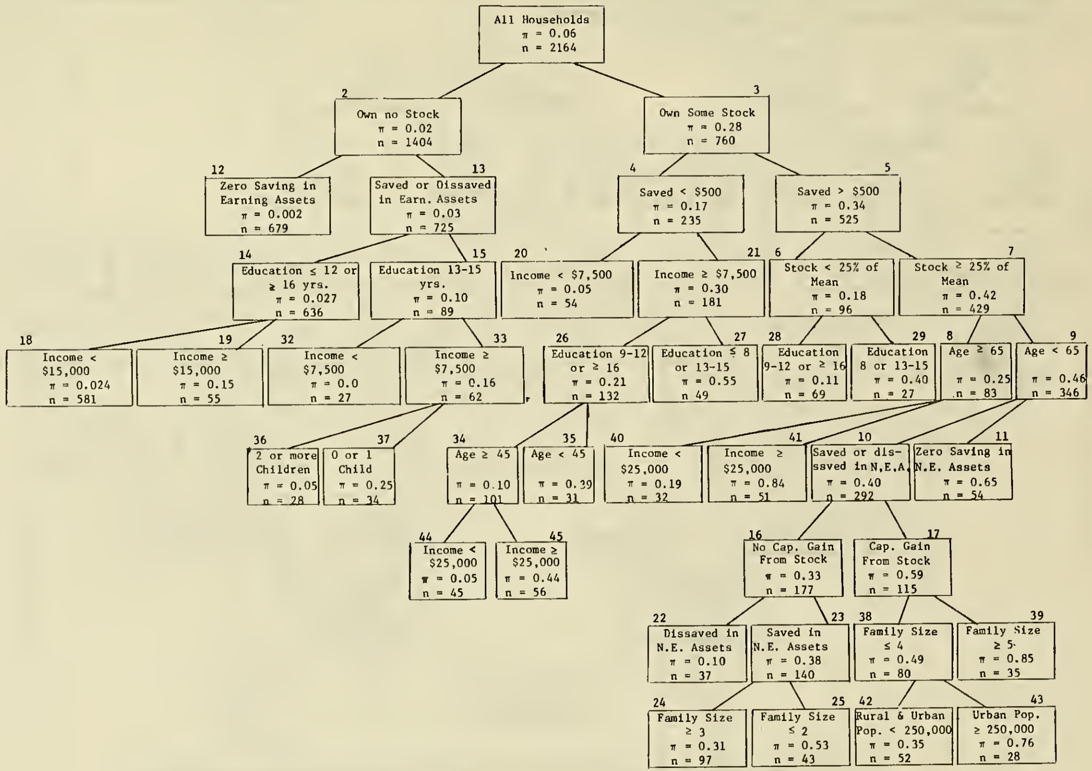
12 final groups account for 45.7% of variance

Chart 14.—*Conditional Probability of Dissaving at Least $100 in Nonearning Assets*



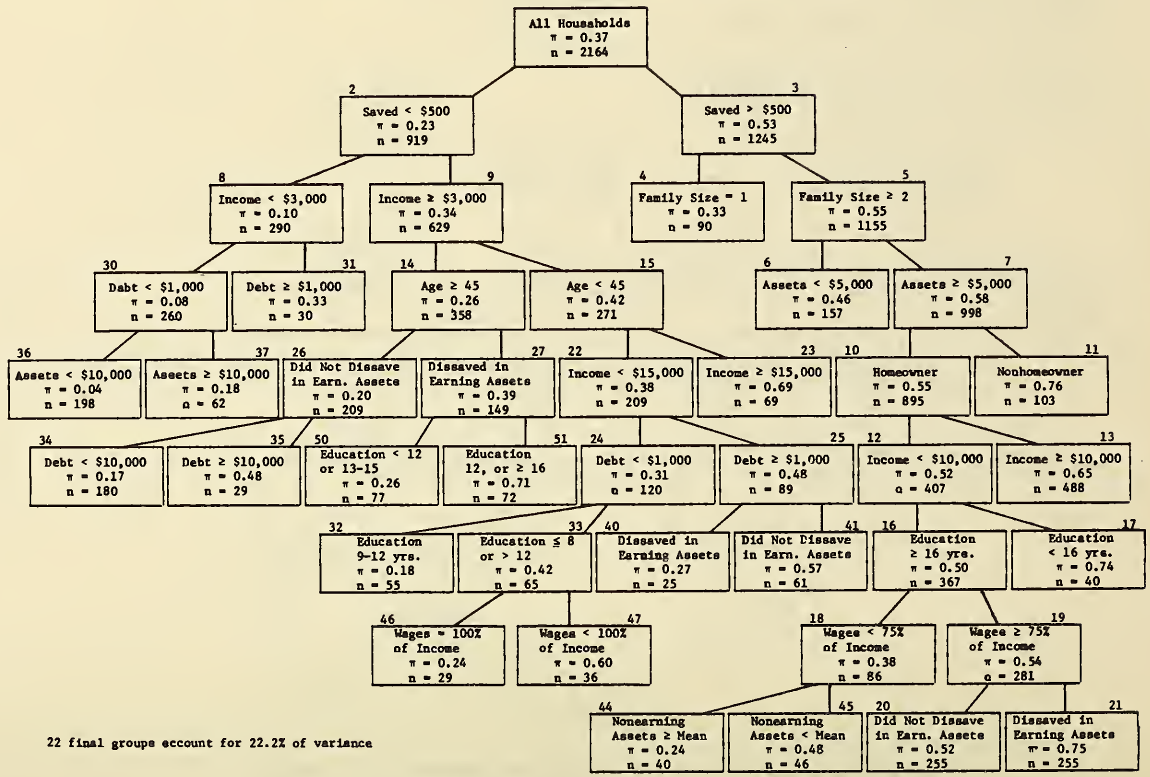
18 final groups account for 29.2% of variance

Chart 15.—*Probability of Saving in Form of Corporate Stock*



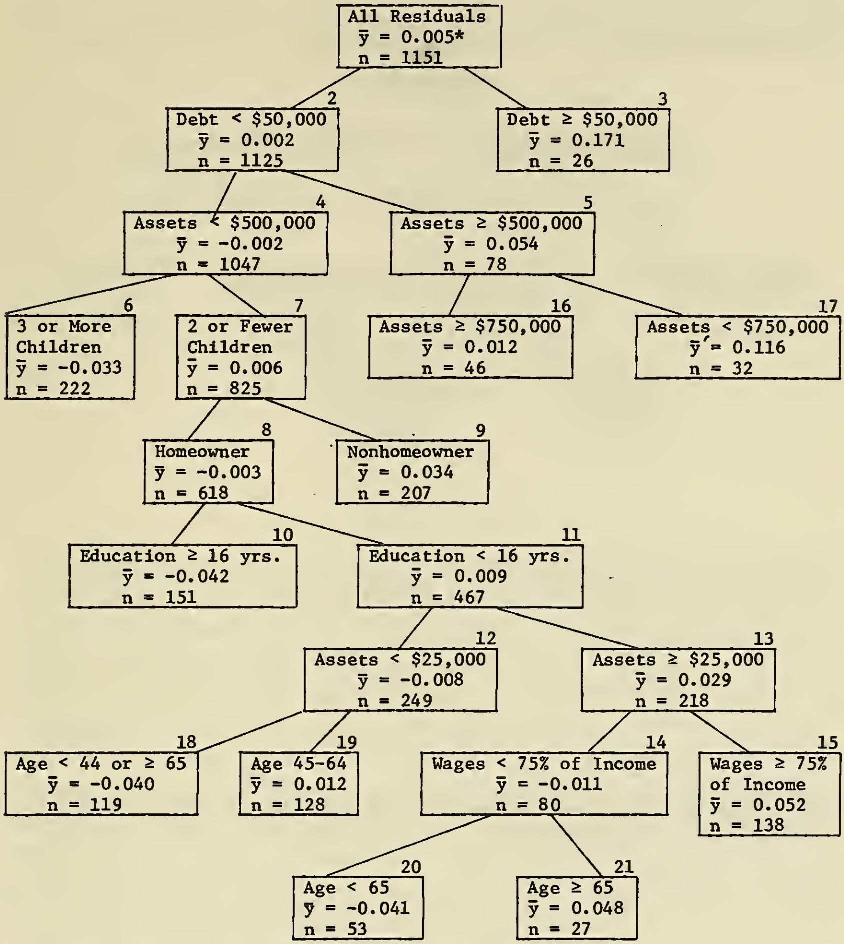
22 final groups account for 34.5% of variance

Chart 16.—*Probability of Saving at Least $100 in Nonearning Assets*



22 final groups account for 22.2% of variance

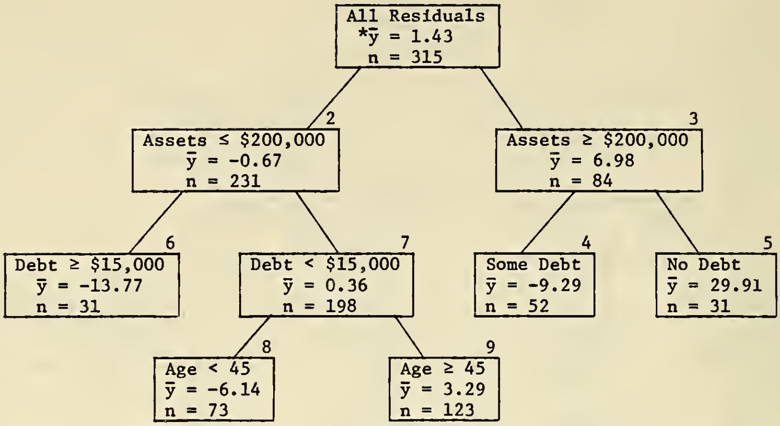
Chart 17.—*Residual Saving Ratio*



11 final groups account for 7.3% of variance.

*Mean residual is not 0 because residuals were calculated for several cases excluded from the regression.

Chart 18.—*Residual Ratio of Assets Plus Income to Dissaving*



5 final groups account for 10.7% of variance.

*Mean residual is not zero because residuals were calculated for several cases excluded from the regression.

Taxing the Intergenerational Transmissions of Wealth: A Simulation Experiment Part II

James D. Smith and Stephen D. Franklin

Pennsylvania State University

Guy H. Orcutt

Yale University

I. Introduction

Questions of how much of the observable cross-sectional distribution can be accounted for by inheritance and how much by saving out of earnings have been matters of speculation and some empirical work by Soltow,[1] Morgan[2] and Projector.[3] However, as Brittain points out research has not been well structured to capture the importance of inheritance.[4] The issue of inheritance has been, for the most part, a peripheral one in studies concerned with other economic behavior.

Data from several sources are used to simulate the transmission of wealth at death. To measure the influence of death taxes and inheritances on intergenerational wealth distributions, simulation experiments were conducted with four death tax systems. Two tax systems use estate wealth as a base and two use inheritances as a base. An experiment without taxes was also run.

In brief, the procedure followed was to: (a) modify the 1962 Survey of Financial Characteristics of Consumers (SFCC) file from a family file to a file of persons identified as members of specific family units; (b) stochastically attribute to families members living away from home; (c) allocate to individuals all wealth not specifically identified with

---

[1] Lee Soltow, *Toward Income Equality in Norway*, Madison, Wisconsin: University of Wisconsin Press, 1965.

[2] Robin Barlow, Harvey E. Brazer and James N. Morgan, *Economic Behavior of the Affluent*, Washington, D.C.: Brookings Institute, 1966.

[3] Dorothy Projector and Gertrude S. Wiess, *Survey of Financial Characteristics of Consumers*, Federal Reserve Technical Papers, August 1966.

[4] John A. Brittain, "The Intergenerational Transmission of Wealth: Prospects for a Research Program," Mimeo, December 1971.

individual family members in the original file; (d) pass the file and subject each individual to a Monte Carlo death process based upon age-sex-race-marital status-specific mortality rates for 1962; (e) distribute estates of decedents stochastically in accordance with estimated probability patterns of bequests and other transfers to family members at home and away from home, and to nonfamily members and to charity; (f) tax estates or inheritances in accordance with four tax "statutes"; and (g) calculate the before and after characteristics of the distribution of wealth and the yield to the Treasury.

The experiment here does not use the capacity of the model to track individuals over time, but focuses on the one-year impact of alternate schemes of taxing the transmission of wealth at death. Although the model's power to grow a population through time is one of its most attractive features, there are many applications which require only a cross-sectional representation of the population. For such applications, policymakers often desire a more current or detailed population representation than is available from a Census (which is on the average always 5 years old) or other data base. In such cases populations with detailed characteristics can be grown up to some specific past or future year and written out onto tape for cross-sectional analysis. It is in the spirit of such cross-sectional analysis that the simulation presented here was conducted.

II. Results of Simulation Experiments

Simulation 1. Current Estate Tax

The first simulation experiment employs a tax statute which approximates the current Federal estate tax law. The tax statute captures the essential features of the present Federal estate tax. It provides for a personal exemption of $60,000 for each decedent's estate and a marital deduction of the actual amount bequeathed a spouse or one-half the estate (whichever is less). Charitable bequests, costs of last illness, legal fees, and administrators' commissions are deductible in arriving at taxable estate. After exemptions and deductions are subtracted from the net worth of estates, the remainder is taxed in accordance with current Federal estate tax rates. (See Table 1.)

Features of the Federal estate tax which were not captured are the credits for State and foreign death taxes and the reduction in rates applicable to assets which have been taxed in another estate within 10 years. Also missed are assets given away in contemplation of death and certain other lifetime transfers which are constructively part of the estate for Federal estate tax purposes.

Using the weighted SFCC sample, slightly modified to better represent the upper tail of the wealth distribution (see section III), a test of the simulation model was made using the Federal estate tax statute.

TABLE 1.—*Federal estate tax schedule*

Taxable estate equal to or more than— (1)	Taxable estate less than— (2)	Tax on amount in column (1) (3)	Rate of tax on excess over amount in column (1) (4)
			(Percent)
0	$5,000	0	3
$5,000	10,000	$150	7
10,000	20,000	500	11
20,000	30,000	1,600	14
30,000	40,000	3,000	18
40,000	50,000	4,800	22
50,000	60,000	7,000	25
60,000	100,000	9,500	28
100,000	250,000	20,700	30
250,000	500,000	65,700	32
500,000	750,000	145,700	35
750,000	1,000,000	233,200	37
1,000,000	1,250,000	325,700	39
1,250,000	1,500,000	423,200	42
1,500,000	2,000,000	528,200	45
2,000,000	2,500,000	753,200	49
2,500,000	3,000,000	998,200	53
3,000,000	3,500,000	1,263,200	56
3,500,000	4,000,000	1,543,200	59
4,000,000	5,000,000	1,838,200	63
5,000,000	6,000,000	2,468,200	67
6,000,000	7,000,000	3,138,200	70
7,000,000	8,000,000	3,838,200	73
8,000,000	10,000,000	4,568,200	76
10,000,000		6,088,200	77

Source: Federal Estate Tax Return, Form 706 (Re. Sept. 1963).

If the simulation model captures behavior in the real world, simulated taxes for decedents with gross assets of $60,000 or more should approach those reported for 1963 by the IRS. Simulated taxes and those reported by the IRS are compared in table 2. The comparison shows similar numbers of returns filed for estates with gross assets above $100,000, but the model generates 64,133 estate tax returns in the range between $60,000 and $100,000 gross assets compared to only 30,999 returns reported by the IRS. Two factors are believed to account for the difference. First, there is a lag in filing, and returns filed with the IRS in 1963 are largely for persons who died before 1963, whereas the returns "filed" in the simulation model are only for persons who "died" in 1963.[5] The population and its mean wealth have increased steadily, consequently, IRS filings in a given year understate the number of returns which will ultimately be filed for decedents who die in that year. Secondly, there is strong evidence of noncompliance with the filing

[5] Conceptually, the model can most accurately be said to reflect death between July 1, 1962 and June 30, 1963. This comes about because the SFCC sample represents the U. S. population on December 31, 1962. Official mortality rates are available using a July 1, 1962 or July 1, 1963 base. We arbitrarily chose the 1962 base.

provision of the law near the filing threshold.[6] Our purpose at this point is not to test for compliance with the law (a potential use of the model), but to establish the credibility of the model for assessing the impact of alternative tax systems on the distribution of wealth. It does appear, however, that the simulated number of tax returns may be closer to the number of estates with gross assets between $60,000 and $100,000 than that reported by the IRS. (See footnote 6.)

TABLE 2.—*Comparison of simulated results with IRS data for returns filed in 1962*

Size of gross estate	Number of returns	
	Simulation	IRS
≥$2,000,000	803	618
$1,000,000 <2,000,000	858	1,151
500,000 <1,000,000	3,493	3,232
100,000 <500,000	52,701	42,989
60,000 <100,000	64,133	30,999
Total returns	121,988	78,989
Tax collected	$2.1 billion	[1] $2.1 billion

[1] Before tax credits. The actual IRS tax collected on 1963 returns amounted to $1.8 billion. See *Statistics of Income: Fiduciary Gift and Estate Tax Returns, 1962*, p. 51.

In the simulation 1.75 million persons died.[7] This compares very favorably with the official reported number of 1.76 million deaths in 1962 or the 1.81 million deaths in 1963.[8]

The assets of the simulated decedents totaled $40.4 billion. Estate taxes collected came to $2.1 billion. After allowance for decedents' debts, attorneys' fees, administration costs, last medical costs, funeral expenses, and estate taxes, $32.4 billion of their assets devolved to their heirs and beneficiaries. The simulation logic of inheritance is described in detail in the methodology section, but basically wealth is transmitted along familial lines revealed in the SFCC data.

The question we would answer is, does this process of death transfers and death taxes alter the distribution of wealth?

It appears (Table 3) that the present estate tax contributes very little to reducing the concentration of wealth. A slight reduction of the number of families at the very bottom of the distribution occurs (see "all units" column in Table 3), but above a net worth of $2,000 no changes are revealed. The changes at the bottom of the distribution are

[6] The number of estates filing tax returns in 1963 are graphed by size of gross assets in the chart page 272. In contrast to all other evidence about the size distribution of assets, the frequency of estates below about $70,000 declines rapidly. The failure of estates to file is believed to result from the fact that most such estates have a zero tax liability. Discussions with employees of the Internal Revenue Service indicate that they are of the problem of nonfiling.

[7] No attempt was made to capture the incidence of fetal deaths or deaths of infants at birth since the SFCC file would not readily provide a basis for such events.

[8] Department of Health, Education and Welfare, *Vital Statistics of the United States*, Mortality, Part A, 1963.

attributable to the deaths of poor older families and the inheritance of wealth by "poor" families from those with substantial wealth.

Distributions by age of head, Table 3, reveal that although the shape of overall distribution is stable, families are moving around within the distribution. For instance, families with a head 65 or older decline in numbers all along the distribution. On the other hand, the numbers of families headed by a person age 30 to 65 and with a net worth of over $15,000 increases. There are also increases in the number of families near the top of the distribution with heads under 30. In the simulation there is no saving function, so all the changes are the result of wealth transfers. It appears then that wealth is transmitted from older to younger families who have at least a minimal net worth (more than $1000) before inheriting. Families also lose wealth because family members die and their wealth is depleted by the cost of dying and taxation before it is inherited by survivors. Just as there is no saving in the model to move families up the wealth distribution, there is no

TABLE 3.—*Distribution of families by net worth and age of head before and after current estate tax, 1962*

[Numbers in thousands]

Net worth	All units	Head's age: <30	30 <65	≥65
		Before taxes		
<1,000	8712.7	2217.3	5314.1	1181.3
1,000 <2,000	5704.6	1859.9	3047.6	797.2
2,000 <3,000	2697.0	727.0	1808.7	161.3
3,000 <4,000	2339.6	405.4	1492.8	441.4
4,000 <5,000	2161.1	266.5	1432.7	461.9
5,000 <6,000	1688.7	179.0	1222.5	287.2
6,000 <7,000	1216.6	139.6	907.8	169.2
7,000 <8,000	1781.5	183.2	1245.8	352.5
8,000 <9,000	1350.4	23.9	1047.0	279.6
9,000 <10,000	1742.3	288.8	972.1	481.4
10,000 <15,000	6381.4	417.6	4549.0	1414.8
15,000 <20,000	4717.8	197.3	3715.2	805.3
20,000 <25,000	3507.6	79.9	2911.0	516.6
25,000 <50,000	7838.8	269.0	5678.2	1891.6
50,000 <100,000	3970.8	6.7	2953.3	1010.8
100,000 <200,000	1226.7	46.1	877.1	303.5
≥200,000	889.4	2.6	556.9	329.8
		After taxes		
<1,000	8464.2	2156.6	5218.0	1091.3
1,000 <2,000	5429.4	1799.8	2898.0	732.2
2,000 <3,000	2646.9	769.6	1718.2	159.2
3,000 <4,000	2261.8	421.1	1436.1	404.6
4,000 <5,000	2198.6	325.7	1449.8	423.1
5,000 <6,000	1727.4	212.5	1246.6	268.3
6,000 <7,000	1238.4	153.8	923.5	161.2
7,000 <8,000	1782.4	195.1	1262.8	324.5
8,000 <9,000	1374.1	48.6	1036.6	288.8
9,000 <10,000	1637.5	266.8	927.3	443.4
10,000 <15,000	6398.4	467.0	4577.9	1354.6
15,000 <20,000	4801.9	221.7	3787.5	793.2
20,000 <25,000	3575.6	105.4	2939.4	530.8
25,000 <50,000	7877.9	287.2	5772.8	1819.3
50,000 <100,000	3964.2	12.8	3016.2	935.3
100,000 <200,000	1230.5	46.2	887.3	297.1
≥200,000	884.3	2.9	561.1	320.4

consumption to move them down. So again, it is the pure effect of death transfers which is observed.

It can be argued that family net worth understates the immediate wealth effect of death on a family. The death of a family member may concentrate wealth in the hands of a smaller number of persons, so average net worth of family members would then increase. The increase in average member wealth could be quite significant if the life of the decedent was well covered with life insurance. The value of human capital is not part of the wealth concept used here. Were human capital to be included, a different view of the wealth effect of death would be in order. To examine the interaction of changing family size and wealth transfers, the before and after tax distributions were tabulated on a per capita family basis, e.g., family net worth divided by family size. The results are shown in table 4.

TABLE 4.—*Distribution of families by per capita new worth and age of head before and after current estate tax, 1962*

[Numbers in thousands]

Net worth	All units	Head's age		
		<30	30 <65	≥65
Before taxes				
<1,000	19551.1	4907.5	12667.8	1975.8
1,000 <2,000	7450.7	1398.4	5098.7	953.5
2,000 <3,000	5371.0	537.2	4317.5	616.2
3,000 <4,000	3622.1	76.6	2791.7	753.8
4,000 <5,000	2514.6	62.4	2068.2	383.9
5,000 <6,000	2084.4	57.0	1284.5	743.0
6,000 <7,000	2011.5	15.5	1592.7	403.3
7,000 <8,000	1613.2	88.3	1008.7	516.2
8,000 <9,000	1395.3	65.5	932.0	397.8
9,000 <10,000	761.5	46.7	446.4	268.4
10,000 <15,000	4005.5	55.7	2715.0	1234.8
15,000 <20,000	2322.6	91.3	1548.3	682.9
20,000 <25,000	1345.8	0.0	855.1	490.7
25,000 <50,000	2446.2	0.0	1628.7	817.5
50,000 <100,000	748.1	6.8	389.5	351.8
100,000 <200,000	460.2	0.5	286.8	172.9
≥200,000	223.3	0.4	99.8	123.0
After taxes				
<1,000	16096.3	4258.3	10342.5	1498.2
1,000 <2,000	6685.3	1321.8	4590.8	774.0
2,000 <3,000	4876.3	706.1	3611.6	559.0
3,000 <4,000	3802.2	413.3	2986.6	402.3
4,000 <5,000	3244.7	167.3	2347.3	730.1
5,000 <6,000	2273.8	99.9	1873.4	300.6
6,000 <7,000	1555.5	46.8	1162.8	345.9
7,000 <8,000	1793.6	43.7	1210.8	539.2
8,000 <9,000	1359.8	9.6	1101.1	249.2
9,000 <10,000	1429.9	77.4	921.1	431.5
10,000 <15,000	4352.1	197.4	2957.2	1197.6
15,000 <20,000	2497.1	33.2	1683.4	780.5
20,000 <25,000	2032.5	94.2	1332.0	606.3
25,000 <50,000	3446.0	15.7	2320.9	1109.5
50,000 <100,000	1119.4	6.8	654.0	458.6
100,000 <200,000	620.4	620.2	391.9	227.7
≥200,000	308.0	0.7	170.3	137.0

Whatever dramatic changes may occur to the average wealth of family members because of changes in family size and inheritances,

they do not reveal themselves in the per capita family wealth tabulations of table 4. If anything, the table suggests more stability on a per capita basis than on an aggregate family basis.

To better examine the dynamics within the distribution suggested by tables 3 and 4, a decile matrix of before and after tax rank was constructed. (Tables 5 and 6.) Families are found to experience considerable decile movement even though the aggregate distribution is stable.

To this point we have demonstrated that our simulation model (a) stochastically generates deaths which are almost identical in number to those reported in the official vital statistics for the United States, (b) generates tax collections by the Treasury (under a tax algorithm which replicates the present Federal estate tax rules) which are very comparable to those reported in the official Treasury statistics, and (c) generates estate sizes which are in essential agreement with the size distribution of estates reported by the IRS in the range above $100,000 gross assets. (Below $100,000 we find many more estates than the IRS reports, and there is strong reason to believe the IRS is wrong and the model's results are very close to correct. (See footnote 6.) On the basis of the above results we are prepared to use the model to form judgements about the process of intergenerational wealth transfers and death taxes.

By measuring events which occur in the simulation population it is possible to gain insights into the intergenerational distribution of wealth which are not available to us from natural data. In Tables 3 and 4 it is shown that the distribution of wealth after one year's deaths and associated taxes is little different from that before the events occurred when saving and consumption were held constant (set to zero). However, there is a widely held belief that the transmission of wealth at death results in increasing its concentration and that the federal estate tax lessens the concentration by whittling down large estates before they devolve to already rich persons. Both beliefs are intuitive since there is no empirical data to support either. To measure the independent effects of death transfers and estate taxes on the distribution of wealth, the simulation was rerun setting the tax rate to zero for all estates. After the simulation, family distributions of net worth were produced and are shown in Table 7 along with the net worth distributions for families before simulation and after the simulation using the federal estate tax. The results are rather startling. Taken alone, the transfer of wealth at death does not tend to increase the concentration of wealth, but to slightly decrease it. In the first three columns of Table 7 the before simulation distribution of net worth on December 31, 1962, the percentage change in the intervals of the distribution due to death transfers in the absence of a death tax, and the resultant distribution on December 31, 1963, are shown. There is a pronounced net movement of families out of the four lowest net worth classes and a slight decrease or no net change in the numbers of families at the top of the size distribution. The overall effect then of death

TABLE 5.—*Before and after death tax family net worth position, current Federal estate tax, 1962*

[Cells contain numbers of families in thousands]

	Decile	After tax family net worth position in thousands of dollars									
		1 ($52.3)	2 ($29.3)	3 ($20.2)	4 ($14.2)	5 ($9.9)	6 ($6.1)	7 ($3.3)	8 ($1.4)	9 ($1.0)	10
$52.3	1	5630.5	92.3	0.5	0.0	9.5	11.3	0.0	4.8	0.0	0.2
$29.3	2	129.0	5381.4	210.3	10.4	0.0	10.9	16.7	0.0	0.0	0.0
$20.2	3	0.0	261.6	5043.9	432.8	0.0	6.4	7.2	0.0	0.0	0.0
$14.3	4	0.0	26.2	431.3	4637.1	627.0	3.9	0.0	8.9	0.0	1.7
$9.9	5	0.0	4.5	47.4	565.6	4338.9	792.3	0.0	0.0	2.3	0.0
$6.1	6	0.0	0.0	18.5	81.7	688.8	4263.5	678.6	12.2	0.0	0.0
$3.3	7	0.0	0.0	11.5	20.2	64.5	575.1	4418.6	654.2	4.7	3.5
$1.4	8	0.0	0.0	0.2	17.1	19.6	65.0	474.4	4389.0	795.1	4.4
$1.0	9	0.0	0.0	0.0	0.0	12.1	30.6	85.0	313.7	4563.1	675.6
	10	0.0	0.0	0.0	0.4	6.5	4.4	85.2	381.5	401.5	4929.3
	Total	5759.5	5766.0	5763.6	5765.3	5766.9	5763.4	5765.7	5764.3	5766.7	5614.7

TABLE 6.—*Before and after death tax family per capita net worth position, current Federal estate tax, 1962*

[Cells contain numbers of families in thousands]

		After tax family per capita net worth position in thousands of dollars									
		$24.4	$13.1	$7.9	$4.9	$3.2	$2.0	$1.1	$0.6	$0.3	
	Decile	1	2	3	4	5	6	7	8	9	10
	1	5570.4	109.7	14.6	6.1	0.0	4.8	0.0	0.0	0.0	0.2
$24.4											
	2	171.2	5328.3	168.6	18.0	9.6	0.0	0.0	0.0	0.0	0.0
$13.1											
	3	13.7	227.0	5172.6	334.0	4.9	0.0	0.0	0.0	0.0	0.0
$7.9											
	4	6.8	64.2	331.6	4922.9	414.9	4.9	5.6	2.3	1.7	0.0
$4.9											
	5	0.0	18.7	39.6	360.2	4685.8	657.0	5.6	0.0	0.0	0.0
$3.2											
	6	0.0	11.9	21.9	49.8	550.6	4571.1	566.6	6.6	0.0	0.0
$2.0											
	7	0.0	5.4	3.5	46.6	75.0	429.0	4582.3	570.9	0.0	0.0
$1.1											
	8	0.0	0.0	7.2	25.8	7.8	57.6	514.1	4537.1	550.1	9.3
$0.6											
	9	0.0	0.0	0.0	0.0	8.3	30.4	65.9	534.1	4519.8	655.9
$0.3											
	10	0.0	0.0	5.0	0.4	8.8	10.9	25.0	112.5	690.7	4953.1
	Total	5762.1	5765.2	5764.6	5763.8	5765.7	5765.9	5765.1	5763.5	5762.3	5618.5

transfers in the absence of a death tax is to lessen concentration. This apparently comes about for several reasons. First, some one-person families, which have a higher probability of being at the bottom of the distribution than do larger families, die off. Secondly, most decedents (over 95 percent in 1962) have estates of less than $60,000 net worth. These estates, when distributed to surviving children and to other family members, are parcelled into rather small bequests so that most inheritors are not moved a long ways up the wealth distribution. Thirdly, inheritors, contrary to popular opinion, are often at the lower end of the wealth distribution. This is particularly relevant for the findings in the case of children of wealthy parents. The children of the rich are likely to have above average levels of human capital because they have longer than average periods of schooling, but that very fact increases the probability that they will inherit wealth before they have accumulated significantly out of their own earnings. In the methodology section it will be found that ones' own wealth is a poor predictor of inheriting; educational level was found to be much better.

TABLE 7.—*Simulation of independent effects of bequests and current estate tax on the size distribution of family wealth, 1962*

Family net worth	Distribution of families before simulation	Change due to bequests	Distribution of families after deaths with zero tax	Change due to current estate tax	Distribution of families after current estate tax	Change due to bequests and estate tax
Dollars	*Thousands*	Δ%	*Thousands*	Δ%	*Thousands*	Δ%
<1,000	8712.7	−2.9	8461.9	0.0	8464.2	−2.9
1,000 <2,000	5704.6	−5.0	5419.0	0.2	5429.4	−4.8
2,000 <3,000	2697.0	−2.5	2630.1	0.6	2646.9	−1.9
3,000 <4,000	2339.6	−3.2	2265.7	−0.2	2261.8	−3.3
4,000 <5,000	2161.1	2.1	2206.9	−0.4	2198.6	1.7
5,000 <6,000	1688.7	1.2	1708.6	1.1	1727.4	2.3
6,000 <7,000	1216.6	2.5	1246.5	−0.6	1238.4	1.8
7,000 <8,000	1781.5	0.1	1783.9	−0.1	1782.4	0.1
8,000 <9,000	1350.4	1.9	1375.8	0.1	1374.1	1.8
9,000 <10,000	1742.3	−4.8	1658.2	−1.2	1637.5	−6.0
10,000 <15,000	6381.4	0.0	6383.2	0.2	6398.4	0.3
15,000 <20,000	4717.8	1.8	4804.4	−0.1	4801.9	1.8
20,000 <25,000	3507.6	1.8	3570.7	0.1	3575.6	1.9
25,000 <50,000	7838.8	0.8	7899.2	−0.3	7877.9	0.5
50,000 <100,000	3970.8	−0.2	3964.2	0.0	3964.2	−0.2
100,000 <200,000	1226.7	0.3	1230.5	0.0	1230.5	0.3
≥200,000	889.4	−0.5	884.3	0.0	884.3	−0.5
Total	57927.0		57493.0		57493.0	

In column 5 of table 7 the distribution of families by size of net worth after simulation with the current estate tax rates in effect is shown. The difference between the distribution shown in column 3 (zero tax rates) and column 5 is the pure distributional impact of the current estate tax, i.e., independent of the pretax devolution pattern. The tax can of course do nothing to move families out of the lower reaches of the distribution so one would not expect to find negative changes (column 4), but there appears to be a slight increase in the numbers of families at the lower

end. This suggests that some of the smaller parcels of wealth intended for families in the range above the lowest three net worth intervals were diminished when the tax was applied. These families who would have ended in richer classes with a zero death tax find themselves at the bottom with the current tax. These families must have come from some other part of the distributions, and they presumably came from the middle range where the net impact of the tax is to reduce the number of families. Oddly enough, the tax has no measurable impact at the top of the distribution. Whatever gross outflows of families occurred from the intervals above a net worth of $50,000 were offset by movement into the classes by inheritors who were nearer the middle of the distribution. There are a number of reasons for this unexpected result. First, bequests are more often made to persons of less wealth than the decedent than the converse. Secondly, the tax on intrafamily wealth transfers is less than the rates alone would suggest. (When the decedent is married up to 50 percent of his estate is exempt. The first $60,000 of net worth of all estates is also exempt.) Thirdly, estate planning removes from the purview of the tax law certain assets and distributes them constructively or in fact prior to death. Fourthly, the base of the current Federal death tax is the net worth after deductions and exemptions of the estate not the wealth of the inheritor. Thus the share of the pretax estate inherited by an indigent heir is diminished by the same percentage as that share inherited by an affluent heir.

We are left then with the paradox that a tax whose philosophical foundation is to mitigate a natural process toward inequality is apparently obstructing a natural process toward greater equality.

At this point we turn to the simulation of three alternative tax structures which differ from the current tax by varying degree.

Simulation 2. Reform Estate Tax

It was noted that the current estate tax had little effect on the distribution of wealth at the top of the distribution because the generous marital and personal exemptions permit the transfer of substantial amounts of untaxed wealth. Also, the marital exemption discriminates among heirs. We have structured a tax which eliminates discriminatory features, increases the personal exemption of all estates to $100,000, and provides for higher rates on estates over $100,000. The rates are 50% on the first $400,000 of taxable estate and 100% on amounts in excess of $400,000. It is assumed, of course, that appropriate measures to insure against tax avoidance, such as placing appropriate taxes on intervivos gifts, would be implemented.

This simulation was run in exactly the manner as the first experiment. The same persons selected to die by the Monte Carlo draws in the first experiment were selected again, so, although the selection of deaths is stochastic, the same stochastic selection was used for all experiments to avoid interexperiment Monte Carlo variation.

This tax reduces inequality on both a straight family basis and on a per capita family basis. (See Tables 8 and 9.) Although the one-year reduction in equality is very slight, it would over a number of years compress the distribution of wealth.

TABLE 8.—*Distribution of families by net worth and age of head before and after reform estate tax, 1962*
[Numbers in thousands]

Net worth	All units	Head's age <30	Head's age 30 <65	Head's age ≥65
<1,000	8712.7	2217.3	5314.1	1181.3
1,000 <2,000	5704.6	1859.9	3047.6	797.2
2,000 <3,000	2697.0	727.0	1808.7	161.3
3,000 <4,000	2339.6	405.4	1492.8	441.4
4,000 <5,000	2161.1	266.5	1432.7	461.9
5,000 <6,000	1688.7	179.0	1222.5	287.2
6,000 <7,000	1216.6	139.6	907.8	169.2
7,000 <8,000	1781.5	183.2	1245.8	352.5
8,000 <9,000	1350.4	23.9	1047.0	279.6
9,000 <10,000	1742.3	288.8	972.1	481.4
10,000 <15,000	6381.4	417.6	4549.0	1414.8
15,000 <20,000	4717.8	197.3	3715.2	805.3
20,000 <25,000	3507.6	79.9	2911.0	516.6
25,000 <50,000	7838.8	269.0	5678.2	1891.6
50,000 <100,000	3970.8	6.7	2953.3	1010.8
100,000 <200,000	1226.7	46.1	877.1	303.5
≥200,000	889.4	2.6	556.9	329.8
After taxes				
<1,000	8464.2	2156.6	5218.0	1091.3
1,000 <2,000	5436.0	1799.8	2904.6	732.2
2,000 <3,000	2644.6	769.6	1715.9	159.2
3,000 <4,000	2310.3	446.2	1454.7	409.4
4,000 <5,000	2192.5	313.6	1460.6	418.3
5,000 <6,000	1720.3	203.1	1244.0	273.3
6,000 <7,000	1253.5	150.2	941.2	162.2
7,000 <8,000	1755.0	195.1	1236.5	323.3
8,000 <9,000	1374.6	61.9	1017.4	295.4
9,000 <10,000	1657.6	253.9	964.8	438.9
10,000 <15,000	6384.9	476.8	4561.1	1348.0
15,000 <20,000	4824.1	220.6	3794.7	809.2
20,000 <25,000	3529.7	96.3	2918.9	514.4
25,000 <50,000	7872.5	287.2	5767.5	1819.3
50,000 <100,000	3959.5	12.8	3011.5	935.3
100,000 <200,000	1233.0	46.3	889.2	297.6
≥200,000	881.0	2.7	558.6	319.9

In Table 10 the pure tax effect on the distribution of family wealth is shown. The tax results in a diminution of persons at the top of the distribution, but also increases the number at the bottom. It also results in an increase in the number of families in the middle range. Although there is only a slight difference in the distribution of wealth resulting from this tax, the yield to the treasury is much greater, $7.2 billion compared to $2.1 billion, under the current federal tax system.

Simulation 3. Inheritance Tax, Modest Reform

The historical justifications of U.S. death taxes have been wealth redistribution and an impediment to plutocracy. In recent years,

TABLE 9.—*Distribution of families by per capita net worth and age of head before and after reform estate tax, 1962*

[Numbers in thousands]

Net worth	All units	Head's age <30	Head's age 30 <65	Head's age ≥65
		Before taxes		
<1,000	19551.1	4907.5	12667.8	1975.8
1,000 <2,000	7450.7	1398.4	5098.7	953.5
2,000 <3,000	5371.0	537.2	4317.5	616.2
3,000 <4,000	3622.1	76.6	2791.7	753.8
4,000 <5,000	2514.6	62.4	2068.2	383.9
5,000 <6,000	2084.4	57.0	1284.5	743.0
6,000 <7,000	2011.5	15.5	1592.7	403.3
7,000 <8,000	1613.2	88.3	1008.7	516.2
8,000 <9,000	1395.3	65.5	932.0	397.8
9,000 <10,000	761.5	46.7	446.4	268.4
10,000 <15,000	4005.5	55.7	2715.0	1234.8
15,000 <20,000	2322.6	91.3	1548.3	682.9
20,000 <25,000	1345.8	0.0	855.1	490.7
25,000 <50,000	2446.2	0.0	1628.7	817.5
50,000 <100,000	748.1	6.8	389.5	351.8
100,000 <200,000	460.2	0.5	286.8	172.9
≥200,000	223.3	0.4	99.8	123.0
		After taxes		
<1,000	16118.6	4265.8	10357.3	1498.2
1,000 <2,000	6687.6	1324.6	4590.2	774.0
2,000 <3,000	4880.8	707.0	3606.9	567.3
3,000 <4,000	3815.3	420.4	3001.0	394.0
4,000 <5,000	3218.0	148.9	2339.0	730.1
5,000 <6,000	2273.0	102.1	1865.5	305.3
6,000 <7,000	1581.9	44.6	1189.8	347.5
7,000 <8,000	1787.5	43.7	1200.6	543.1
8,000 <9,000	1345.7	12.9	1094.0	238.8
9,000 <10,000	1436.0	74.4	916.6	445.0
10,000 <15,000	4340.8	197.0	2959.8	1184.0
15,000 <20,000	2499.4	33.2	1675.9	790.3
20,000 <25,000	2015.1	94.2	1324.3	596.6
25,000 <50,000	3450.7	15.7	2325.6	1109.5
50,000 <100,000	1114.9	6.9	649.0	459.1
100,000 <200,000	623.3	0.7	395.4	227.2
≥200,000	304.4	0.5	166.9	137.0

TABLE 10.—*Percentage change in family wealth size distribution due to pure tax effect of reform estate tax, 1962*

Net worth	Δ % due to tax
<1,000	0.0
1,000 <2,000	0.3
2,000 <3,000	0.6
3,000 <4,000	2.0
4,000 <5,000	−0.7
5,000 <6,000	0.7
6,000 <7,000	0.6
7,000 <8,000	−1.2
8,000 <9,000	−0.1
9,000 <10,000	0.0
10,000 <15,000	0.0
15,000 <20,000	0.4
20,000 <25,000	−1.1
25,000 <50,000	−0.3
50,000 <100,000	−0.1
100,000 <200,000	0.2
≥200,000	−0.4

however, the annual Treasury yield from the tax has approached $2 billion. Whether redistributive or revenue raising, the burden of the tax would best be distributed on the ability to pay of natural persons with a beneficial interest in the estate. Clearly, there is no beneficial interest of a decedent in his estate. The only persons having beneficial interests are the potential heirs. Two simulation experiments taxing heirs were run.

The first experiment taxes *heirs* using the current estate tax schedule, (Table 1), but the base of the tax is the sum of one's inheritance plus his own net worth. An inheritor is not taxed on any part of his own wealth but the tax rates applicable to his inheritance begin with the marginal rate applicable to the first dollar of inherited wealth in excess of his own net worth. For instance, if the inheritor had a net worth of $60,000, the first $5,000 of his inheritance would be taxed

TABLE 11.—*Distribution of families by net worth and age of head before and after modest reform inheritance tax, 1962*
[Numbers in thousands]

Net worth	All units	Head's age <30	Head's age 30 <65	Head's age ≥65
Before taxes				
1,000	8712.7	2217.3	5314.1	1181.3
1,000 <2,000	5704.6	1859.9	3047.6	797.2
2,000 <3,000	2697.0	727.0	1808.7	161.3
3,000 <4,000	2339.6	405.4	1492.8	441.4
4,000 <5,000	2161.1	266.5	1432.7	461.9
5,000 <6,000	1688.7	179.0	1222.5	287.2
6,000 <7,000	1216.6	139.6	907.8	169.2
7,000 <8,000	1781.5	183.2	1245.8	352.5
8,000 <9,000	1350.4	23.9	1047.0	279.6
9,000 <10,000	1742.3	288.8	972.1	481.4
10,000 <15,000	6381.4	417.6	4549.0	1414.8
15,000 <20,000	4717.8	197.3	3715.2	805.3
20,000 <25,000	3507.6	79.9	2911.0	516.6
25,000 <50,000	7838.8	269.0	5678.2	1891.6
50,000 <100,000	3970.8	6.7	2953.3	1010.8
100,000 <200,000	1226.7	46.1	877.1	303.5
≥200,000	889.4	2.6	556.9	329.8
After taxes				
1,000	8461.9	2154.3	5218.0	1091.3
1,000 <2,000	5422.4	1802.1	2888.8	732.2
2,000 <3,000	2639.7	753.2	1724.0	162.5
3,000 <4,000	2258.7	425.5	1431.9	401.3
4,000 <5,000	2207.1	330.2	1453.8	423.1
5,000 <6,000	1725.7	220.0	1237.5	268.3
6,000 <7,000	1244.6	150.0	933.4	161.2
7,000 <8,000	1791.4	198.9	1267.9	324.5
8,000 <9,000	1369.6	48.6	1025.5	295.4
9,000 <10,000	1645.4	266.8	941.9	436.8
10,000 <15,000	6407.3	467.0	4586.8	1354.6
15,000 <20,000	4818.1	226.6	3789.9	802.1
20,000 <25,000	3560.2	102.3	2931.6	526.3
25,000 <50,000	7900.1	285.4	5801.3	1814.9
50,000 <100,000	3936.2	12.8	2985.3	938.2
100,000 <200,000	1224.3	46.4	883.3	294.6
≥200,000	880.7	2.7	558.2	319.9

at a 3 percent rate, the next $5,000 at a 7 percent rate, and so on. (See Table 1, page 251.) If the total inheritance of this heir were to amount to $20,000, his total tax bill would amount to $1,600 on the $20,000 inheritance. A richer individual receiving the same $20,000 would be taxed at higher rates. For instance, should the inheritor have a net worth of $500,000, entry into the tax table would be at the 35 percent rate. Since the tax rate for amounts between $500,000 and $750,000 is 35 percent, the entire $20,000 would be subject to a 35 percent rate, or a total of $6,000. An heir whose net worth was $12,000 would pay no tax on his $20,000 inheritance. Tables 11 and 12 show the before and after distributions of the tax.

The pure tax effect of this inheritance tax results in greater reductions at the top and smaller increases at the bottom of the wealth distribution than do the present or reform estate tax. Apparently, the high marginal rates on wealthy heirs have a significant impact on

TABLE 12.—*Distribution of families by per capita net worth and age of head before and after modest reform inheritance tax, 1962*

[Numbers in thousands]

Net worth	All units	Head's age <30	30 <65	≥65
		Before taxes		
<1,000	19551.1	4907.5	12667.8	1975.8
1,000 <2,000	7450.7	1398.4	5098.7	953.5
2,000 <3,000	5371.0	537.2	4317.5	616.2
3,000 <4,000	3622.1	76.6	2791.7	753.8
4,000 <5,000	2514.6	62.4	2068.2	383.9
5,000 <6,000	2084.4	57.0	1284.5	743.0
6,000 <7,000	2011.5	15.5	1592.7	403.3
7,000 <8,000	1613.2	88.3	1008.7	516.2
8,000 <9,000	1395.3	65.5	932.0	397.8
9,000 <10,000	761.5	46.7	446.4	268.4
10,000 <15,000	4005.5	55.7	2715.0	1234.8
15,000 <20,000	2322.6	91.3	1548.3	682.9
20,000 <25,000	1345.8	0.0	855.1	490.7
25,000 <50,000	2446.2	0.0	1628.7	817.5
50,000 <100,000	748.1	6.8	389.5	351.8
100,000 <200,000	460.2	0.5	286.8	172.9
≥200,000	223.3	0.4	99.8	123.0
		After taxes		
<1,000	16096.3	4258.3	10342.5	1498.2
1,000 <2,000	6690.1	1321.8	4590.7	778.8
2,000 <3,000	4868.9	706.1	3599.1	564.2
3,000 <4,000	3799.1	401.4	3005.4	392.3
4,000 <5,000	3245.9	171.8	2344.0	730.1
5,000 <6,000	2282.0	109.1	1872.3	300.6
6,000 <7,000	1566.9	41.2	1179.8	345.9
7,000 <8,000	1785.8	47.5	1199.2	539.2
8,000 <9,000	1374.1	11.3	1113.7	249.2
9,000 <10,000	1431.0	78.2	912.4	440.4
10,000 <15,000	4346.6	194.9	2958.5	1193.3
15,000 <20,000	2501.0	35.6	1689.4	776.1
20,000 <25,000	2029.7	91.8	1326.8	611.1
25,000 <50,000	3451.0	15.7	2330.8	1104.5
50,000 <100,000	1105.2	6.8	636.9	461.5
100,000 <200,000	614.9	0.7	389.3	224.9
≥200,000	304.5	0.5	167.1	136.9

TABLE 13.—*Percentage change in family wealth size distribution due to pure tax effect of modest reform inheritance tax, 1962*

Net worth	Δ % due to tax
<1,000	0.0
1,000 <2,000	0.1
2,000 <3,000	0.4
3,000 <4,000	−0.3
4,000 <5,000	0.0
5,000 <6,000	1.0
6,000 < 7,000	−0.2
7,000 <8,000	0.5
8,000 <9,000	−0.5
9,000 <10,000	−0.8
10,000 <15,000	0.4
15,000 <20,000	0.3
20,000 <25,000	−0.3
25,000 <50,000	0.0
50,000 <100,000	−0.7
100,000 <200,000	−0.5
≥200,000	−0.4

affluent heirs within the same family as the decedent while the exemption for heirs results in a smaller bite being taken from relatively less affluent inheritors. (See Table 13.)

Simulation 4. Severe Inheritance Tax Reform

To test the effect of a severe inheritance tax, a $50,000 limit was placed on the amount one could inherit from one estate. The tax has almost no direct effect on the distribution of wealth except at the very highest wealth levels. (See Table 14.) The top two wealth classes are slightly diminished and the third from the top class picks up the few families which are bumped down. The reasons for the small direct redistributive effect of such a severe tax are that (a) very few people inherit amounts in excess of $50,000 and (b) there is no provision in the

TABLE 14.—*Percentage change in family wealth size distribution due to pure tax effect of severe inheritance tax, 1962*

Net worth	Δ % due to tax
<1,000	0.0
1,000 <2,000	0.0
2,000 <3,000	0.0
3,000 <4,000	0.0
4,000 <5,000	0.0
5,000 <6,000	0.0
6,000 <7,000	0.0
7,000 <8,000	0.0
8,000 <9,000	0.0
9,000 <10,000	0.0
10,000 <15,000	0.0
15,000 <20,000	0.0
20,000 <25,000	0.0
25,000 <50,000	0.0
50,000 <100,000	0.4
100,000 <200,000	−0.7
≥200,000	−0.6

model for a behavioral change in the bequesting practices of individuals.

If wealthholders were confronted with either of the inheritance plans we have simulated, they would presumably change their wills to minimize tax erosion of their estates. If they carried this behavior to its limit they would avoid all inheritance taxes by bequeathing amounts no greater than $60,000 and $50,000 to individual heirs respectively in the third and fourth simulation experiments. But if they were to do so, it would achieve substantial redistribution.

In the modest reform inheritance tax system inheritors are subject to progressive tax rates when the sum of the heirs prior wealth and inheritance exceed $60,000. The rate will never, however, exceed 77 percent. Under such a tax, testators would evaluate how much they were willing to have their total distribution diminished by inheritance taxes in order to benefit specific heirs. If testators' aversion to having their bequests taxed were stronger than their preference to benefit specific heirs, they would distribute their estate so that no inheritor ended up with more than $60,000 when his prior wealth and inheritance were summed. It is unlikely that all testators have such strong aversions, and many would accept some diminution of their distributed estates in order to benefit preferred heirs at lower marginal tax rates. At higher marginal rates, which come about with increasing size of bequests and increasing prior wealth of heirs, it is suspected that testators' aversion to taxes would overtake their preference for benefitting specific heirs, and they would parcel out some of their bequests in smaller amounts and to less affluent heirs.

The manner in which the modest reform inheritance tax simulation (experiment 3 above) was run has decedents' estates distributed according to patterns observed under the present federal estate tax, that is, without penalty for giving bequests in excess of $60,000 or benefitting heirs, whose prior net worth equalled or exceeded $60,000. Consequently, we can think of the experiment as reflecting a limit at which testators were insensitive to a tax penalty for contributing to the concentration of wealth.

At the other limit testators would not make bequests which resulted in an heir's after-inheritance wealth exceeding $60,000. We can approximate the results of such behavior by redistributing the taxes collected by the Treasury in experiment 3. We have no empirical basis for estimating how testators would parcel out their assets, but will arbitrarily assume that testators are indifferent to the wealth of heirs so long as the total inheritance tax remains zero for heirs other than those they would favor under an estate tax. It is also assumed that all taxes are redistributed on a per capita basis. These assumptions make it easy to compare the results of all four tax systems. The effect of the taxes under these assumptions can be measured by redistributing on a per capita basis taxes collected in each of the experiments.

A tax which redistributes net worth toward a greater equality will reduce the numbers of family units in the intervals at the tails of the distribution and increase them near the mean of the distribution. In Table 15 the percentage change in the number of families within intervals due to the joint effect of taxation and redistribution of Treasury collections is shown. The "no tax" column shows the changes due solely to the devolution choices made by testators, and the law, for intestate decedents.

TABLE 15.—*Percentage change in number of families in intervals of the distribution of net worth due to the taxation of death transfers and the redistribution of treasury yield*

	Before simulation distribution	No tax	Current estate tax	Reform estate tax	Modest inheritance tax	Severe inheritance tax
	000					
<1,000	8712.7	−2.9	−43.6	−52.1	−52.0	−52.0
1,000 <2,000	5704.6	−5.0	55.6	61.2	61.7	61.7
2,000 <3,000	2697.0	−2.5	−1.4	10.6	9.5	9.2
3,000 <4,000	2339.6	−3.2	−3.7	−6.5	−7.3	−7.3
4,000 <5,000	2161.1	2.1	4.1	0.8	1.1	−0.5
5,000 <6,000	1688.7	1.2	−6.7	1.7	−2.3	−0.8
6,000 <7,000	1216.6	2.5	18.2	23.5	27.7	28.4
7,000 <8,000	1781.5	0.1	0.5	−3.7	−3.4	−5.1
8,000 <9,000	1350.4	1.9	−0.7	6.7	6.2	6.9
9,000 <10,000	1742.3	−4.8	−7.0	−22.0	−20.5	−19.8
10,000 <15,000	6381.4	0.0	0.0	4.0	3.7	3.4
15,000 <20,000	4717.8	1.8	1.1	2.2	2.3	1.9
20,000 <25,000	3507.6	1.8	2.1	−0.2	0.6	1.1
25,000 <50,000	7838.8	0.8	1.4	1.9	2.1	2.2
50,000 <100,000	3970.8	−0.2	0.5	0.4	−0.1	0.9
100,000 <200,000	1226.7	0.3	0.3	0.5	−0.2	−0.4
≥200,000	889.4	−0.5	−0.5	−0.9	−0.9	−1.2
Mean net worth	$29,714	$29,930	$29,930	$29,932	$29,934	$29,923
Standard deviation	$446,823	$446,060	$445,615	$445,879	$445,566	$446,544
Relative standard deviation (percent)	15.04	14.90	14.89	14.90	14.88	14.92

The three hypothetical tax systems which we structured result in greater equality than the current estate tax. It will be recalled that the pure tax effect of the current estate tax is toward inequality, so what is observed in column 3 is mostly the redistribution of Treasury yield (about $2 billion). We have assumed that all citizens benefit equally from Treasury expenditures, so families benefit in proportion to their size. In the case of the modest inheritance tax or the severe inheritance tax, the redistribution of Treasury tax collections is equivalent to a behavioral change on the part of testators to bequest to heirs as to avoid death taxes. Since the tax yield of the modest inheritance tax, the reform estate tax and the severe estate tax are all about three and one-half times larger than the current estate tax yield (see Table 17) they all produce greater redistributive effects than does the current estate tax. Both of the inheritance taxes result in greater redistribution than does the current reform estate tax. This is apparent from the percentage changes in the number of families within net worth intervals at the tails of the distribution.

When the redistributive effect of government expenditures is taken into account, it becomes apparent that reforms in death taxes can perceptibly change the distribution of wealth even in the short run. Longer run simulations using the full Urban Institute model will permit tests of the longer run consequences of these prototype death tax reforms.

As noted in the beginning of this section, the model takes into account the costs of dying—medical expenses, lawyer's fees, executor's commissions and funeral expenses. Because these expenses are greater than the total taxes collected under the present estate tax system, some importance attaches to them in considering the impact of death on the distribution of wealth. Many low-wealth families may be driven into debt to bury a family member and settle his estate. In Table 16 we show the simulated costs of dying and related information.

The cost of last illness is estimated at $507 million or about $301 per decedent. Attorneys' fees amount to $1 billion or over 3 percent of the wealth passing from decedent to beneficiary at death. Funeral expenses were even a greater share of the total wealth transmitted at death, $1.8 billion out of $36.6 billion, or almost 5 percent. Taking the total estimated cost of dying, $3.8 billion, we find that it amounts to about 10 percent of the total assets left for distribution.

TABLE 16.—*Characteristics of decedents and simulated costs of dying, charitable bequests and assets available for distribution*

Number of decedents:	
Gross estate <$60,000	1,625,245
Gross estate ≥$60,000	121,988
All decedents	1,747,333
Gross assets of decedents	$40,394,000,000
Net assets of decedents	38,524,000,000
Costs of dying:	
Last illness medical expenses	$ 529,000,000
Attorney's fees	1,009,000,000
Executor's commissions	544,000,000
Funeral expenses	1,944,000,000
Total	4,026,000,000
Charitable contributions	1,706,000,000
Net assets available for distribution and taxes	32,792,000,000

TABLE 17.—*Treasury yield under four simulated death tax systems, 1962*
[In millions]

Current estate tax	$2,127
Reformed estate tax	7,214
Modest reform inheritance tax	6,572
Severe reform inheritance tax	6,672

III. Methodology

To operate the simulation model a suitable organized sample representation of the U.S. population and a set of behavioral relations

characterizing the devolution of wealth at death, by bequest or otherwise, were required. The Survey of Financial Characteristics of Consumers (SFCC) file was modified to that end. Patterns of wealth devolution were estimated using federal estate tax files and files of Washington, D.C. estate tax returns. We turn first to the modification of the SFCC file and then to a discussion of the simulation logic, bringing in the behavioral estimates in about the order they are used in the simulation.

Modifying the SFCC file

The SFCC file contains observations on 2557 sample families representing 57.9 million family units in the population on December 31, 1962. The survey in which the information was gathered is the most detailed survey inquiry into family financial data available. Nevertheless, it contains a number of deficiencies which had to be remedied for the purpose to which we put it.

1. Family composition—The SFCC file contains limited information on family members living at home, but none for children who have left home. Since the main intergenerational flow of wealth is from parent to child it is important to have a basis of simulating the flow of wealth to children who have left home.

Information in the SFCC file combined with information available about the number of children ever born to married women permitted a rough assignment of the number of children living away from home.

The SFCC file contained the following relevant information.

1. Marital status of head.
2. Age of head.
3. Age of spouse.
4. Age of youngest child at home.
5. Age of oldest child at home.
6. Number of years since marriage.

From the 1960 Census of Population the numbers of children ever born by age of married women were obtained:

Age of woman	Number of children ever born
15–19	1.3
20–24	1.8
25–29	2.5
30–34	2.8
35–39	2.9
40–44	2.9
45–49	2.9
50+	3.4

Source: Tauber and Tauber, *People of the United States in the 20th Century*, Census Monograph Series, p. 429.

Above age 35 the average number of children born appears to be very close to 3.

The following rules were used to expand the family composition information on the SFCC record:

i. It was assumed the first child was never born before mother's age 17.

ii. Mothers under 35 were assumed to have all children living at home. Those 35 and over were eligible to have children living away from home.

iii. All mothers 35 and older were assumed to have no away-from-home children older than the number of years since their marriage.

iv. The number of children over 18 living away from home were assumed to be equal to the number of years since last marriage minus 18, but women were assumed *never more than 3 living children including those at home.*

v. The number of children living at home was set equal to the number of persons in the family minus 2 for "married couples" and minus 1 in families in which the head was widowed, divorced or separated.

vi. For all families headed by a divorced, widowed or separated head, the number of children away and at home was calculated as though there were a wife present of the same age as the head. Further it was assumed that the marriage took place at the "wife's" age 20.

vii. When a family was assigned children living away from home, a shadow record for each such child was created in the file.

2. Treatment of asset composition—The SFCC file provides far more detail of the composition of family wealth than is needed in the model. Consequently, assets were compressed into two categories: (1) life insurance face value minus policy loans and (2) all other assets. All debts except life insurance policy loans (which were netted out of gross assets), were lumped together. The only reason for identifying life insurance as a separate category is the particular role that it plays in intergenerational transfers. Only the cash surrender value of the policy is appropriately considered a part of the assets of the living, but the event of death creates an asset equal to the face value less policy loans subject to claim by the insured's beneficiaries. (In reality the owner of a life insurance policy and the insured need not be the same person, but we have not tried to deal with this distinction in the model.)

The SFCC file identifies three classes of persons: (1) family head (always male in a family including a husband and wife), (2) wife of head, (3) other family members. The file identifies separately the values of the following assets and debts belonging to heads or wives:

1. Checking account balances.
2. U.S. savings bonds (face value).
3. Mortgage assets.
4. Nonmortgage loans to individuals.
5. Life insurance (face value).
6. Savings account balances (Includes amounts in savings and loan

associations, credit unions, commercial banks, mutual savings banks and other savings institutions not specifically identified.)

Each of the above assets held by other family members as a group was also identified. Other family members included both children and other persons living with the family. For our purposes we assumed that all other family members were children of the head and wife and that they shared equally in the ownership of all assets and debts designated as belonging to other family members.

The following assets were not reported in the file as belonging to a class of persons, but simply as family assets:

1. Treasury bills (par value).
2. Treasury notes (par value).
3. Treasury certificates (par value).
4. Treasury bonds (par value).
5. State and local bonds (par value).
6. Foreign government and corporation bonds (par value).
7. Domestic corporation bonds (par value).
8. Loans to businesses.
9. Corporate stock (market value).
10. Value of business assets (book).
11. Loans to business n.e.c.
12. Withdrawable amounts in profit sharing plans.
13. Value of family's residences.
14. Value of investment real estate.
15. Net value of brokerage accounts.
16. Automobile value (market).
17. Oil royalties, patents and commodity contracts.

Lacking any data with which reliable estimates of the relative shares of total family assets held by husbands and wives could be made, we summed the values of the above 17 categories and arbitrarily assigned 65 percent of the sum to the head and 35 percent to the wife, if present, or allocated them entirely to the family head when no wife was present. This procedure reduces some of the variation in relative shares of total family wealth held among husbands and wives and increases each of their shares relative to that of other family members. Since the value of assets held by other family members is relatively very small when compared to that held by the head and wife, the latter adjustment appears to be of little consequence.

Loans against life insurance policies were identified in the original file as obligations of the head, wife, or other family members as a group. The same treatment was applied to these debts in our model as was given assets similarly identified.

The following were designated family debts in the original file:

1. Loans secured by stock (other than margin accounts).
2. Loans secured by bonds.
3. Installment debt.
4. Noninstallment debt n.e.c.

The same treatment was accorded these debt items as was accorded family assets.

3. Pareto adjustment of upper tail of SFCC wealth distribution—The Survey of Financial Characteristics file is a stratified sample of U.S. noninstitutional population on December 31, 1962. High income families were relatively overselected in the sample design, a feature particularly appropriate to our use of the file. Nevertheless, it is extremely difficult for a sample survey to capture the elongated upper tail of the wealth distribution. In practice what happens is that one ends up with a sample of a truncated tail. Because the tail of the distribution is of critical importance in a model of death transfers, we fitted Pareto function to the weighted observations of families with net worth over $25,000 and spread the 57 (unweighted) richest families out across the function retaining their original weights so they continued to represent the same proportion of the total number of U.S. families as in the original file, but they were assigned the mid-point of the net worth interval in which they fell on the net worth argument of the function.

The Pareto function, $P_{nw} = b(NW)^{-\alpha}$ was estimated to have the following parameter values: $b = 213.8$ and $\alpha = 0.74$, where P_{nw} is the percent of the population with net worth (measured in 1000s) in excess of NW. Once the parameters of the function were estimated it was possible to derive the value of NW which is exceeded by any proportion of the population.

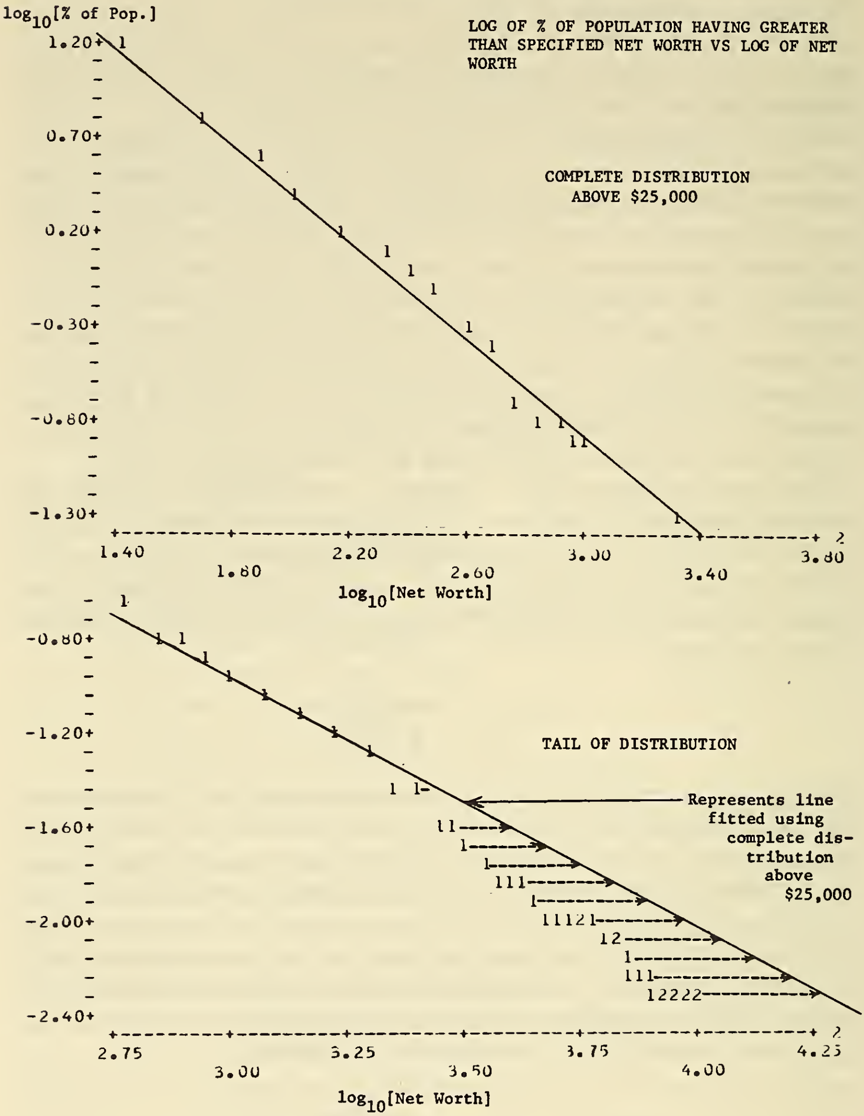
$$NW = \sqrt[-\alpha]{\frac{P_{nw}}{b}}$$

The sum of the weights in the file are equal to the 1963 population of families. If the distribution of wealth were Paretian and the sample were adequate, we would expect the observations on net worth to follow the estimated values of the function. The richest 57 cases in the file had a combined weight of 23,501. This represented 0.0406 percent of the total sum of weights, and hence the lower end of the interval in which the entire 57 cases fell was estimated as $2.24 million. Taking each of the cases and sequentially cumulating their weights, the lower limit of the interval on the Pareto function was calculated.

$$NW_i = \sqrt[-\alpha]{\frac{1 - Z_n - \sum_{i=n+1}^{m} \gamma_m / \Sigma \gamma}{b}}$$

where Z_n is the ratio of the sum of the weights of the first 2500 cases to the total sum of weights $\sum \gamma$. i runs from 2501 to 2557 (the 57 richest cases). The midpoint of the interval within which each case fell was substituted for the reported net worth of the case, and all the asset values of the cases were adjusted in accordance with the ratio of the assigned/original net worth values. Chart 1 and Table 18 show the details of this adjustment.

Chart 1.—*Log of % of Population Having Greater Than Specified Net Worth vs. Log of Net Worth*



4. Adjustment of SFCC wealth to national balance sheet totals—It is characteristic of field surveys that they underestimate the value of assets held in the society. In the case of the SFCC file consumer durables were not measured and had to be assigned using 1962 balance sheet figures. The value of consumer durables in 1962 was $150.3 billion. This value was distributed among families as follows: One-half

of the $150.3 billion was distributed in proportion to all other assets. The other half was distributed in equal shares to each family.

The value of assets other than consumer durables reported in the SFCC file was $1.41 trillion, while that reported in national balance sheets was $1.92 trillion. The Pareto function adjustment, described above, brought the SFCC total to $1.45 trillion. The difference, $510 billion, was assigned to families (heads and wives) in the same proportion as reported assets were held.

TABLE 18.—*Adjustment of upper tail to conform to Pareto function, 1962 SFCC file* $000

	Original value	Predicted value		Original value	Predicted value
1.	23,438	180,504	51.	2,433	2,569
2.	14,862	77,327	52.	2,381	2,406
3.	14,491	53,647	53.	2,293	2,392
4.	10,084	11,729	54.	2,254	2,361
5.	9,846	11,115	55.	2,249	2,352
6.	7,943	10,625	56.	2,225	2,319
7.	7,690	10,285	57.	2,221	2,308
8.	7,378	9,318	58.	2,198	2,165
9.	7,098	8,797	59.	2,183	2,156
10.	6,951	8,236	60.	2,178	2,143
11.	6,923	7,722	61.	2,158	2,133
12.	6,714	7,563	62.	2,120	2,112
13.	6,610	7,418	63.	2,120	2,102
14.	6,380	7,262	64.	2,101	2,086
15.	6,013	6,976	65.	2,060	2,075
16.	5,503	6,807	66.	2,060	1,980
17.	5,010	6,694	67.	2,057	1,975
18.	4,886	6,330	68.	2,024	1,964
19.	4,673	6,099	69.	1,986	1,954
20.	4,667	6,016	70.	1,967	1,947
21.	4,389	5,184	71.	1,934	1,928
22.	4,362	5,051	72.	1,892	1,921
23.	4,266	4,981	73.	1,871	1,912
24.	4,228	4,914	74.	1,866	1,904
25.	4,163	4,819	75.	1,854	1,898
26.	4,139	4,723	76.	1,842	1,888
27.	3,903	4,672	77.	1,825	1,881
28.	3,574	4,608	78.	1,811	1,875
29.	3,571	4,552	79.	1,788	1,789
30.	3,450	4,492	80.	1,788	1,783
31.	3,401	4,433	81.	1,783	1,776
32.	3,351	4,390	82.	1,762	1,770
33.	3,341	3,963	83.	1,755	1,756
34.	3,228	3,851	84.	1,729	1,744
35.	3,143	3,818	85.	1,729	1,737
36.	3,112	3,787	86.	1,726	1,731
37.	3,073	3,688	87.	1,663	1,726
38.	3,054	3,657	88.	1,658	1,710
39.	3,010	3,560	89.	1,629	1,705
40.	3,005	3,190	90.	1,611	1,688
41.	2,870	3,163	91.	1,597	1,682
42.	2,851	3,139	92.	1,585	1,611
43.	2,738	3,068	93.	1,573	1,605
44.	2,718	3,046	94.	1,572	1,591
45.	2,695	2,993	95.	1,567	1,580
46.	2,598	2,740	96.	1,556	1,574
47.	2,591	2,699	97.	1,556	1,561
48.	2,533	2,649	98.	1,548	1,547
49.	2,486	2,626	99.	1,546	1,534
50.	2,454	2,611	100.	1,483	1,529

Simulation Procedures

Chart 2 is a schematic of the overall simulation model. The following procedures were carried out in the simulation process.

1. Replication of file—The SFCC is a small sample, 2,557 cases. To overcome such Monte Carlo variability, all cases in the file with net worth under $2,000,000 were replicated 10 times and all cases with a net

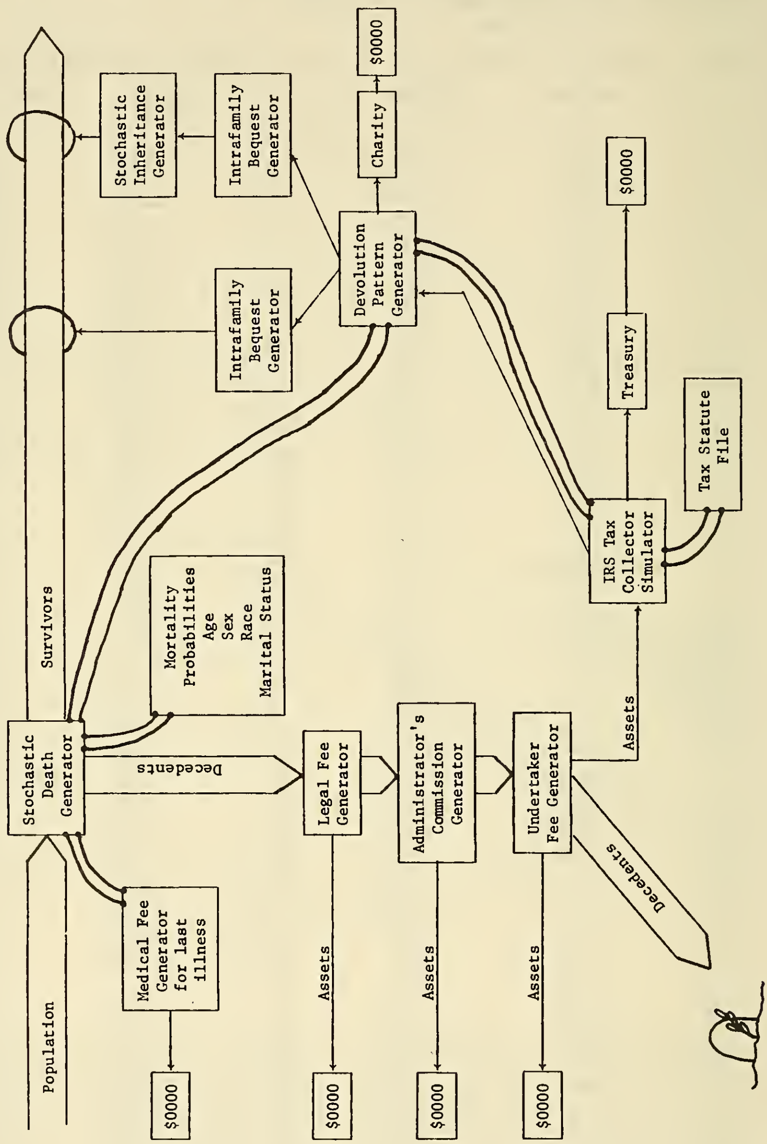


Chart 2.—*Generalized Schematic of Simulation Model*

worth of $2,000,000 or more were replicated 100 times and the weights were adjusted accordingly.

2. Mortality probabilities and death—The modified SFCC file was passed and each person's record was interrogated by the Stochastic Death Generator to determine the age, sex, race, and marital status of the person. From a set of 1962 age-sex-race-marital status-specific mortality rates the probability of death was determined. A Monte Carlo draw and the ascertained probability determined if the person died or lived. If the person was to live, the next person's record in the family was interrogated. If it was determined a person was to die, death was effected immediately and his family's record was reconstructed to reflect only the surviving members. If more than one death took place in a family, the family record was reconstructed after the last death. Following the death of a family member as much of the economic process of dying and transferring wealth was captured as data permitted.

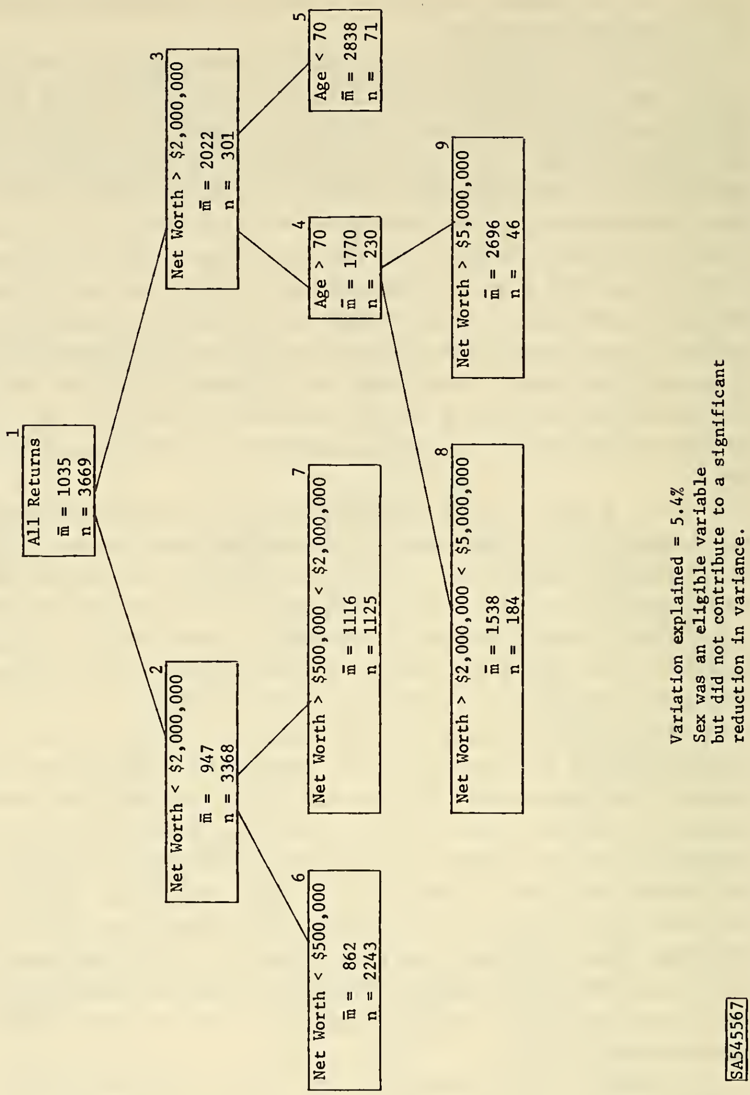
3. Cost of last illness—Nearly all deaths impose medical costs on the estates of the decedents. In cases where there is a terminal illness of prolonged length, the medical costs may be substantial. The deductibility of these costs for purposes of calculating taxable estate on the Federal estate tax return provided a data base to estimate the relation of the cost of last illness to other characteristics of decedents. The cost of last illness was analyzed using AID-III.[9]

In chart 3 the result of the AID analysis is shown. The five final groups explain 5.4 percent of the variance in the cost of terminal illnesses as reported on Federal estate tax returns. One would not expect to explain a great deal of the variance with the variables available to us, but there is a systematic, positive relationship between net worth and cost of last illness. The only other variable which contributed significantly to reducing the original variance was age of decedent. Thus only these two characteristics of decedents were used in the attribution of last illness costs. The actual attribution of the cost was unsophisticated, the expected value was assigned within each characteristic class.

4. Attorneys' fees—Attorneys' fees are a deductible item in the Federal estate tax. Consequently, they are available from the estate tax return. Using AID to split the population into groups such that a regression of attorneys' fees on gross estate within groups would produce

[9] AID-III is a data searching algorithm which sequentially splits a population into pairs such that the sum of the variance around the mean of the pair or the expected value of a regression is the smallest possible proportion of the variance around the expected values of the group from which the pair was derived. The technique has the advantage over regression in not requiring an additive set of independent variables. It also imposes no linearity restrictions on relations between variables. For a detailed discussion of AID-III see, Sonquest, Baker and Morgan, *Searching for Structure*, Institute for Social Research, Ann Arbor, Michigan, 1971.

Chart 3.—*Medical Expenses of Last Illness*
$\bar{m}$ = mean cost in dollars



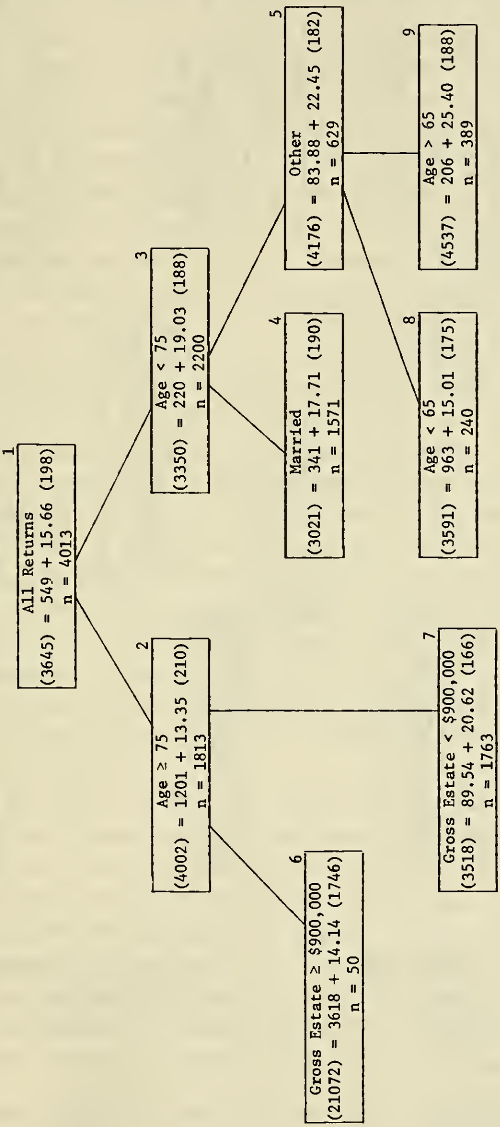
the greatest reduction of variance relative to a regression on the total set of observations, 51.1 percent of the variance was explained. Age and marital status of decedent were the only other variables which were able to provide a basis for splitting the population with a significant reduction in variance. In chart 4 it can be seen that a simple regression of attorneys' fees on gross assets (measured in thousands of dollars) would produce coefficients of a = \$549, b = 15.66. The predicted value \$3,645 is the expected attorney's fee when the mean value of the group's gross assest (\$198,000) is plugged into the equation.

5. Executors' fees—The cost of executors' fees was estimated using two regression equations and data from the 1962 Federal estate tax file.

$$\text{EXCOM} + = a + b_1\,(\text{NETWORTH}) + b_2(\text{MS1}) + b_3(\text{MS2}) + b_4(\text{MS3}), \text{ where}$$

net worth is measured in $1,000s, MS1 is a dummy for married decedents, MS2 is a dummy for never married decedents and MS3 is a dummy for all other marital status. The equation was fitted separately for decedents with net worth under $200,000 and those with net worth of $200,000 or more. The estimated coefficients for the two equations are:

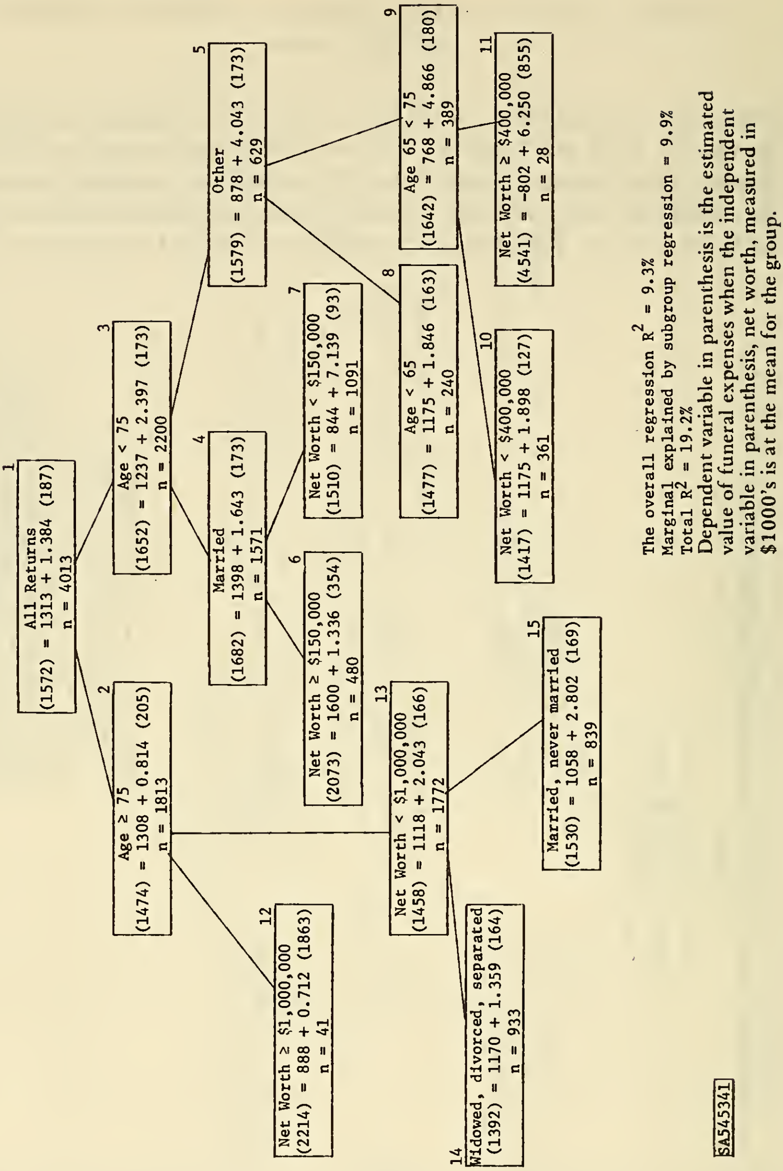
Chart 4.—*Attorneys' Fees AID with Regression on Gross Estate (dollars)*



The overall regression R^2 = 46.8%
Marginal variance explained by subgroup regression = 4.3%
Total R^2 = 51.1%
Sex was also an eligible variable, but could not produce a significant reduction in variance.
The predicted value of the equation in each group is the value of attorneys' fees estimated when gross estate measured in 1000's was at its mean for the group.

SA545575

Chart 5.—*Funeral Expenses AID with Regression on Net Worth*
(dollars)



	Net Worth < \$200,000	Net Worth ≥ \$200,000
a	\$172.50	\$2,517.80
b_1	14.8	17.3
b_2	–843.5	-3,575.0
b_3	575.5	4,223.4
b_4	268.0	–648.1
	R_2 = 17.9	R_2 = 32.2

6. Funeral expenses—In the simulation they are attributed to decedents' estates on the basis of 8 regression equations fitted in the process of an AID run on the 1962 estate tax file. The combined splitting of the population into eight final groups and the simple regression of funeral expenses on net worth within each final group explained 19.2 percent of the variance of funeral expenses. In Chart 5 we show the results of the AID run with group regressions.

In some cases the total costs of dying exceeds the assets of the decedent. This is frequently the case with children. Although they will not generally incur legal or administration fees of any significance, the cost of last illness and funeral will diminish estates of children as well as of adults. Whether a child or an adult, the cost of last illness, administration fees, lawyers' fees and funeral expenses are all deducted from their estates in accordance with the AID analyses above. When these costs result in a negative estate, it is transferred to the decedent's heirs in the same manner as a positive valued estate. This conceptualization is consistent with the actual process of cost bearing for decedents.

7. Bequest patterns—Little information is available about the pattern of transfers set in motion by death. Data available from the IRS identify amounts going to charity and in some years to spouse, but no information about the division of bequests between members of the decedent's family living at home or for that matter the total amount remaining in the decendent's family vs. that going to noncharitable legatees outside the immediate family. In order to estimate the pattern of estate distribution, a file constructed by Smith from estate tax returns filed in the District of Columbia in 1969 was used.[10] The statutes of the District of Columbia require an estate tax return to be filed for the estates of all decedents with gross assets of $1,000 or more. Thus the file provided nearly the complete range of estate sizes. Further, the file was constructed to provide information about the distribution of assets among spouse, children, other relatives, nonrelatives and charities (including gifts to governments). The processes of estimating the pattern of estate distribution in a form suitable for Monte Carlo applications is depicted in chart 6.

The first step was to use AID to estimate the probability that a bequest was made outside the family. A family is defined for this purpose as a head, wife and children wherever living. All never married persons were excluded from the estimation on the grounds that we would follow the arbitrary rule that never married persons had neither spouse nor children and that all their wealth would flow outside the family as defined. The results of the AID analysis is shown in chart 7. The combination of being married and having a net worth of under $100,000 resulted in a probability of 0.142 of making a bequest outside

[10] James D. Smith, "White Wealth and Black People: The Distribution of Wealth in Washington, D.C., 1967," in James D. Smith, *Personal Distributions of Income and Wealth*, National Bureau of Economic Research.

Chart 6.—*Estimation Sequence for Identifying Patterns of Wealth Flows at Death*

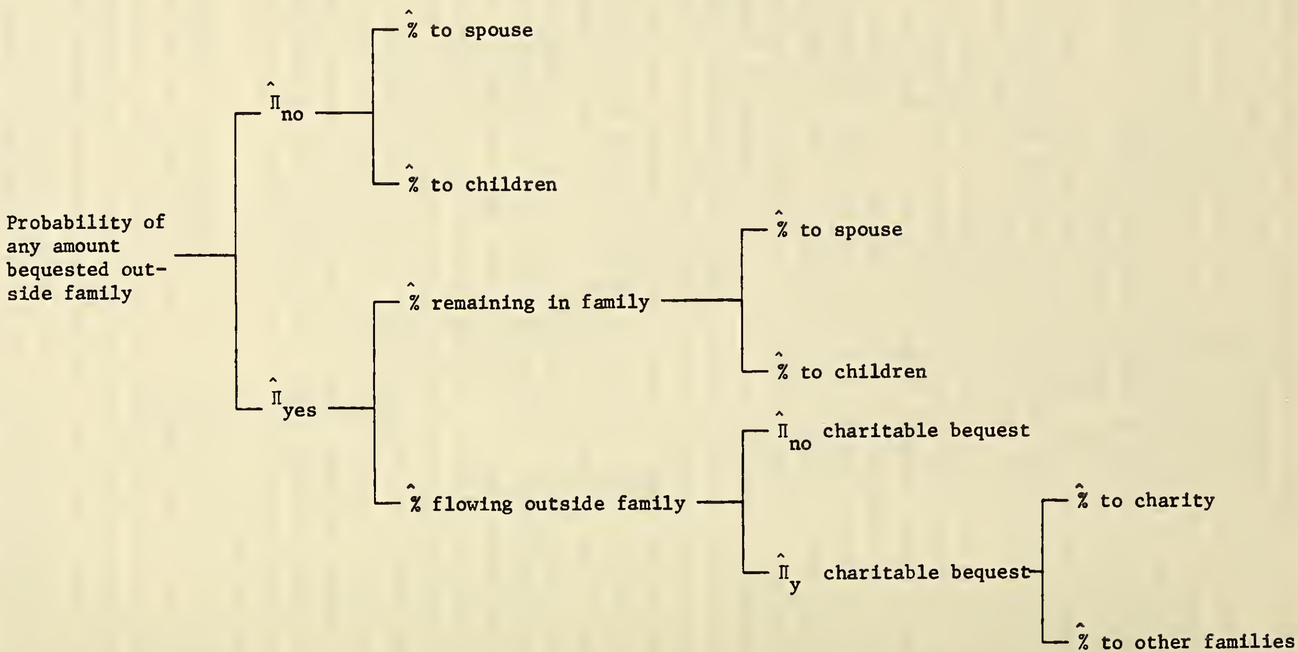
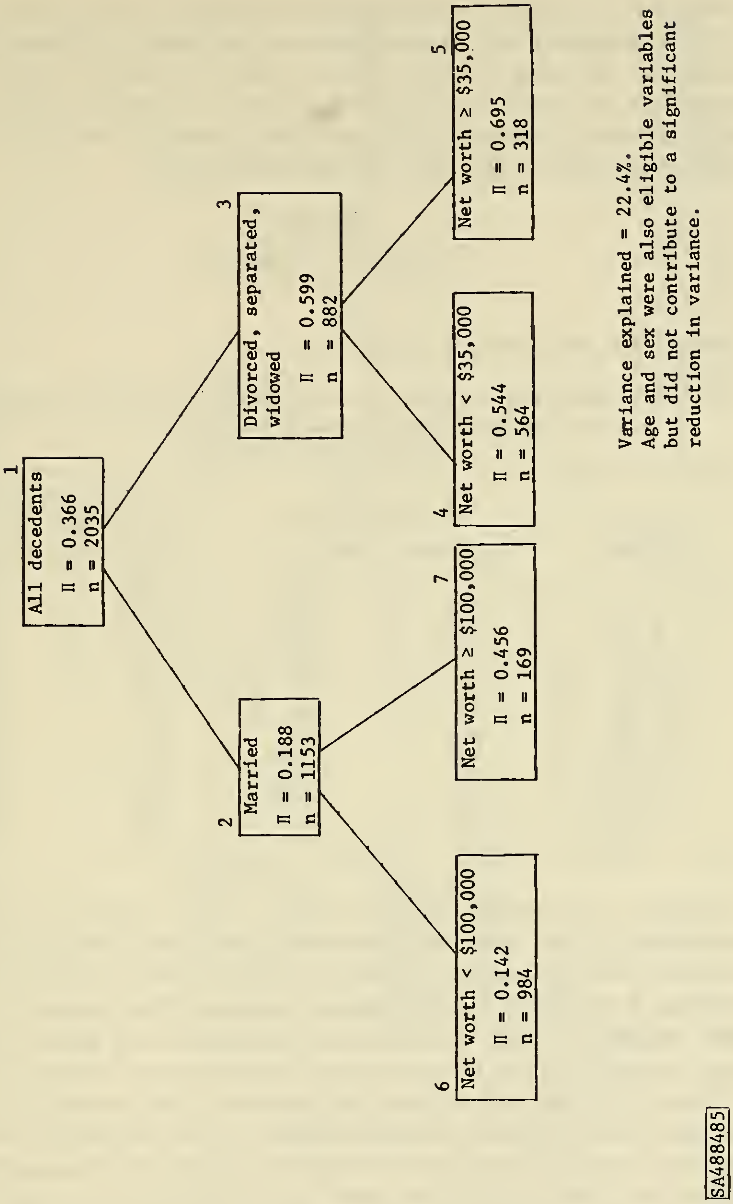


Chart 7.—Probability of Making a Bequest Outside Family



the family. If a decedent were not married and had $35,000 or more, on the other hand, the probability of making an outside bequest was 0.695. The four end groups shown in chart 7 accounted for 22.4 percent of the variance around the mean probability of outside bequests for all decedents.

In order to estimate the mean share of assets going to spouse a sample of 1,090 decedents who were residents of the District of Columbia and married at the time of their death in 1967, was used to calculate the proportion that the value of assets passing to spouse was of the total value of assets passing to children plus spouse, by sex and value of estate. The mean of the ratio was also calculated:

$$\frac{\sum_{i=1}^{n} \frac{\text{spouse}_i}{\text{spouse}_i + \text{children}_i}}{n}$$

These values are shown in table 19.

TABLE 19.—*Wealth bequeathed to spouse as a percent of total wealth bequeathed to spouse and children by sex and value of total net estate of decedents, Washington, D.C., 1967*

Value of net estate $000	Mean of ratio—Spouse / Spouse + Children		Ratio of aggregates of wealth Σ Spouse / Σ Spouse + Children	
	Males	Females	Males	Females
<5	86.7	79.0	85.7	82.7
5 <10	90.4	87.3	90.9	86.1
10 <15	91.7	82.3	92.1	82.9
15 <20	90.6	86.3	90.9	86.9
20 <25	95.5	79.9	95.1	80.5
25 <30	90.7	90.9	91.0	89.7
30 <35	97.7	83.4	97.7	82.9
35 <50	96.0	81.3	96.3	81.6
50 <75	82.9	75.2	82.3	75.6
75 <100	91.0	66.6	90.9	63.0
100 <250	79.4	59.0	79.8	55.8
≥250	72.1	52.7	75.3	41.8

Females consistently transfer less to their surviving spouse, as proportion of total transfers to spouse and children, than do males and the distribution of estates between wives and children shifts in favor of children as the total value of estates increase. The observed patterns with respect to value of estate are consistent with estate planning strategies to minimize repeated taxation of the same bundle of wealth.

Given that we had a basis for predicting the probability that a bequest would be made outside the family, it was necessary to estimate the share of one's estate which would flow outside (or conversely remain inside) the family. AID was used again, this time to estimate the proportion of a decedent's assets which were bequeathed outside the family if any outside bequests were made. See chart 8.

As might be expected married decedents give a smaller share outside their family than do decedents who are other than married. The proportion of one's assets given beyond the bonds of the family decreases with increased wealth, but the absolute amount most likely increases given the relative small change in the percentage increase associated with increased wealth.

Chart 8. – *Percent of Decedent's Assets Bequeathed Outside Family for All Decedents Making Outside Bequests*

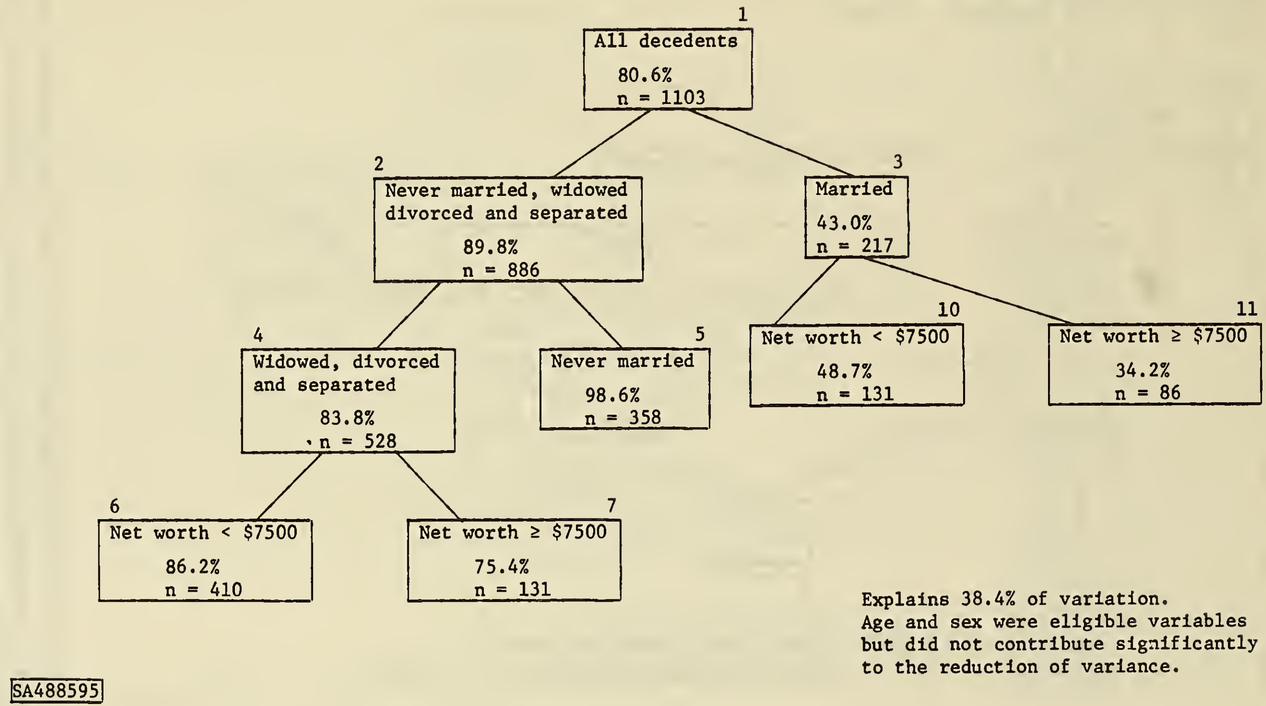
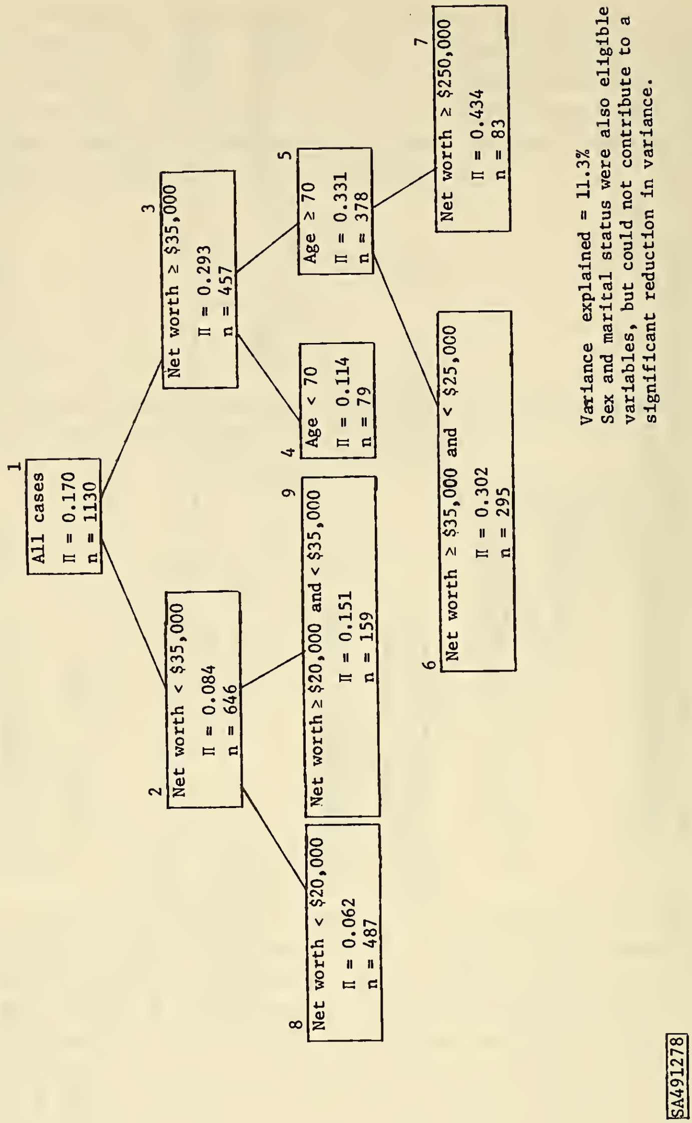


Chart 9. — *Probability that Bequest is Made to Charity or Government Given That Some Wealth Was Bequeathed Outside Family*



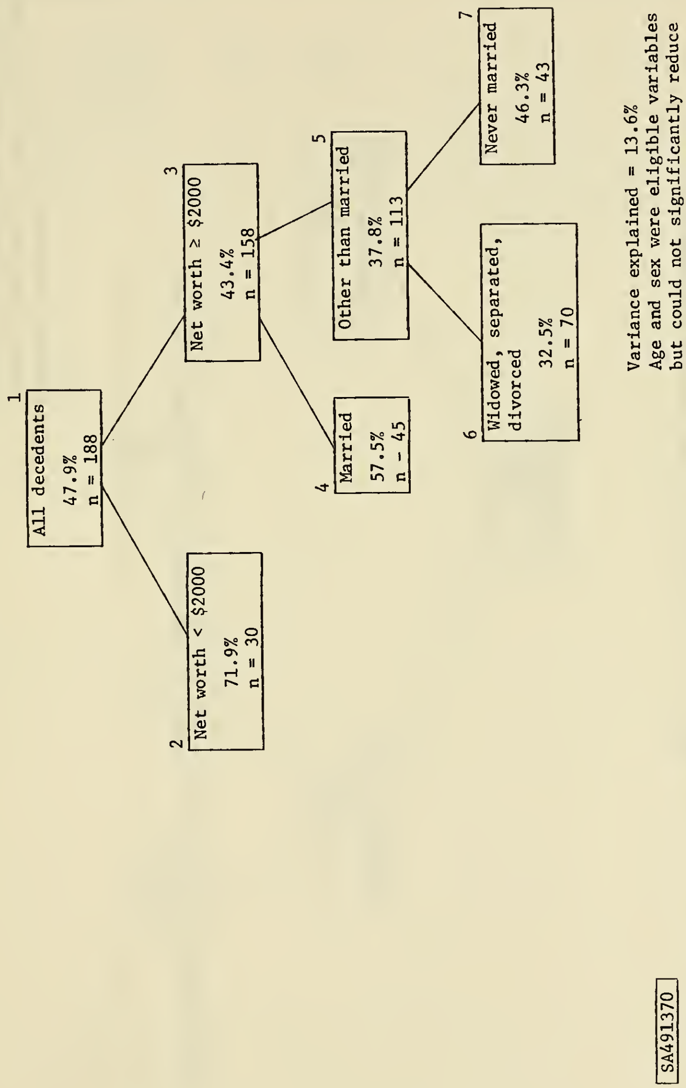
We next estimated the probability that those decedents transferring wealth outside their family made a bequest to charities or governments. The AID tree shown in chart 9 explained 11.3 percent of the variance in the probability that a charitable transfer took place. The second step in the estimation of charitable bequests was to estimate the share of assets leaving the family which went to charities (including gifts to government) for all decedents making charitable bequests. We were able to explain 13.6 percent in the variance in the share of wealth flowing away

from decedents which went to charities. The AID tree of chart 10 shows the result of the AID analysis.

Estimation of inheritances from outside family unit

The sum of inheritances and gifts received by 113 families are contained in the SCFF file. The ratio of inheritances generated by estates over $60,000 to the value of gifts greater than $3,000 reported in *Statistics of Income, 1962,* is about seven to one. On this basis, the

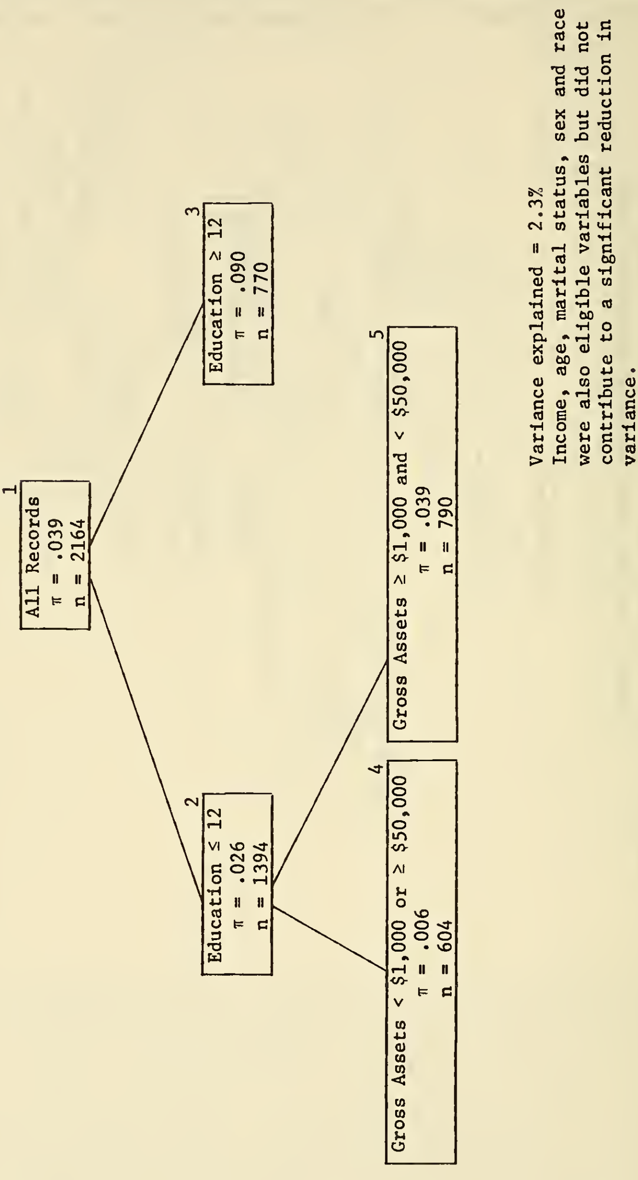
Chart 10. — *Percent of Estate Leaving Family Which is Bequeathed to Charity * for Decedents Making Bequests to Charity*



*Includes donations to government

value of gifts and inheritances in the SCFF file was treated as inheritances only. Estimation of an inheritance pattern was done in two stages. First, the probability that a family will receive an inheritance during the simulated year was estimated using AID. The results of the AID analysis shown in chart 11 indicate that inheritance from outside the family unit cannot be predicted very well ($R^2 = 2.3\%$) with the information that was available. For the simulation, each family was

Chart 11.—*Probability of Receiving an Inheritance*



assigned an inheritance probability π equal to one of four values in the final AID groups. The second stage was to estimate an equation to predict the size of inheritance using the 113 SCFF families which recieved an inheritance. The resulting equation has the form:

$$\log_e (\text{INHERITANCE}) = a + b_1 (\text{GROSS ESTATE}) + b_2 (\text{AGE}) + b_3 (\text{ED})$$

where GROSS ESTATE is measured in \$1,000's, AGE and ED are the age and number of years of education of the head respectively. The estimated coefficients are:

$$\begin{aligned} a &= 3.58 \\ b_1 &= 0.0007 \qquad R^2 = 21.9\% \\ b_2 &= 0.027 \\ b_3 &= 0.17 \end{aligned}$$

Tax Revenue and Economic Effects of Tax-Exempt State and Local Bonds: A General Equilibrium Approach to Tax Policy Analysis

Robert M. Spann

Department of Economics,
Virginia Polytechnic Institute and State University

Tax Revenue and Economic Effects of Tax-Exempt State and Local Bonds: A General Equilibrium Approach to Tax Policy Analysis

Robert M. Spann*

Department of Economics, Virginia Polytechnic Institute and State University

Abstract

In this report, a general equilibrium model of tax policy analysis is developed in which all input prices, output prices, factor utilization, and output (including the governmental sector) are endogenous variables. Such a general equilibrium model indicates that the marginal costs of providing governmental services are not independent of the tax policies chosen by the Federal Government. Alternative tax policies affect the marginal costs of Federal Government services, and in the long run, the quantity of such services demanded by voters.

The aspect of tax policy examined in this report is the effects of eliminating tax-exempt state and local bonds. The general equilibrium model developed here indicates that elimination of tax-exempt State and local bonds would reduce the cost of capital† to the Federal sector by .7–1.9 percent, reduce the marginal cost of Federal Government output by .3–.4 percent, and increase the demand for output of the Federal sector by .1 to .3 percent.

Other tax policies can be analyzed within the framework developed in this paper as well. Tax policies discussed in this paper in addition to elimination of State and local bonds include elimination of the personal income tax, elimination of the corporate income tax, and alternative personal income tax rates. In the final section of the paper, some additional extensions of the model are suggested.

The general equilibrium model used in this paper is, of course, subject to a number of important qualifications. The U.S. economy consists of numerous sectors and is subject to a relatively complex tax code. The model developed here is a four sector model—the four sectors

---

* The research on which this paper is based was supported by U.S. Treasury Contract No. OS-705. The author is indebted to G. Tolley, H. Galper, G. Robbins and O. Brownlee for helpful suggestions on earlier drafts. Responsibility for errors, opinions and omissions rests with the author.

† The cost of capital in this model is the rental rate on capital, which would include interest, taxes, depreciation, etc., not merely the interest rate on government bonds.

being Federal Government, State and local government, business and privately owned housing and consumer durables—and applies numerous simplifying assumptions to the U.S. tax code. In addition, the model is a long run equilibrium model and does not include the short run effects of tax policies or an explicit adjustment mechanism which traces out intermediate or short run effects of tax policies. Thus, the model developed in this paper should be treated as a framework for long-run equilibrium analysis, illustrating the general equilibrium nature of tax policy effects rather than a detailed or precise representation of the economy.‡

Even though the model does not provide a detailed representation of the economy as a whole, it does illustrate a number of important points. The model develops the relationships between changes in tax policy and changes in all sectors, including the demand for government services. In addition, the model illustrates the effects of tax policies in various factor markets, including the effect of Government-owned capital.

I. Introduction

The majority of tax policy or tax incidence analysis is partial equilibrium in nature. One generally examines the effects of tax policies on factor prices, output, and factor utilization in various components of the private sector, holding total Government spending or output constant, or one analyzes the effects of various tax policies on Government revenues and output, ignoring the effects of tax policies on the allocation of resources in the private sectors of the economy. Even studies which attempt to combine both the effects of tax policy on Government revenues and factor prices generally do not include the effect of tax policies on the cost of government services and hence the amount of Government services which will be demanded.[1]

Tax-exempt state and local bonds provide an excellent example of this point. The tax exemption for interest on State and local government bonds lowers the cost of borrowing to State and local governments

‡ In Appendix I of this report, the simplifying assumptions employed and the caveats that should be placed on the model as a result of those assumptions are discussed in detail.

[1] The analysis presented below is similar in many ways to Harberger's (1962) classic analysis of the taxation of corporate income. In that paper, Harberger utilized a general equilibrium model in which rates of return on capital were equated between taxed and nontaxed industries in the private sector. Changes in taxes on corporate capital lead to flows of capital between the taxed and nontaxed sectors. In this paper I assume that changes in tax policy not only lead to reallocations of capital within the private sector, but reallocations of capital between the public and private sectors. This distinction is not negligible when one realizes that the public sector is probably more capital intensive than the private sector [Orzechowski (1974)]. In 1973, 23% of total debt was public debt [*Economic Report of the President* (1975)] and in 1966, approximately 25% of all capital was owned by the public sector (see below).

and raises the cost of borrowing to the Federal Government. Initially, such a tax policy would have two effects:[2]

(1) Since the tax exemption lowers the cost of State and local services relative to Federal Government services, it leads to a substitution of State and local government services for Federal services.

(2) Since the tax exemption lowers the cost of borrowing to State and local governments, it leads State and local governments to choose more capital-intensive projects (or more capital intensive methods of producing a given good or service).

The effects of tax-exempt State and local bonds are not limited to the public (Federal, State and local government) sectors of the economy. To the extent that the tax exemption and resulting lower cost of capital to State and local governments acts as a substitute for local property taxes, tax exemptions for interest paid on State and local bonds raises the demand for privately owned housing. To the extent that the tax exemption granted for interest paid on state and local government bonds acts as a substitute for State and local sales taxes, such a tax exemption alters the demand for products subject to sales taxation. Since the tax revenues to the Federal sector are altered by granting an exemption for interest paid on state and local bonds, other Federal tax rates are altered. This alteration in other Federal tax rates may lead to changes in input prices and hence input usage and output prices to the private sector of the economy.

The tax policy effects described above are best represented in a general equilibrium model in which input prices, input usage, outputs, and output prices (or marginal costs in the case of the public sector) are treated as endogenous variables.

This same general equilibrium framework used to discuss the effects of tax-exempt bonds can be used to estimate the effects of other tax policies as well. Other policies discussed in this report include the effects of elimination of either the personal income tax or corporate taxation and the effects of alternative personal or corporate tax rates.[3] The model also indicates that the effects of eliminating a tax privilege, such as tax-exempt bonds, are not independent of which tax rates are

[2] The analysis of the tax exemption granted interest paid on State and local bonds centers primarily on the effects of such a tax exemption on the allocation of resources between State and local governments and the Federal sector and the allocation of resources between the public and private sectors. The majority of previous studies of the effects of tax-exempt State and local bonds have focused on the effects of this tax privilege on the interest rates paid by the Federal Government and by State and local governments.

[3] In the model presented below, all corporate taxes and business taxes (both the corporate income tax and indirect business taxes) are included as a single tax on corporate or business capital. Thus the estimates concerning corporate taxation are not completely analogous to estimates of the effects of the corporate income tax, holding indirect business tax rates constant.

altered in order to make up the revenue surplus or deficit which results from changes in tax preferences.

In what follows, I first discuss, in general terms, the effects of elimination of a tax privilege or an alteration in tax rates on the various public and private sectors of the economy. I then describe a general equilibrium model that can be used to simulate these effects and report the results of simulating the effects of the elimination of tax-exempt bonds and the effects of various alterations in tax policy for both 1966 and 1973.[4] The final section of the paper discusses the policy implications of the model and possible extensions of the model developed in this report.

II. Tax Incidence Within a General Equilibrium Model

In order to examine tax incidence within a general equilibrium framework, first consider the case of tax-exempt State and local bonds. Individual investors are interested in the after-tax rate of return earned on funds loaned to either Federal or State and local governments. If individuals are willing to invest in Federal securities yielding an interest rate r, and if they view Federal securities and State and local securities as equally risky,[5] they would be willing to buy State and local bonds at an interest rate of $(1\text{-}\gamma)r$ where γ is the personal income tax rate.

Assume that the tax exemption granted state and local bonds is removed. Since it is assumed that individuals view Federal securities as perfect substitutes for State and local bonds, the interest rate paid by State and local governments will now be equal to the interest rate paid by the Federal Government. Assume that the new equilibrium interest rate for Federal, State and local bonds is r^*, where

$$(1\text{-}\gamma)r < r^* < r.$$

The initial effect of eliminating the tax exemption granted State and local bonds is to lower the cost of capital to the Federal Government and to raise the cost of capital to State and local governments. Such a change in factor prices would have two effects. The increase in the cost of capital (at any given wage rate) to the State and local sector will lead State and local governments to use less capital relative to labor in

---

[4] 1966 is the most recent year for which accurate data on capital utilized in the public sector is available. The 1973 data is an updated version of the 1966 data. Thus, both years are included in this report.

[5] State and local debt is not viewed as equally risky by investors. The assumption of equal riskiness is made to simplify the calculations made below. Inclusion of risk premia on various securities would probably not significantly alter the results of the model. In addition, I shall be concerned with the rental rate on capital, rather than interest rates alone, so the assumption of equal riskiness is dampened somewhat in the actual simulations. The rental rate on capital is total returns to capital including interest, depreciation, taxes, etc.

producing State and local government services. In the Federal sector, where the cost of capital has fallen, capital will be substituted for labor. In addition, the cost of providing Federal services has fallen (due to the reduced cost of capital to the Federal sector), whereas the cost of providing State and local government services has risen (due to the increase in the cost of capital to the State and local sector). One would expect that an increase in the cost of State and local services would lead voters to desire fewer State and local government services. The decrease in the cost of providing Federal services would lead voters to desire more Federal services. These output effects would lead to a decrease in the usage of both capital and labor in the State and local sector, an increase in the usage of both capital and labor in the State and local sector, and an increase in the usage of both capital and labor in the Federal sector.[6]

Elimination of tax-exempt State and local bonds would affect other sectors of the economy as well. Two effects are of relevance here: possible changes in factor prices and changes in other taxes in order to make up for revenue deficiencies or surpluses created in the Federal, State and local sectors as a result of the elimination of a tax preference. Consider the effects of factor price changes first. Corporate bonds or equities and investments in real, personal assets (such as residential housing, consumer durables, etc.) are viewed as substitutes for Federal, State and local bonds on the part of investors. The lower interest rate on Federal bonds would lower the interest rate on corporate securities as well. This would lead to an expansion of the corporate sector capital demands as the cost of capital to the corporate sector was lowered. Similarly, the reduction in the interest rate on Federal securities would lower the opportunity cost of investing in capital assets such as residential structures, consumer durables, etc., and lead to increased demand for those assets.

In addition to changing factor prices, elimination of tax-exempt state and local bonds must change other tax rates. If all tax rates remained unchanged, the taxation of a previously tax-exempt source of income (interest on State and local bonds) will lead to a revenue surplus in the Federal sector. Elimination of the tax exemption granted interest paid on State and local bonds raises the costs of providing State and local services; thus state and local governments, at any given level of output, will have to increase taxes to have sufficient tax revenues to pay for services provided.[7]

---

[6] The arguments presented here concerning resource flows between the State and local sector and Federal sector due to differential State and local taxation is analogous to the type of resource flows between taxed and nontaxed industries that result from differential taxation of corporate versus noncorporate income in a Harberger (1962) type model.

[7] Throughout this paper I assume that both the Federal budget and the State and local government budgets must be balanced. Government expenditures (defined to be the sum of payments to capital utilized by the Government and payments to Government employees) must equal tax revenues collected by the Government.

These tax changes will affect the demand for various private sector outputs and factor usage in the private sector. For example, if the State and local property taxes are increased, the demand for residential structures is reduced. Alternatively, if the State and local sales taxes are increased, the demand for private sector outputs is reduced since at least some part of the sales tax is passed on in the form of higher prices to consumers. The Federal Government can either reduce income taxes or reduce corporate taxes in order to compensate for the increased revenues raised by taxing interest on state and local bonds. A decrease in the corporate tax rate would lower the cost of capital to the corporate sector (since corporations make decisions based on the cost of funds to them, including corporate taxes which are essentially taxes on corporate equity capital), leading to increased capital-labor ratios in that sector.[8] This would not be the case if the personal income tax were lowered in order to compensate for the increased revenues to the Federal sector generated by State and local bonds.

The effects described above are the "first round" effects of eliminating the tax exemption granted interest paid on State and local bonds. Equilibrium would require that all factor and product markets be cleared (i.e., no excess demand for any product or input). If the increase in Federal and private sector capital usage described above exceeds the decrease in capital usage in the State and local government sector, the interest rate or the rental rate on capital would have to rise until there was no excess demand for capital (and vice versa).[9] The long run result would be similar to the effects described above as "first round effects."

The second factor not explicitly discussed above is that the marginal cost of various Government services depends on the tax structure chosen by the Government. Thus, as tax rates change, the marginal cost of various Government services change, leading to changes in the demand for Government services and input usage in the Government sector. This point is discussed in more detail below.

Tax Structure and the Marginal Cost of Government

The general equilibrium model described above assumed that Government output was a product produced and consumed much the same as the output of other sectors. The only additional requirement placed on the Government sector was the total revenues raised from taxes must equal total expenditures since there is no explicit unit price charged for Government services. Rather, voters pay for those services via various forms of taxes.

There is a demand curve for Government services much the same as there exists a demand curve for any private sector service. The higher

---

[8] See Harberger (1962).

[9] By Walras's Law, if the capital market clears at some set of factor prices, the labor market must clear as well. Alternatively, if there is excess demand in one factor market, there must be excess supply in another factor market.

the price of private sector services, the less the quantity demanded. A similar argument applies to Government goods and services. The greater the cost of those services, the less of those services demanded by voters, *certibus partibus*.[10] Government output is produced by combining capital and labor, purchased (or rented) from the owners of factors of production in much the same manner as the output of the private sector is produced.[11]

At any given level of Government output, the Government combines capital and labor (presumably in the most efficient manner) to produce the output of the Government sector. Thus, the total costs of Government output are a function of the level of Government output and factor or input prices. The total costs of producing Government output are raised via various forms of taxes. The type taxes chosen to raise the costs of providing Government services affects factor prices. Thus, changes in tax policies which alter factor prices change the cost of providing Government services. In summary, tax policies influence factor prices, which determine the cost of Government services. This implies that the cost of Government and the amount of Government services demanded are not independent of the manner in which the Government chooses to raise tax revenues.

This point can be illustrated by a simple example involving the corporate income tax. Suppose that capital is perfectly mobile between the public and private sectors and that investors are willing to purchase Government bonds or corporate equities at an interest rate r.[12] Labor is also perfectly mobile between sectors and both the private and public sectors face a wage rate w. The corporate tax is primarily a tax on capital (in particular, equity capital) in the corporate sector. Thus, the relevant cost of capital for Government decisionmakers choosing the least cost input combinations is the interest rate r. In the corporate sector, firms will make input choices based on the cost of capital adjusted for corporate income taxes or $r/(1\text{-}\tau)$ where τ is the rate of corporate taxation.

Suppose that the corporate income tax is eliminated. The cost of capital relevant to corporate decision makers now falls from $r/(1\text{-}\tau)$ to r. This reduction in the cost of capital to the corporate sector (before taxes) would lead to increases in the demand for capital on the part of

---

[10] For estimates of the elasticity of demand for Government services, see Borcherding and Deacon (1972) or Bergstrom and Goodman (1973).

[11] For simplicity I assume in the discussion that follows that the Government minimizes the cost of producing any given level of output and that the size of the public sector is determined by the intersection of the marginal cost function for public services and the demand function for Government services, or that the public sector behaves similarly to the private sector. For models of public sector behavior which assume that the public does not cost minimize, or include alternative behaviorial assumption, see Niskanen (1971).

[12] For simplicity, it is assumed the corporate equities and government bonds are equally risky. Inclusion of risk premia would not alter this argument.

the corporate sector. At the existing interest rate, r, there is an excess demand for capital as a result of the increased private sector demand for capital. So long as the supply of capital to the economy as a whole is not infinitely elastic, the interest rate on Government bonds must increase to some interest rate that exceeds r (but is less than $r/(1-\tau)$; otherwise aggregate demands for capital would not increase). The increase in the cost of capital to the Federal sector must increase the cost of providing Federal sector services and lead to a decreased demand for those services.[13]

The proposition being advanced here is quite simple. If a tax policy leads to different factor prices in the public sector versus the private sector, substitution of an alternative tax which does not lead to such a differential will alter factor prices in both sectors and thus change the cost of Government services.[14] This proposition is not limited to the corporate taxes. It applies to all taxes except a comprehensive income tax with no tax privileges or deductibility of other taxes. For example, the personal property tax is a tax on assets other than assets held by governmental bodies or other tax-exempt institutions. Thus, the effects of the property tax might be much the same as the effects of corporate income taxation (except for the fact that one must include residential structures and consumer durables in the analysis of the property tax).[15] A similar analysis might apply to sales taxes. State and local sales taxes require payment of sales taxes on goods produced by the private sector but not on the implicit consumption of public sector output.

III. A General Equilibrium Model of Tax Incidence

The tax incidence effects and the effects of various tax policies described in the previous section are best described within a general equilibrium model which includes both the public and private sectors. In this section, I describe such a model and the manner in which equilibrium in such a model is computed. In section IV of this report, I present a detailed account of the calibration of the model and the computation of equilibrium prices and outputs. Section V of this report presents estimates of the effects of eliminating the tax exemption granted state and local bonds and the effects of altering various tax policies, using data from both 1966 and 1973.

---

[13] I assume in the text that the incidence of corporate taxes are borne by capital. For recent empirical evidence on this point, see Oakland (1972).

[14] This result must follow since the cost of Government services is a function of input prices.

[15] This point is implicit in Netzer's (1971) discussion of the incidence of the property tax in which he points out that, due to tax exemptions granted Government-owned property and property owned by certain other institutions, only 78% of total capital is subject to the property tax.

The Model

The economy consists of four sectors: the Federal Government sector, the State and local government sector, the business sector,[16] and privately owned structures (primarily housing and land). The first three sectors each produce a homogenous output utilizing two factors of production capital and labor. For simplicity, a constant returns to scale Cobb-Douglas production of the form:

$$Q_i = C_i K_i^{\alpha_i} L_i^{1-\alpha_i} \tag{1}$$

where:

Q_i = output of the ith sector
C_i = a constant in the production function of the ith sector
K_i = capital utilized in the ith sector
L_i = labor utilized in the ith sector

Implicit output of the privately owned structures and land sector is proportional to capital utilized in that sector.

The demand for the output of each sector is a function of price and income. For sectors 1, 2, and 4 (all sectors but the business sector), the demand function for output is defined to be:

$$Q_i = A_i P_i^{\eta_i} Y_i^{\delta}$$

where:

Q_i = quantity demanded of the ith sector's output
A_i = a constant in the demand function
P_i = price of the ith sector output[17]
Y_i = aggregate income
η_i = the price elasticity of demand in the ith sector
δ_i = the income elasticity of demand in the ith sector

The demand for the output of the corporate sector is determined by the constraint:[18]

$$\sum_{i=1}^{4} P_i\, Q_i = Y$$

where

Y = total GNP

[16] The business sector consists of both corporate and noncorporate businesses. Corporate and noncorporate sectors were combined into one sector for simplicity since this paper is primarily concerned with resource flows between the public and private sectors and within public sectors.

[17] The fact that no explicit price (i.e., so many dollars per unit) is charged for Government output, and how this problem is solved in the model is discussed below.

[18] In a system of n markets, only n-1 demand functions can be independently defined. The demand for sector 3 is $Q_3 = (Y - \sum_{i \neq 3} P_i Q_i)/P_3$. Thus, the implicit own price elasticity of demand is unity for the corporate sector.

Equilibrium in the Model

Equilibrium in the model is determined by two sets of first order conditions and two sets of constraints. The first order conditions in each sector require that price equal marginal costs and that the cost of producing any given level of output in any given sector is minimized. These well known first order conditions are:

(1) Prices:

$$P_i = MC_i$$

where:

$$MC_i = \text{marginal costs}$$

(2) Capital:

$$\frac{\alpha_i P_i Q_i}{K_i} = r_i$$

where:

r_i = the cost of capital or rental rate on capital in the ith sector.[19]

(3) Labor:

$$(1\text{-}\alpha_i)\frac{P_i Q_i}{L_i} = w$$

where:

w = the wage rate, assumed to be the same in all sectors.

The requirement that price equal marginal cost in all sectors provides a convenient method of introducing the demand for Government services into the model without assuming an explicit price is charged for Government services. One merely assumes that citizens via their voting decisions, respond to the cost of Government services. Increases in the cost of Government services lead citizens to demand fewer services.

The constraints on the model are constraints on the usage of capital and labor in each sector:

$$\sum K_i = K_o$$

$$\sum L_i = L_o$$

where:

K_o = the total stock of capital, and L_o = total labor force,[20]

---

[19] The cost of capital is the rental rate on capital and includes interest, depreciation, taxes, etc. It is assumed that after-tax rental rates on capital are equalized at the margin. This point is discussed in more detail below.

[20] For simplicity, a perfectly inelastic supply of capital and labor to the economy as a

or that all capital and labor are fully employed; and the constraints that total Federal Government expenditures equal Federal taxes raised from all sources, and that State and local government expenditures equal State and local taxes raised from all sources. The taxes included in the model are:

Federal Sector:
- Personal Income Tax
- Corporate Taxation
- Social Security Tax

State and Local Government Sector:
- Personal Income Tax
- Corporate Taxation
- Sales Tax
- Property Tax

The following provisions on the Federal and State and local income tax codes are included in the tax constraints: The deductibility of State and local taxes from Federal income taxes, the tax exemption granted interest paid on State and local bonds, and the deductibility of sales and property taxes from taxable income in computing State and local income taxes.

Computation and Simulation of the Model

In order to compute the effects of various tax policy changes, one needs as input data the price and income elasticities of demand for each sector, the constant C_i in the production function for each sector, the parameter α_i in the production function for each sector, and the constant A_i for the demand function for each sector.

The price and income elasticities of demand in each sector are variables subject to parametric variation and sensitivity analysis. The constants, C_i, A_i and the parameter of the Cobb-Douglas production function are calculated in the following manner: All output prices are assumed to be equal to unity. One then solves for values of C_i, A_i and α_i such that the distribution of capital and labor among sectors and the proportion of revenues raised via various types of taxes duplicates some base period. In the simulations discussed below, 1966 is taken as the base period.[21] Results for 1973 are reported and discussed as well.

Due to the complexity of the tax code, the model cannot be solved directly in closed form. Rather, the model is simulated for various values of the elasticities of demand for each of the four sectors in the model.

---

whole is assumed. A more realistic assumption would be to include supply functions for capital and labor.

[21] 1966 results are reported since this is the most recent year for which detailed capital stock data for the Federal and State and local government sectors are available.

IV. Calibration and Simulation of the Model

The general equilibrium model described in the previous section is calibrated and simulated in two steps. First, the model determines a set of tax rates and other parameters such that there are no excess supplies or demands in any input or output market, Federal expenditures equal Federal revenues, State and local expenditures equal State and local revenues, the usage of capital and labor in the various sectors is duplicated for a base period, and the shares of tax revenues from various sources in some base period is duplicated. In addition, the model requires that after-tax yields from all assets are equated at the margin.

The effects of tax policy changes are simulated by changing a tax policy and recomputing all taxes, prices, inputs usage, and outputs subject to the constraints that all markets are in equilibrium, total revenues from taxes equal total costs in each of the governmental sectors, and after-tax yields on various assets are equated.

Initial Input Data Used to Calibrate the Model

The initial data required to calibrate the model are the shares of Federal, State and local government revenues from various sources, salaries and wages paid by various sectors, capital utilized in various sectors, total revenues or sales in the corporate sector, and employment in the corporate sector. Price and income elasticities of demand are initial input data as well but are subject to parametric variation. The initial tax or revenue shares from various tax sources for both the Federal sector and the State and local government sector are listed in Table 1. Table 2 lists the initial allocation of capital in the four sectors of the model. Table 2 is derived from tables 2a (reproduced from Netzer (1971) and table 2b (reproduced from Olzechowski (1974)). Table 3 lists salaries and wages paid by various sectors. Corporate salaries and wages are limited to the private sector. Federal salaries and wages include compensation for members of the armed forces. Initial total

TABLE 1.—*Initial shares of revenues raised by various taxes in the Federal, State and local sectors, 1966*

[In percentages]

Tax	Share of revenue
Federal sector:	
Corporate income tax	31
Personal income tax	44
Social security taxes	25
Total	100
State and local sector:	
Corporate income tax	4.0
Personal income tax	9.4
Sales tax	37.7
Property tax	48.9

Source: *Economic Report of the President*, 1975.

TABLE 2.—*Initial allocation of capital, 1966*

Sector	Capital[1]
Federal government	346.0
State and local government	258.7
Corporate sector	698.0
Privately owned housing[2]	1,100.0

[1] In billions of dollars.
[2] Includes consumer durables.

TABLE 2a*.—*Estimated tangible national wealth, 1966, by census of governments property-use categories, in current dollars*[1]
[In billions of dollars]

	(1)	Total value of residential structures	$593.0	
less	(2)	Publicly-owned	14.7	
less	(3)	Farm	20.5	
less	(4)	Private nonhousekeeping	11.1	
less	(5)	Mobile homes	4.4	
equals	(6)	Privately-owned nonfarm housekeeping buildings		542.3
	(7)	Business and farm nonresidential structures	276.4	
less	(8)	Institutional structures	[2]41.3	
plus	(9)	Farm residential structures, from line (3)	20.5	
plus	(10)	Private nonhousekeeping residential structures, from line (4)	11.1	
equals	(11)	Business and farm structures, excluding institutional		266.7
	(12)	Business and farm equipment	252.3	
less	(13)	Institutionally-owned producer durables	[3]2.5	
plus	(14)	Business and farm inventories	181.5	
equals	(15)	Business and farm tangible personal property		431.3
	(16)	Consumer durables	[4]196.9	
plus	(17)	Mobile homes, from line (5)	4.4	
equals	(18)	Household-owned tangible personal property		201.3
	(19)	Privately-owned land		[5]508.4
	(20)	Public nonresidential structures	395.8	
plus	(21)	Publicly-owned residential structures, from line (2) 14.7	14.7	
plus	(22)	Publicly-owned producer durables	[6]10.1	
plus	(23)	Publicly-owned inventories	12.9	
plus	(24)	Publicly-owned land	127.4	
plus	(25)	Institutional structures, from line (8)	41.3	
plus	(26)	Institutionally-owned producer durables, from line (13)	2.5	
equals	(27)	Tangible wealth in tax-exempt ownership		604.7
	(28)	Total tangible wealth sum of lines (6), (11), (15), (18), (19), and (27)		2,554.7

[1] Lines (1) through (6) are from Allan H. Young, John C. Musgrave and Claudia Harkins. "Residential, Capital in the United States, 1925–70." *Survey of Current Business*, November 1971. Lines (7) and (12) are based on the data in U.S. Department of Commerce, *Fixed Nonresidential Business Capital in the United States*, 1925–70, November 1971. National Technical Information Service, Com-71-01111; the series used is variant 1 assuming service lives at 85 per cent of Bulletin F and straight-line depreciation. Lines (8), (14), (16), (19), (20), (23) and (24) are based on the national wealth estimates in U.S. Congress, *Institutional Investor Study, Report of the Securities and Exchange Commission, Supplementary Volume I*, House Document 92-64, Part 6, March 10, 1971, as shown in *Statistical Abstract of the United States*, 1971.

[2] The national wealth estimates in the S.E.C. study for this category have been adjusted downwards by roughly 10 per cent, for consistency with the Commerce Department estimate in line (7) from which line (8) is subtracted.

[3] Estimated at 1 percent of line (12), on the basis of data for earlier periods in Raymond W. Goldsmith, Robert E. Lipsey and Morris Mendelson, *Studies in the National Balance Sheet of the United States*, Vol. II, Princeton University Press, 1963.

[4] Includes some institutionally-owned consumer durables.

[5] Includes some institutionally-owned land, possibly in the $10–15 billion range.

[6] Estimated at 4 percent of line (12), on the basis of data for earlier periods in Goldsmith *et al.*

* Reproduced from Netzer (1971).

TABLE 2b*.—*Real and personal property of the Federal government*

BOOK VALUE OF FIXED ASSETS PER FEDERAL EMPLOYEE EXCLUDING DEPARTMENT OF DEFENSE[1]

Category	1963	1966[2]
Machinery and equipment[3]	13,042	14,608
Real property (excluding public domain)	28,489	34,322
Total	41,531	48,930
Employment (excluding Department of Defense)	1,477,953	1,633,209
Book value of asset per employee	28,252	30,018
INCLUDING DEPARTMENT OF DEFENSE		
Total Fixed Assets (excluding Department of Defense)	41,531	48.930
Personal property (Department of Defense)	132,577	143,714
Real property (Department of Defense)	43,043	46.148
Employment (including civilian and military of Department of Defense)	5,140,000	5,734,000
Book value of assets per employee	50,232	50,213
INVESTMENT PER FEDERAL EMPLOYEE[4]		
Real and personal property of Federal agencies	315,235	346,947
Employment (military and civilian)	5,149,000	5,734,000
Investment per Federal employee	61,239	60,558
BOOK VALUE OF FIXED ASSETS PER STAFF MEMBER: VIRGINIA PUBLIC SCHOOL SYSTEMS[5]		
For 1965 = 26,456 (approximately)		

[1] Source: U.S. Congress, House Committee on Government Operations, Federal Real and Personal Property Inventory Report, June 30, 1964.
[2] Figures are acquisition costs.
[3] All figures given in millions of dollars.
[4] Includes: Investments (other than public debt), other assets, accounts and loans receivable, cash of U.S. Treasury, work in progress, machines and supplies, and public domain.
[5] Source: Virginia Statistical Abstract. Presently this is the only index of capital intensity in the local government sector that I could find.

* Reproduced from Orzechowski (1974).

TABLE 3.—*Initial allocation of salaries and wages, 1966*

Sector	Salaries
Federal government	31.75[1]
State and local government	45.94
Corporate sector	316.8

[1] In billions of dollars.

Source: *Economic Report of the President*, 1975.

sales of the corporate sector are assumed to be equal to total sales of manufacturing corporations less sales tax payments.[22]

Initial Calibration of the Model

The initial base case of the model is calibrated in a series of steps. The first step is to determine the allocation of labor in the Federal and

[22] The approach utilized here is not without problems and simplifying assumptions. Output of the business sector is limited to manufacturing concerns whereas the capital data includes both businesses and farms. Employment and salary data are from the private sector as a whole. These seemingly inconsistent uses of data were included to account for a number of factors which could not be modeled without significantly

State and local government sectors. Rather than use actual employment, employment was adjusted for wage differentials between the private and public sector. This adjustment was made by assuming that employment in the government sector was equal to salaries paid by the public sector, divided by the computed wage rate in the private sector. (The computed wage rate in the private sector was salaries and wages of 316.8 billion divided by employment of 56.71 million in 1966.) This procedure allows the use of the simplifying assumption that wages are the same in both the public and private sectors.

The next step in the calibration of the model is to determine the differential yields on various assets such that after tax yields are equated at the margin. These differential yields are dependent on the tax structure. Consider first the differential yield between federal securities and state and local securities. Federal securities are subject to State and local income taxes and Federal income taxes. State and local income taxes are deductible from Federal income taxes. Thus, assuming that State and local bonds are perfect substitutes for Federal Government bonds, one must have

$$r_2 = (1.0 - \gamma_1 + \gamma_2\gamma_2)\, r_1$$

where

r_1 = the rental rate on capital loaned to the Federal sector,
r_2 = the rental rate on capital loaned to the State and local sector,
γ_1 = the Federal personal income tax rate, and
γ_2 = the State and local personal income tax rate.

The tax calculations required on corporate and business capital are somewhat more complex. Capital utilized in the business sector is subject to property taxation and state and local corporate income taxes. The recipients of income from corporate bonds or equities are subject to State and local personal income taxes. Finally, State and local taxes are generally deductible from income for Federal tax purposes. In order to develop the yield differential on the cost of capital to corporations, define,

T_1 = Taxes paid by corporations on a unit of capital to State and local governments,

---

increasing the scope of the model. GNP accounts net out purchases by the Federal sector from the private sector in computing final output. Output of manufacturing corporations includes purchases from intermediate sectors which would be netted out in calculation of GNP. Total sales of manufacturing corporations is somewhat similar (in numerical magnitude) to personal consumption expenditures plus fixed investments of nonresidential and nongovernmental sectors. Thus, use of aggregate employment data plus use of aggregate capital data leads to a reasonable proxy for the corporate or business sector. In addition, the results presented below indicate that the data reasonably duplicate the actual results of 1966. Thus, the simplifying assumptions made to derive a production function for the corporate sector, while not completely consistent, lead to reasonable results.

T_2 = Taxes paid by corporations to the Federal Government,
T_3 = Personal income taxes paid to State and local governments by recipients of income from corporate bonds and equities, and
T_4 = Personal income taxes paid to the Federal Government by recipients of income from corporate bonds and equities.

Using these definitions, one then has

$$T_1 = p + \tau_2 (r_3 - p),$$
$$T_2 = \tau_1 (r_3 - T_1)$$
$$T_3 = \gamma_2 (r_3 - T_1 - T_2),$$
$$T_4 = \gamma_1 (r_3 - T_1 - T_2 - T_3), \text{ and}$$
$$r_2 - \delta\gamma_1 r_2 = (r_3 - T_1 - T_2 - T_3 - T_4)$$

where:

δ = 1 if state and local bonds are taxed, otherwise
δ = 0
p = the property tax rate,
τ_2 = the state and local corporate tax rate,
τ_1 = the federal corporate tax rate, and
r_3 = the cost of capital to corporations[23]

The differential yield between capital invested in residential structures and consumer durables is determined by noting that the latter are subject to property taxation in most states, whereas capital held by state and local governments is not subject to property taxation. Property taxes are generally deductible from income for federal and state and local tax purposes. Thus, one has:

$$r_4 = r_2 + p(1\text{-}\gamma_1 - \gamma_2 + \gamma_1\gamma_2)\ \delta\gamma_1 r_2$$

where:

r_4 = the rental rate on capital invested in residential structures and consumer durables.

The model then computes a cost of capital or rental rate on capital in the business sector such that total costs are equal to total revenues, or[24]

$$r_3 = (R_3 - wL_3)/K_3$$

---

[23] The cost of capital to corporations would be the rental rate of capital which is the total returns to capital, not simply return to capital for tax purposes. While this raises some problems, it does not raise enough problems to invalidate the approach utilized here. See below.

[24] Several assumptions are made in the model described in the text. It is assumed that all investments are equally risky. In addition, the tax laws which apply primarily to interest payments or corporate dividends are applied to the rental rate on capital or the total returns to capital. These assumptions simplify the model somewhat and avoid a number of problems beyond the scope of this report. For example, the appropriate risk premia on various assets are not independent of the tax structure. The debt-equity ratio

where:

R_3 = total revenues of the business sector,
w = the wage rate
L_3 = labor utilized in the business sector, and
K_3 = capital utilized in the business sector.

Given the rental rate on capital utilized in the corporate sector, the set of differential yields described above can be used to determine the rental rates on capital in all other sectors at any given set of tax rates.

Total expenditures of the Federal and State and local government sectors are calculated as the rental rate on capital times the amount of capital used plus the wage rate times the amount of labor utilized in that sector, or

$R_1 = wL_1 + r_1K_1$, and
$R_2 = wL_2 + r_2K_2$

where:

R_1 = total required revenues to cover costs in the Federal sector
L_1 = total labor utilized in the Federal sector
K_1 = capital utilized in the Federal sector
R_2 = total revenues required to cover costs in the State and local government sector
L_2 = labor utilized in the State and local government sector, and
K_2 = capital utilized in the State and local government sector.

Total tax receipts of the Federal and State and local sectors are determined by applying the appropriate tax rates to the corresponding tax base. The only taxes included in determining tax receipts not mentioned previously are the social security tax and the sales tax. Sales tax revenues are simply the rate of sales taxation times sales of the corporate sector. Social security taxes are the social security tax rate times total wages and salaries (in this model, the product of the wage rate and total labor supply).

The model iterates on the various tax rates (using Newton's method for finding the root of an equation) until all the equilibrium conditions are met.

Computation of Constants in Production and Demand Functions

The second stage of the initial calibration of the model computes the Constants C_i and α_1 of the Cobb-Douglas production function and the

---

in corporations is determined by the tax structure. In order to include these factors, one would have to include relationships between risk premia and tax structure and between capital structure in corporations and the tax structure. Since the model simulates reasonably well, these factors were not included in this initial general equilibrium model. As noted above, the model simulates reasonably well and thus, these simplifying assumptions are not overly severe.

constant A_i of the demand function for sectors 1, 2, and 4 (all sectors but the corporate sector). These constants are computed in the following manner:

α_i: the parameter α_i of a Cobb-Douglas production function is equal to capital's share and is thus computed as $\alpha_i = r_iK_i/(r_iK_i + wL_i)$

C_i: the constant in the production functions computed by assuming all prices are equal to unity.[25] It is easy to show that if firms (and governments) minimize the cost of producing any given level of output, marginal costs are given by

$$MC_i = \frac{r_i\alpha_i\, w^{1-\alpha_i}}{C_i\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}}$$

Assuming that price is set equal to marginal costs in each sector, one has

$$C_i = \frac{r_i^{\alpha_i} w^{1-\alpha_i}}{(\alpha_1^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}}$$

A_i = the constant in the demand function is computed by noting that

$$A_i = Q_i\, Y_i^{-\delta_i}$$
$$\text{if } P_i = 1\; A_i$$

The final step in the calibration of the model is to compute total output of the economy or GNP. Total output is simply the sum of the outputs of all sectors or

$$Y = \sum_i r_iK_i + \sum_i wL_i = \sum_i R_i$$

Initial Base Data

Table 4 lists the inital values of the constants in the production and demand functions for all four sectors. Of particular interest here is the fact that the public sectors exhibit much more capital intensity than the private sector. The estimate of capital's share in the corporate sector is .4, which is not exceedingly different from the estimates of capital's share for the economy as a whole of .3 usually reported in the literature. The larger capital share for the public sector is not investigated in detail here, but is consistent with data reported in other studies.[26]

---

[25] The assumption that all prices are equal to unity allows one to compute the constants in both the production and demand functions. Since the model is concerned with changes in prices and other variables due to tax policies, this assumption does not affect the results of the model.

[26] See Spann (1975) or Orzechowski (1974).

TABLE 4.—*Estimated parameters of demand and production functions, 1966*

Sector	A_i	C_i	α_i
Federal	1.34	1.78	.71
State and local	1.19	2.00	.54
Corporate		3.07	.41
Private housing	.25	.23	1.00

TABLE 5.—*Initial values of endogenous variables in the model, 1966*

Tax		Percent
Federal sector:		
Corporate income tax		17.47
Personal income tax		8.71
Social security tax		6.98
State and local sector:		
Corporate tax rate		1.99
Personal income tax rate		1.91
Sales tax		7.00
Property tax		2.70
ALLOCATION OF FACTORS OF PRODUCTION		
Sector	Capital[1]	Labor[2]
Federal Government	346	5.68
State and local government	258	8.22
Corporate sector	698	56.71
Privately owned housing	1,100	0
INITIAL FACTOR PRICES		
Sector	Capital	Labor[3]
Federal Government	.2267	5.586
State and local government	.2076	5.586
Corporate sector	.3129	5.586
Privately owned housing	.2312	5.586
INITIAL OUTPUT AND REVENUES		
Sector	Output	Total revenues[4]
Federal Government	110.21	110.21
State and local government	99.48	99.48
Corporate sector	535.20	535.20
Privately owned housing	254.218	254.218

[1] In billions of dollars.
[2] In millions of employees.
[3] In thousands of dollars per man-year.
[4] In billions of dollars.

In table 5, the initial values of all the endogenous variables in the model are listed. Given the simplicity of the model used (relative to an economy with several hundred sectors, various transfer payments, etc.), the base results, in many cases, are surprisingly good. The base corporate tax rate is 17.7 percent, which is relatively similar to a rough estimate of the effective corporate tax rate on the total returns to capital of 20 percent in 1966.[27] The base income tax rate of 8.8 percent

[27] Calculated by dividing total returns to capital (sales of manufacturing corporations less wage and salary payments) into taxes paid by corporations in 1966. The effective tax rate on the rental rate on capital is considerably less than the statutory tax rate of 48%

is relatively similar but somewhat below the effective income tax rate of 11.6 percent in 1966.[28] In the State and local government sectors, the sales tax rate of 7.0 percent is considerably above the actual sales tax rate of 3.44 percent observed in 1966.[29] Similarly, the base property tax rate of 2.7 percent is somewhat above the observed property tax rate of 1.7 percent in 1966.[30]

The employment data in table 5 are very similar to actual employment data observed in 1966. The figure 5.68 million employees is almost equal actual Federal employment. State and local employment of 8.22 is also almost equal to State and local employment in 1966.[31]

Base Federal expenditures of $111.5 billion are below observed Federal expenditures of $131 billion in 1966. Base State and local government expenditures of $99.4 billion are above observed State and local government expenditures of $83 billion in 1966.[32]

The model overestimates total GNP. The model in the base case yields Gross National Product of $1000.39 billion whereas actual 1966 GNP was $749.91 billion. The most likely source of this error is the definition of GNP which, in the model, is not completely consistent with the National Income Accounts definition of GNP. The model includes in the Government sector input return on capital, transfer payments, and purchases from the private sector. The private sector is included in the model separately. Thus, there is some "double counting" in the model in the sense used in GNP accounting.

These errors are not overly severe for the purposes of the model. One would not expect the model to exactly replicate the economy unless it included all sectors instead of four sectors. This paper is more concerned with comparative statics and the effects of tax policy changes than absolute values of the variables. Thus, so long as the initial base data are reasonably accurate, comparative static inferences concerning the size and magnitude of tax changes can be made.

Simulation of the Effects of Tax Changes

Tax change simulations are conducted by changing the tax policy to be analyzed. The model first computes the differential rental rates on

---

on equity capital. The corporate tax in the text is applied to the rental rate on capital which includes equity capital returns, interest payments, depreciation, etc. Thus the effective tax rate on the rental rate on capital is less than the effective tax rate on equity capital.

[28] The observed tax rate of 11.6% was calculated by subtracting transfer payments and income to corporations from total personal income and dividing this figure ($495.5 billion) into Federal tax revenues of $57.6 billion.

[29] Calculated by sales tax revenues into personal consumption expenditures. This procedure biases the estimate of the tax rate downward since it does not take into account the fact that not all personal consumption expenditures are subject to sales taxation.

[30] The property tax rate of 1.7% is taken from Netzer (1971).

[31] *Economic Report of the President*, 1975.

[32] *Economic Report of the President*, 1975.

capital for all four sectors at any given level of r_3, the rental rate on capital in the corporate sector. The model then computes the prices and marginal for each sector using the formula[33]

$$P_i = MC_i = \frac{r_i \alpha^i w^1 - \alpha^i}{C_i \alpha_i \alpha_i (1 - \alpha_i) \; 1 - \alpha_i}$$

Next, the model computes the demands for final outputs, using the demand curves (and the residual demand function for the corporate sector). The model then computes input demands, using the first-order conditions

$$K_i = \frac{P_i Q_i \alpha_i}{r_i}$$

$$L_i = \frac{P_i Q_i (1 - \alpha_i)}{w}$$

Finally, the model computes the aggregate demand for capital and compares total expenditures in each of the governmental sectors ($P_i Q_1$ for the Federal sector, $P_2 Q_2$ for the State and local sector) and compares these expenditures with total revenues raised via taxes in each governmental sector. If the aggregate demand for capital exceeds K_o, the initial capital stock, the rental rate on capital is adjusted upward and vice versa. The model takes all taxes except for one tax in each sector as given and adjusts the tax left free to vary until total revenues equal total costs in both government sectors. A modified version of Newton's method for finding the root of an equation is used in adjusting tax rates and the rental rate on capital. The model continues to iterate until there is no excess demand for capital (which, by Walras's law, implies no excess demand for labor) and both the Federal and State and local government sectors have balanced budgets.[34]

V. Estimated Effects of Eliminating Tax-Exempt Municipal Bonds and Effects of Other Tax Policy Changes

The effects of eliminating tax-exempt state and local bonds were estimated using the general equilibrium model described in sections III and IV of this report. Since the price elasticity of demand for various sectors is an exogenous variable in the model, simulations of the effects of eliminating tax-exempt State and local bonds were conducted for various price elasticities of demand for Federal and State and local government output. As noted earlier, the model can be used to estimate the effect of other tax policies as well. Later in this section, the results of

[33] This approach is equivalent to assuming perfect competition in the corporate sector and that both the corporate sector and the public sector combine inputs so as to minimize the costs of producing any given level of output.

[34] The computer program used for the simulations reported in the text is available from the author on request.

simulating the effects of the elimination of the corporate income tax, eliminating the personal income tax, and the effects of various levels of personal income taxation on the corporate tax rate and the price of Government services are reported.

The first set of simulations reported is for 1966. This year was used since it was the most recent year for which detailed estimates of public sector capital was available. Later in this section, the data are updated to 1973, and similar simulations are reported.

The initial set of simulations assumes fairly low elasticities of demand for State and local government services. The assumed elasticities for the initial estimations are −.25 for both the Federal and State and local government sectors.[35]

The following assumptions were made in the initial simulations:

(1) Interest paid on state and local bonds is subject to Federal income tax but not to State and local income tax,

(2) The only Federal income tax that changes as a result of the elimination of the tax exemption for interest in State and local bonds is the personal income tax, and

(3) The only State and local tax which changes as a result of elimination of the tax exemption granted interest received from State and local bonds is the property tax.

Other assumptions could be made with attendant alterations in the results. The assumptions made above were considered the most plausible set of assumptions for the policy responses to elimination of tax-exempt state and local bonds.

The results of the elimination of the tax-exempt state and local bonds for 1966 are reported in table 6, in which the percentage changes in output prices, input prices, input utilization, tax rates, output, and total revenues by sector are listed.

The principle effects of eliminating tax-exempt State and local bonds contained in table 6 are:

(1) Output of the Federal sector increases by 0.3 percent whereas output of the State and local government sector decreases by 0.9 percent.

(2) The price (or marginal cost) of Federal Government services falls by 1.4 percent whereas the price of State and local government services increases by 3.94 percent.

(3) Total revenues or expenditures of the Federal sector *decline* by 1.09 percent whereas total revenues of expenditures of State and local

---

[35] Only the price elasticity of demand was altered during simulations. The model computes GNP for the base case (see Section IV) and assumes that tax policy does not affect GNP. This is equivalent to assuming a quantity theory of money and no changes in the stock of money.

TABLE 6.—*Effects of elimination of tax exempt municipal bonds (Low demand elasticity case)*

[Percentage change in endogenous variables by sector, 1966 data]

	Percent
Implicit output:	
Federal government	0.9
State and local government	−.9
Corporate sector	.19
Privately owned housing	.07
Implicit prices:	
Federal government	−1.4
State and local government	3.94
Corporate sector	.5
Privately owned housing	.15
Total revenues:	
Federal government	−1.09
State and local government	2.94
Corporate sector	−.3
Privately owned housing	−.1
Capital utilization:	
Federal government	.9
State and local government	−4.19
Corporate sector	.9
Privately owned housing	.07
Labor utilization:	
Federal government	−1.02
State and local government	2.94
Corporate sector	−.3
Privately owned housing	[1] 0.0
Cost of capital:	
Federal government	−1.9
State and local government	7.4
Corporate sector	−1.22
Privately owned housing	−.14
Wage rate:	
All sectors	0.0
Tax rates	
Federal income tax	−9.9
State and local property tax	6.06

[1] Privately owned housing consists solely of capital.

governments *increase* by 2.94 percent. The reason elimination of a tax preference leads to a decline, rather than an increase, in Federal Government revenues is the assumption of an inelastic demand curve for Governmental services. Were an elasticity of demand in excess of unity assumed for Government services, total revenues or expenditures of the Federal sector would increase as a tax preference was eliminated.[36]

(4) The Federal personal income tax rate falls by 9.9 percent and State and local property tax rates rise by 6.06 percent.

(5) Capital utilized in the Federal sector increases by 0.9 percent, capital utilized in the State and local sector declines by 4.19 percent,

[36] See below in which the effects of alternative demand elasticity assumptions are presented. This does not imply that there is no "tax loss" to the Federal treasury as a result of tax exempt state and local bonds. Any tax loss calculation would assume a given set of factor utilization across sectors. Changes in tax policy change factor utilization. Once this factor is included in a model, the effect of tax preference elimination depends on whether the elasticity of demand for federal services is elastic or inelastic. Other federal tax rates are lowered as state and local bonds become subject to tax, however. See below.

and capital in the privately owned structures, land and consumer durables sector, increases by 0.07 percent.

(6) The cost of capital or rental rate on capital to the Federal sector declines by 1.9 percent and the cost of capital or rental rate on capital in the State and local sector increases by 7.4 percent.[37]

(7) Labor utilized in the Federal sector declines by 1.02 percent whereas labor utilized in the State and local government sector increases by 2.94 percent.

The percentage change in the property tax is small relative to the percentage change in the federal income tax, because the property tax applies to a much broader base than the income tax. Thus, small changes in the property tax lead to large changes in State and local government revenues. Much larger changes in the federal income tax are required to generate any given increase in Federal Government revenues.

The price of Government services falls slightly more in percentage terms than the price of State and local government services increases. This is primarily a result of the fact that elimination of tax-exempt State and local bonds reduces the cost of capital to the Federal sector substantially. This fall in the cost of capital in the Federal sector (as opposed to a substantial rise in the cost of capital to the State and local sector) is the result of changes in tax policy and changes in the demand for capital. It is assumed that State and local governments adjust property taxes in response to the elimination of tax-exempt State and local bonds. The increased rate of property taxation initially reduces the demand for capital in the housing and consumers durables sector. Initially, this reduction in the demand for capital along with the reduction in the demand for capital in the State and local government sector leads to an excess supply of capital. Thus, the rental rate on capital must fall. The rental rate on capital does not fall in all sectors. The rental rate on capital is increased in the State and local sector. The combined effects of a lower rental rate on capital and a higher rate of property taxation is to increase the rate of capital utilization in the business or corporate sector and increase the rate of capital utilization slightly in the private housing and consumer durables.

Effects of Alternative Price Elasticities of Demand for Government Services

The simulations reported in table 6 assume a particular set of price elasticities of demand for Federal and State and local government services. Tables 7 through 11 report the results of simulating the effects

---

[37] Recall that the rental rate of capital to any sector is the cost of capital to that sector adjusted for tax law provisions. After-tax returns to renters of capital are equal in all sectors.

TABLE 7.—*Effects of alternative tax policies on income tax rate at alternative demand elasticities, 1966 data*

[In percentages]

η_1	η_2	Tax-exempt state and local bonds	Elimination of tax-exempt bonds	Elimination of corporate tax
−.25	−.25	8.715	7.85	15.24
−.25	−.75	8.715	7.80	15.22
−.25	−1.25	8.715	7.74	15.18
−.75	−.25	8.715	7.99	14.49
−.75	−.75	8.715	7.92	14.46
−.75	−1.25	8.715	7.84	14.62
−1.25	−.25	8.715	8.11	13.89
−1.25	−.75	8.715	8.02	13.79
−1.25	−1.25	8.715	7.93	13.75

TABLE 8.—*Effect of alternative tax policies on cost of capital to Federal Government sector under various elasticity assumptions, 1966 data*

			Elimination of tax-exempt bonds		Elimination of specific taxes	
η_1	η_2	Tax-exempt state and local bonds	Income tax adjusts	Corporate tax adjusts	Income tax	Corporate tax
−.25	−.25	[1]0.22676	[1]0.22242	[1]0.22545	[1]0.19656	[1]0.25296
−.25	−.75	.22676	.22242	.22630	.19623	.25307
−.25	−1.25	.22676	.22234	.22718	.18586	.25327
−.75	−.25	.22676	.22269	.22535	.19499	.25120
−.75	−.75	.22676	.22311	.22622	.19453	.25141
−.75	−1.25	.22676	.22362	.22721	.19400	.25165
−1.25	−.25	.22676	.22285	.22525	.19301	.24970
−1.25	−.75	.22676	.22334	.22619	.19237	.24993
−1.25	−1.25	.22676	.22375	.22726	.19167	.25022

[1] Cost of capital is the rental rate on capital in this model.

TABLE 9.—*Effects of alternative tax policies on price of Federal Government services at alternative elasticities of demand, 1966 data*

			Elimination of tax-exempt bonds		Elimination of specific taxes	
η_1	η_2	Tax-exempt state and local bonds	Income tax adjusts	Corporate tax adjusts	Income tax	Corporate tax
−.25	−.25	1.0	0.986	0.995	0.907	1.077
−.25	−.75	1.0	.988	.998	.906	1.078
−.25	−1.25	1.0	.990	1.001	.904	1.078
−.75	−.25	1.0	.987	.995	.901	1.073
−.75	−.75	1.0	.989	.998	.899	1.074
−.75	−1.25	1.0	.991	1.001	.897	1.074
−1.25	−.25	1.0	.987	.995	.894	1.069
−1.25	−.75	1.0	.989	.998	.891	1.070
−1.25	−1.25	1.0	.991	1.001	.889	1.071

of alternative elasticities of demand for Federal and State and local government services on the income tax rate, the cost of capital to the Federal sector, the price of Government services, Federal Government revenues, and Federal Government employment. The first column of each of these tables lists the value of the relevant variable assuming tax-exempt State and local bonds. The second column of each table

TABLE 10.—*Effect of alternative tax policies on Federal Government revenues at alternative demand elasticities, 1966 data*

			Elimination of tax-exempt bonds		Elimination of specific taxes	
η_1	η_2	Tax-exempt state and local bonds	Income tax adjusts	Corporate tax adjusts	Income tax	Corporate tax
−.25	−.25	[1]110.21	[1]109.07	[1]109.83	[1]102.42	[1]116.56
−.25	−.75	110.21	109.24	110.08	102.32	116.61
−.25	−1.25	110.21	109.40	110.33	102.20	116.61
−.75	−.25	110.21	109.85	110.07	107.37	112.17
−.75	−.75	110.21	109.90	110.06	107.32	112.20
−.75	−1.25	110.21	109.95	110.25	107.26	112.21
−1.25	−.25	110.21	110.55	110.35	113.34	108.37
−1.25	−.75	110.21	110.50	110.26	113.42	108.34
−1.25	−1.25	110.21	110.45	110.15	113.50	108.32

[1] In billions of dollars, 1966.

TABLE 11.—*Effect of alternative tax policies on Federal Government employment at alternative demand elasticities, 1966 data*

			Elimination of tax-exempt bonds		Elimination of specific taxes	
η_1	η_2	Tax-exempt state and local bonds	Income tax adjusts	Corporate tax adjusts	Income tax	Corporate tax
−.25	−.25	[1]5.68	[1]5.51	[1]5.61	[1]5.21	[1]6.07
−.25	−.75	5.68	5.51	5.61	5.21	6.07
−.25	−1.25	5.68	5.51	5.61	5.21	6.07
−.75	−.25	5.68	5.61	5.63	5.47	5.82
−.75	−.75	5.68	5.61	5.63	5.47	5.82
−.75	−1.25	5.68	5.61	5.63	5.47	5.82
−1.25	−.25	5.68	5.70	5.69	5.79	5.61
−1.25	−.75	5.86	5.70	5.69	5.79	5.61
−1.25	−1.25	5.68	5.70	5.69	5.79	5.61

[1] Millions of employees.

lists the value of the relevant variable assuming that tax-exempt State and local government bonds are eliminated. The remaining columns of table 6 refer to the effects of alternative tax policies and are discussed in more detail below. The effects of eliminating tax-exempt State and local bonds (and the effects of other tax policies) were simulated for elasticities of demand ranging from −.25 to −1.25. In the far left hand column of tables 7 thru 22, η_1 refers to the price elasticity of demand for Federal services and η_2 refers to the price elasticity of demand for State and local services.

The effects of alternative price-elasticity assumptions on the select endogenous variables in the model can be summarized as follows:

Income Tax Rate: The greater the elasticity of demand, the less the fall in the income tax rate due to the elimination of tax-exempt State and local bonds;

Cost of Capital to Federal Sector: The greater the elasticity of demand, the less the fall in the cost of capital to the Federal sector due to the elimination of tax-exempt State and local bonds;

Price of Federal Government Services: There is an insignificant increase in the price of Government services as the elasticity of demand increases;

Federal Government Revenues: The greater the elasticity of demand, the greater Federal Government revenues; and

Federal Government Employment: The greater the elasticity of demand, the greater the level of Federal Government employment.

These relationships can be explained quite simply. Eliminating tax-exempt State and local bonds reduces the cost (or the implicit price) of Federal Government services. If the demand for Federal Government services is inelastic, this tends to reduce Federal Government revenues.[38] If the demand for Federal Government services is elastic, reducing the price of Federal Government services leads to higher Federal Government revenues. The greater federal government expenditures, the greater the required tax rate for a balanced budget.

Elimination of tax-exempt State and local bonds lower the cost of capital to the Federal sector and the price of Federal Government services. The Federal Government's elasticity of demand for capital is greater, the higher the elasticity of demand for Federal Government services. Thus, the increase in the Federal Government's demand for capital due to the elimination of tax-exempt State and local bonds is greater, the greater the elasticity of demand for Federal service. The greater the increase in the demand for capital in the Federal sector as a result of eliminating tax-exempt State and local government bonds, the greater the aggregate demand for capital. The greater the aggregate demand for capital, the higher the rental rate on capital required to clear the capital market.

Elimination of tax-exempt State and local bonds leads to a decrease in Federal Government employment if the demand for Federal Government output is less than unity and vice versa. The reduction in the cost of capital to the Federal sector and the reduction in the price of Federal Government services as a result of eliminating tax-exempt State and local bonds generate two effects with regard to the quantity of labor demanded for the Federal Government. The reduced cost of capital to the Federal sector leads to a substitution of capital for labor, reducing the demand for labor. The increased demand for Federal Government output as a result of the reduced price of Federal Government services increases the demand for labor. Substitution effects dominate output effects if the elasticity of demand for Federal Government output is less than unity and vice versa.

Effects of Other Tax Policies

The general equilibrium model developed in this report can be used to simulate the effects of other tax policies. The effects of three

[38] This is the familiar rule that price and total revenue are positively related if demand is inelastic and vice versa.

additional policy changes are described in tables 8 through 11. These policies are:

(1) Elimination of tax-exempt State and local government bonds assuming the corporate income tax rather than the personal income tax adjusts to meet any Federal surplus resulting from eliminating tax-exempt State and local bonds;
(2) Elimination of the personal income tax;
(3) Elimination of the corporate taxation.[39]

In all three cases, it is assumed that the property tax adjusts to meet any revenue surpluses or deficits in the State and local sector. In the case of the latter two policy changes, it is assumed that State and local bonds are tax exempt.

If the revenue surplus in the Federal sector as a result of eliminating tax-exempt State and local bonds is corrected by adjusting the corporate tax rate instead of reducing the income tax rate, the following differential effects emerge:

(1) The reduction in the cost of Federal Government services is lowered;
(2) The reduction in the cost of capital to the Federal Government sector is reduced.

These effects are due to the fact that the Federal Government competes with the corporate and State and local government sector for funds. Reductions in the corporate tax rate, at any given level of interest rates reduce the rental rate on capital in the corporate sector. A reduction in the rental rate on capital increases the demand for capital in that sector. The increased demand for corporate capital, in conjunction with an increased demand for capital in the Federal Government sector, as a result of eliminating tax-exempt bonds, leads to a greater excess demand (or a smaller excess supply) of capital when tax-exempt State and local bonds are eliminated. This increases the rental rate on capital to the federal sector and increases the price of Federal Government services relative to the prices of Federal Government services that would have been observed had the income tax been adjusted as tax-exempt State and local bonds been eliminated.

[39] The estimates reported below should not be considered detailed estimates of the effects of eliminating the corporate income tax since corporate taxes in the model presented here include both indirect business taxes and the corporate income tax. In addition, the estimates concentrate primarily on the flows of resources between the public and private sectors as opposed to flows of resources between taxes and nontaxed industries in the private sector. Thus the results presented below should be considered indicative of the qualitative effects of including the public sector in general equilibrium models of tax incidence rather than precise estimates of the effects of specific tax alteration.

Elimination of the Personal Income Tax

The second tax policy change simulated was the elimination of the personal income tax. Elimination of the personal income tax leads to the following changes:

(1) A substantial increase in corporate taxation,
(2) A reduction in the price of Federal Government services, and
(3) A reduction in the rental rate or cost of capital to the Federal sector.

The increased corporate tax as a result of the elimination of the income tax raises the rental rate on capital in the corporate sector. This reduces the demand for capital in the corporate sector, creating an excess supply of capital to other sectors. The excess supply of capital to other sectors reduces the market clearing rental rate on capital in other sectors including the Federal sector. The reduced cost of an input, in this case capital, to the Federal sector reduces the marginal cost of Federal Government output and increases the quantity of Federal Government services demanded.

Another interpretation of the effects of eliminating the personal income tax, which is analogous to the analysis of tax-exempt State and local bonds is as follows: Tax-exempt State and local bonds are a subsidy to State and local governments since they reduce the cost of capital to State and local governments. Alternatively, tax-exempt State and local bonds are an implicit tax on capital usage in the Federal sector since they raise the cost of capital to the Federal sector. The corporate income tax is a tax on capital utilized in the private sector. Using the analogy discussed concerning tax-exempt State and local bonds, the tax on corporate capital can be viewed either as a tax on capital utilized in the corporate sector or as a subsidy to capital utilized in sectors not subject to corporate income taxation, in this case the Federal and State and local government sectors.

Elimination of the personal income tax raises the corporate income tax and thus increases the implicit subsidy to the usage of capital in the Federal Government sector. Increasing the implicit subsidy to the usage of capital in Federal Government sector reduces the marginal cost of Federal Government output and increases the demand for Federal Government services.

The analysis presented here is identical to the standard textbook analysis of taxes and subsidies in private markets. A subsidy to the usage of a factor in any one market shifts the marginal cost curve to the right, reducing price and increasing output. There is no intrinsic or logical reason why the public sector should be analyzed any differently. The public sector produces an output by combining capital and labor much the same as do private firms. The demand curve for Federal Government services is the same as the demand curve in any other

TABLE 12.—*Sign of effects of alternative tax policies (low elasticity case), 1966 data*

	Tax policy			
	Elimination of tax-exempt bonds		Elimination of specific taxes	
Endogenous variable	Income tax adjusts	Corporate tax adjusts	Income tax	Corporate tax
Cost of capital to Federal Government	–	–	–	+
Price of Federal Government services	–	–	–	+
Federal Government revenues	–	–	–	+
Federal Government employment	–	–	–	+

market.[40] While no explicit price for Federal Government services is charged, voters pay for the output of the Federal Government via taxes. The higher the cost of governmental services, the less the quantity demanded.

Elimination of the Corporate Taxation

The effects of eliminating the corporate income tax (with corresponding adjustments in the personal income tax rate) are symmetric to the effects of eliminating the income tax rate. For the reasons outlined in the discussion of the personal income tax, elimination of the corporate income tax raises the cost of capital to the Federal Government and increases the cost of Federal Government services.[41]

Summary of Effects of Tax Policy Changes

Table 12 summarizes the effects of the various tax policy changes discussed so far. A negative sign in a cell in table 12 indicates that a particular policy change reduces the level of that endogenous variable and vice versa. The results in table 12 refer to the low elasticity of demand case discussed in table 6. The primary change in table 12 at alternative elasticities would be a reversal in signs on the effects of tax policy changes on Federal Government revenues if a demand elasticity greater than unity were assumed. Table 13 presents a summary of the effects of alternative policies on the corporate tax rate.

Effects of Alternative Income Tax Rates

Table 14 presents an analysis of the effects of alternative income tax rates on price of Government services, Federal expenditures, and the corporate tax rate. It is assumed that State and local bonds are tax exempt, that changes in the personal tax rate lead to compensating

[40] The analogy is not perfect. The demand for Government services is derived from a voting equilibrium, not private exchange. This does not destroy the analogy discussed in the text.

[41] See also Section II.

TABLE 13.—*Effect of alternative tax policies on corporate tax rate at alternative demand elasticities, 1966 data*

[In percentages]

η_1	η_2	Tax-exempt state and local bonds	Elimination of tax-exempt bonds	Elimination of income tax
−.25	−.25	17.47	13.79	36.67
−.25	−.75	17.47	13.68	36.78
−.25	−1.25	17.47	13.55	36.89
−.75	−.25	17.47	14.42	39.34
−.75	−.75	17.47	14.25	39.52
−.75	−1.25	17.47	14.04	39.76
−1.25	−.25	17.47	15.15	42.61
−1.25	−.75	17.47	14.91	42.94
−1.25	−1.25	17.47	14.65	43.39

TABLE 14.—*Effect of alternative income tax rates on price of government services, corporate tax rate and federal expenditures, 1966 data*

Income tax rate	Price of government services	Federal expenditures	Corporate tax rate
0.00	0.905	[1]103.53	0.3699
0.05	.958	107.98	.2654
0.10	1.014	112.69	.1472
0.15[2]	1.074	117.69	.0122

[1] Billions of dollars.
[2] At income tax rates above 16%, corporate tax rate must be negative to equate total federal expenditures and revenues.

changes in the corporate tax rate and that the demand for Federal Government output is −.25. Since compensating adjustments in the corporate tax rate are assumed, the higher the personal income tax rate, the higher the marginal cost (and price) of Federal Government services. If the demand for Federal Government services is inelastic, this implies that Federal Government revenues are greater, the greater the portion of revenues raised via income taxes. Conversely, if the demand for Federal Government services is elastic, Federal Government revenues fall as the income tax rate increases.

Updating the Model to 1973

The estimated effects of tax policy changes reported above dealt with 1966 data. This year was used for initial calibration of the model since it was the most recent for which detailed capital stock estimates for the public sector were readily obtainable. In updating the model to 1973, the primary problem is estimating the capital stock in the various sectors of the economy. All the other input data are readily obtainable from published sources.

Capital Stock Estimates for 1973

The capital stock in each sector for 1973 was estimated assuming that the implicit debt-to-equity ratio in each sector was approximately the

TABLE 15.—*Adjustment of 1966 capital stock to 1973 levels*
[In billions of dollars]

Sector	1966 Debt	1966 Capital	1973 Debt	1973 Capital (estimates)
Federal	271.8	346.1	349.1	444.53
State and local	104.8	258.7	184.5	455.44
Corporate	506.6	698.0	1111.1	1530.88
Private housing, etc.	401.8	1100.00	744.0	2036.83

Source: Columns 1–3 from *Economic Report of the President*, 1975.

TABLE 16.—*Initial shares of revenue raised by various taxes in federal, state and local sectors, 1973*
[In percentages]

Tax	Share of revenue
Federal sector:	
Corporate income tax	25.7
Personal income tax	44.4
Social security taxes	29.9
Total	100.0
State and local sector:	
Corporate income tax	4.8
Personal income tax	16.2
Sales tax	37.1
Property tax	41.9
Total	100.0

Source: *Economic Report of the President*, 1975.

TABLE 17.—*Initial allocation of salaries and wages, 1973*

Sector	Salaries
Federal	[1]45.3
State and local	101.3
Corporate	545.0

[1] In billions of dollars.

Source: *Economic Report of the President*, 1975.

same in 1973 as in 1966. Table 15 lists outstanding debt for each sector in both 1966 and 1973 and the 1966 stock of capital in each sector. The 1973 capital stock in each sector was estimated by multiplying 1973 debt outstanding in each sector by the ratio of capital to debt in 1966. This leads to the capital stock estimates shown in column 4 of table 15.[42]

Input Data 1973

The remaining input data for the 1973 estimates of the effects of tax policy is contained in 16 and 17. Table 16 lists the shares of revenues

[42] This procedure is not without problems. Debt-equity ratios do change over time. The method used here is relatively straightforward and, even though not perfectly accurate, I argue below that the comparative static implications of the model are not highly dependent on the initial values of the endogenous variables (factor prices, output prices, factor usage, and output levels).

raised by various types of taxes in 1973. The two principal changes in the Federal sector (relative to the 1966 data) are the reduced share of Federal revenues raised from corporate income taxes and the increased share of revenues raised via social security taxes. In the State and local sector, the primary changes are the smaller share of State and local tax revenues raised via property taxes and the increased share of revenues raised via personal income taxes.

Table 17 lists the allocation of wages and salaries by sector. Initial sales of the business sector was taken as $974.99 billion and initial employment in the Federal sector was taken as $70.4 million.[43]

Initial Values of Endogenous Variables, 1973

Table 18 lists the initial values of the parameters C_i, α_i of the Cobb-Douglas production functions and the initial value of the constant A_i of the demand functions by sector for 1973. Table 19 lists the initial values of various endogenous variables for 1973, as computed by the model. The accuracy with which the model duplicates actual 1973 data is much poorer than the duplication of 1966 data. In particular, the 1973 estimations severely underestimate the personal income tax rate and Federal Government expenditures. The model estimates a personal tax rate of 6.36%. The effective personal tax rate in 1973 was 12.62 percent.[44] The model estimates Federal expenditures of $146.33 billion in 1973 whereas actual Federal expenditures in 1973 were $255.5 billion. The model correctly predicts a fall in the effective corporate tax rate from 1966 to 1973 but underestimates the effective 1973 corporate tax rate. The estimated corporate tax rate is 4.48 percent whereas the actual effective corporate tax rate in 1973 was 15.12 percent.[45]

The most likely cause of the low estimate of 1973 Federal expenditures and tax rates is the increased percentage of Federal Government expenditures which are transfer payments. In 1966, transfer payments accounted for 24 percent of Federal expenditures.[46] In 1973, transfer payments accounted for 34 percent of Federal Government expenditures.[47] In the model, transfer payments are treated as an output of the Federal Government. GNP accounting would not include transfer payments in the output or contribution of Government to GNP. The most appropriate approach is probably somewhere between these two extremes. Transfer payments (or perhaps more appropriately, income redistribution) are clearly an output of the Government to the extent that there is an expressed demand within a society for income

---

[43] Initial data for 1973 were calculated in much the same manner as were the 1966 data. See Section IV above.

[44] The effective tax rate is calculated in the same manner for 1973 as for 1966. See Section IV above.

[45] The effective corporate tax rate is calculated in the same manner for 1973 as for 1966. See Section IV above.

[46] *Economic Report of the President*, 1975.

[47] *Economic Report of the President*, 1975.

TABLE 18.—*Estimated parameters of demand and production functions, 1973*

Sector	A_i	C_i	α_i
Federal	1.22	1.97	.69
State and local	1.62	2.64	.48
Corporate		3.32	.44
Private housing	.26	.23	1.00

TABLE 19.—*Initial values of endogenous variables in the model, 1973*

Tax	Percent
Federal sector:	
Corporate income tax	11.48
Personal income tax	6.36
Social security tax	5.11
State and local sector:	
Corporate tax rate	1.98
Personal income tax	2.01
Sales tax	7.48
Property tax	2.65

ALLOCATION OF FACTORS OF PRODUCTION

Sector	Capital[1]	Labor[2]
Federal government	444.50	5.85
State and local government	455.40	13.08
Corporate sector	1,530.88	70.40
Privately owned housing	2,036.80	0.0

INITIAL FACTOR PRICES

Sector	Capital	Labor[3]
Federal government	.21622	7.74
State and local government	.20247	7.74
Corporate sector	.28088	7.74
Privately owned housing	.22680	7.74

INITIAL OUTPUT AND REVENUES

Sector	Output	Revenues[4]
Federal government	146.33	146.33
State and local government	195.00	195.00
Corporate sector	974.99	974.99
Privately owned housing	461.32	461.32

[1] In billions of dollars.
[2] In millions of employees.
[3] In thousands of dollars per man-year.
[4] In billions of dollars.

redistribution programs. However, the production function for transfer payments may be quite different than that for other Government goods and services. In addition, it could be argued that the appropriate measure of output when transfer payments are included may not be the total volume of transfer payments, but some measure of the effect of transfer payments on the distribution of income or income redistributed.

The model predicts local government expenditures much more accurately than Federal Government expenditures. The initial calibration of the model leads to State and local government expenditures of

$195.00 billion, whereas actual State and local government expenditures were $181.086 billion in 1973.

The inaccuracies which result from the initial calibration of the model are not overly serious however. The model is primarily concerned with comparative static results and the effects of changes in various tax policies. As indicated below, the effects of various tax policy changes in general terms are not overly sensitive to the initial values of the endogenous variables in the model.

Effects of Tax Policy Changes, 1973

Table 20 lists the effects of eliminating tax-exempt state and local bonds using the 1973 data. In most cases, the effects of eliminating tax exempt bonds are similar, using the 1973 data as those calculated using the 1966 data. The magnitude of the effects is somewhat less, primarily due to the initial underestimation of the personal income tax rate in the calibration of the model for 1973. Some upward adjustment to the numbers could be made to take account of having underestimated the

TABLE 20.—*Effects of elimination of tax-exempt municipal bonds (low demand elasticity case) percentage change in endogenous variables by sector, 1973*

	Percent
Implicit output:	
Federal government	0.103
State and local government	−.73
Corporate sector	0.0
Privately owned housing	.3
Implicit prices:	
Federal government	−.14
State and local government	2.95
Corporate sector	0.0
Privately owned housing	.7
Total revenues:	
Federal government	−.314
State and local government	−2.16
Corporate sector	0.0
Privately owned housing	−.33
Capital utilization:	
Federal government	.4
State and local government	−3.59
Corporate sector	.4
Privately owned housing	.33
Labor utilization:	
Federal government	−.55
State and local government	1.96
Corporate sector	−.47
Privately owned housing	[1]0.0
Cost of capital:	
Federal government	−.7
State and local government	6.015
Corporate sector	−.66
Privately owned housing	−.65
Wage rate:	
All sectors	.2
Tax rates:	
Federal income tax	−8.63
State and local property tax	4.45

[1] Privately owned housing consists solely of capital.

TABLE 21.—*Effects of elimination of corporate or personal income tax on select endogenous variables, 1973 (low elasticity case)*

[In percentages]

Variable	Tax policy	
	Elimination of income tax	Elimination of corporate taxation
Federal government revenues	−5.31	+3.29
Price of federal government services	−6.78	+4.42
Cost of capital to federal government	−10.0	6.68
Federal government employment	−5.91	+3.76
Personal income tax rate		[1]+60.91
Corporate tax rate	+146.25	

[1] The increase in the personal income tax rate is much greater in this model than calculations which only include elimination of the corporate income tax since all corporate taxes (both corporate income taxes and the indirect business taxes) are assumed to be eliminated.

personal income tax rate, corporate tax rate, and the size of the Federal budget. In all probability, such adjustments would lead to estimations of the effects of eliminating tax-exempt bonds somewhat similar to those contained in table 6. The effective personal income tax rate was only slightly higher in 1973 than in 1966. Since the primary source of the difference in the rental rate on capital between the federal and state and local sectors is the tax exemption granted State and local bond interest, any adjustments to the 1973 data would yield results approximately the same as those obtained using the 1966 data.

Table 21 lists the effects of two alternative tax policies for the 1973 data: elimination of the personal income tax and elimination of the corporate taxation. Again, the effects of tax policy changes, using the 1973 data, are roughly similar to the effects of tax policy changes using the 1966 data. The primary difference is a slightly smaller impact of eliminating the corporate income tax. This is due to the smaller share of Federal Government revenues derived from corporate taxation in 1973 as opposed to 1966. No parametric variations on the elasticities of demand are shown for 1973 since these effects would be roughly the same as the 1966 results reported earlier.

Summary of Tax Policy Effects

The general equilibrium model of tax incidence developed in this report was used to simulate the effects of three alternative tax policies: elimination of tax-exempt State and local bonds, elimination of the personal income tax, and elimination of the corporate income tax. The principal effects of eliminating tax-exempt State and local bonds are felt in the Federal sector. Elimination of tax-exempt State and local bonds reduced the cost of capital to the Federal sector by 5–7 percent, reduced the marginal cost of Federal Government output by 4–5 percent, and increased the demand for output of the Federal sector by 1–5 percent.

The marginal cost of Government services is higher when the corporate income tax is eliminated and greater, the lower the corporate income tax (or the higher the personal income tax). This is due to the fact that the corporate income tax creates a differential yield between the rental rate on capital in the public and corporate sectors, lowering the rental rate on capital to the Federal sector. Elimination of the corporate income tax could raise the rental rate on capital to the Federal sector by as much as 7 percent, leading to a 4.5 percent increase in the marginal cost of Federal Government services. If one assumes an inelastic demand curve for the output of the Federal Government, raising the marginal cost of Government output (via eliminating the corporate income tax) increases Federal Government revenues and expenditures whereas reducing personal income taxes (and increasing corporate income taxes) lowers the marginal costs of Government output and leads to smaller total expenditures.

VI. Summary and Conclusions

In this report, a general equilibrium model consisting of four sectors (the Federal Government, State and local government, the corporate sector, and privately owned housing and consumer durables) was constructed to estimate the effects of various tax policies. The aspect of tax policy examined was elimination of tax-exempt State and local bonds. The model indicates that the primary effect of eliminating tax-exempt State and local bonds would be in the State and local government sector. Elimination of tax-exempt State and local bonds would, however, lower the cost of capital to the Federal sector and the marginal cost of Federal Government services, and would raise substantially the marginal costs of State and local government services and the cost of capital to State and local governments.

The model developed in this paper can be used to analyze other tax policies, including elimination of specific taxes or adjustments in basic tax codes. The general equilibrium approach taken in this paper also indicates that the marginal cost of federal government services faced by voters is not independent of the manner in which taxes are raised. Different Federal taxes lead to different factor prices which affect the marginal costs of Government output.

The general equilibrium model developed in this report is a long run equilibrium model with only four sectors. Even within such a simplified model, numerous tax policies can be analyzed. In addition to the policies described in the report, the effects of creating additional tax-exempt sectors (such as tax-exempt utility bonds) could be analyzed by adding additional sectors to the model. One of the equilibrium conditions in the model is that total revenues of each governmental sector equal total expenditures. Essentially, this con-

straint defines a tax code and alternative definitions of taxable income or alternative forms of taxes (such as a value added tax) can be analyzed by altering the tax constraints in the model.

The results of this report indicate that the long-run effects of alternative tax policies can be analyzed within a general equilibrium framework in which one explicitly recognizes the effects of tax policies on the level of Federal and State and local government services demanded. Tax policies affect not only factor prices and the allocation of resources within the private sector, but, in long-run equilibrium, the quantity of Government services demanded by voters.

References

Ackerman, S., and Ott, D., "An Analysis of the Revenue Effects Proposed Substitutes for Tax Exempt State and Local Bonds", *National Tax Journal*, December 1970.

Bergstrom, T. C., and Goodman, R. P., "Private Demand for Public Goods", *American Economic Review*, June 1973.

Borcherding, T. E., and Deacon, R. A., "The Demand for the Services of Non-Federal Governments", *American Economic Review*, December 1972.

Economic Report of the President, Washington, D.C., Government Printing Office, 1975.

Fortune, P., "The Impact of Taxable Municipal Bonds: Policy Simulations with Larger Econometric Models", *National Tax Journal*, March 1973.

Galper, H., and Peterson, J., "An Analysis of Subsidy Plans to Support State and Local Borrowing", *National Tax Journal*, June 1971.

Harberger, A. C., "The Incidence of the Corporation Income Tax", *Journal of Political Economy*, 1962.

Netzer, D., "The Incidence of the Property Tax Revisited", *National Tax Journal*, December 1973.

Niskanen, W. A., *Bureaucracy and Representative Government*, Chicago: Aldine-Atherton, 1971.

Oakland, W. H., "Corporate Earnings and Tax Shifting in U.S. Manufacturing Corporations, 1930–1968", *Review of Economics and Statistics*, August 1972.

Orzechowski, W. P., "Labor Intensity Productivity and Growth of the Federal Sector", *Public Choice*, Fall 1974.

Spann, R. M., "Rates of Productivity Change in the Growth of State and Local Government Expenditures", T. E. Borcherding (editor), *Budgets and Bureaucrats: A Public Choice Approach to Government*, Duke University Press, forthcoming, 1975.

Appendix I

Caveats, Qualifications and Assumptions Utilized in the Model

The model developed in this report utilizes a number of simplifying assumptions and simplifies the tax code considerably. In this appendix I discuss some of these assumptions in more detail.

The model applies to the rental rate on capital which would include debt capital, equity capital, depreciation, etc. It is assumed that all capital is rented. Further, it is assumed that renting capital to all

sectors is equally risky; i.e., Government securities have the same risk characteristics as business securities. Taxes which apply to either equity capital or debt capital are applied to the entire rental rate on capital. In the corporate sector, this is equivalent to applying the corporate tax rate to debt capital, equity capital, and other capital charges. In the governmental sectors, there is implicit equity capital to the extent that the amount of capital possessed by the Government exceeds the level of Government debt. Income taxes in actuality apply only to interest payments (or debt capital) of the governmental sector.

The model ignores purchases (other than labor) of the governmental sector from the private sector. It is assumed that Government output is produced from a two input production function (the inputs being capital and labor). This is one reason why the model leads to a value of total output in excess of recorded GNP.

The model utilizes a rather simple representation of the demand side of the economy. In all sectors the demands for final outputs depend only on own prices, and do not include cross-price elasticities of demand. The only exception to this rule is the business sector which is treated as a residual since only n-1 demand functions are independently determined within a system of n markets. Thus, the Hicks-Slutsky conditions for systems of demand equations are not completely satisfied in the model.

Finally, the model includes indirect business taxes in corporate income taxes and ignores transfer payments either to individuals or to State and local governments.

Each of these problems could be resolved with a much more detailed representation in the economy. The treatment of debt versus equity capital and the taxation of debt versus equity capital could be expanded in either or both of two ways. One could include detailed sets of equations determining the debt equity ratios in all sectors, the risk premia between alternative forms of investments, the relationship between debt-equity ratios, risk premia and tax policy and the separate treatment of debt and equity capital payments for tax purposes. One could also (or alternatively) utilize a multiperiod model which explicitly included a capital market with savings and investment decisions.

Purchases of goods and services from the private sector by the government sector could be included by including private sector outputs in the production function for government goods and services. In such a model, one would probably want to include the government output in the production function for private goods and services since the government provides a number of goods and services to private businesses. Highways traveled by trucks and police protection in business districts are two examples of such transactions.

Inclusion of the Hicks-Slutsky conditions would require more detailed data on cross elasticities of demand between public sector and private sector outputs. In addition, the Hicks-Slutsky conditions result

in nonlinear constraints on demand elasticities which would require additional equations and iterations to solve the model.

Indirect business taxes could be treated as separate taxes if the capital side of the model were expanded to include the factors mentioned above. Again, this would require additional tax equations and a more explicit representation of indirect business taxes.

Transfer payments could be included in the model by inclusion of a mechanism of transfer payments within a general equilibrium system which recognized the fact that transfer payments are not costless in terms of real resources. In addition, one would have to separate the level of transfer payments from resources used to effect transfer payments. This would include not only transfer payments to individuals, but transfer payments from the Federal Government to State and local governments.

All of these additions would improve the "reality" of the model. They are to a large degree somewhat beyond the scope of the present paper which is primarily a preliminary attempt to include a supply-demand model of Government and a general equilibrium framework in the analysis of tax policy. Thus, this effort has been limited to the major taxes and the utilization of numerous simplifying assumptions. The general conclusions concerning the nature of tax policy incidence and the effects of tax policies on the size of the Government output are probably still valid within the caveats stated above. These caveats and assumptions are detailed here primarily as an indication of the possible limitations of the model in its present form, the nature of the assumptions which might be made in this (or any other) analysis and to outline possible areas of further investigation in the use of general equilibrium models of tax policy incidence.

☆ U.S. GOVERNMENT PRINTING OFFICE : 1977 O—200-508

U.S. Dept. of the
Treasury.

Conference on tax
research 1975.

ARTMENT OF THE TREASURY
WASHINGTON, D. C. 20220

OFFICIAL BUSINESS
ENALTY FOR PRIVATE USE, $300

POSTAGE AND FEES PAID
DEPARTMENT OF THE TREASURY
TREAS - 551

U.S.MAIL

THE DEPARTMENT OF THE TREASURY
1789